Exploring Newton's Method

ID: XXXX

Time required
45 minutes

Activity Overview

In this activity, students build an understanding of Newton's Method for finding approximations for zeros of a given function. They use a variety of tools, graphical, numerical, algebraic and programming, to observe the process and limitations of this important method.

Concepts

- *Gradient, tangent, derivative, iteration.*

Teacher Preparation

This investigation offers opportunities for review and consolidation of key concepts related to gradient of a tangent at a point and approximation methods. As such, care should be taken to provide ample time for ALL students to engage actively with the requirements of the task, allowing some who may have missed aspects of earlier work the opportunity to build new and deeper understanding.

- *This activity can serve to consolidate earlier work on gradient. It offers a suitable introduction to itertaive methods and approximation.*
- *Begin by discussing the importance of approximation methods for dealing with the majority of functions encountered in real-world situations.*
- *The screenshots on pages X–X (top) demonstrate expected student results. Refer to the screenshots on page X (bottom) for a preview of the student .tns file.*
- ***To download the .tns file, go to http://education.ti.com/exchange and enter "XXXX" in the search box.***

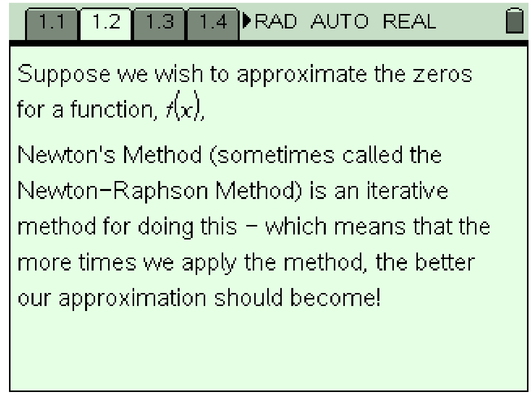
Classroom Management

- *This activity is intended to be* ***student led.*** *You should seat your students in pairs so they can work cooperatively on their handhelds. Use the following pages to present the material to the class and encourage discussion. Students will follow the steps using their handhelds, although some of the ideas and concepts are only presented in* ***this*** *document; be sure to cover all the material necessary for students' total comprehension.*
- *Students can either record their answers on the handheld or you may wish to have the class record their answers on a separate sheet of paper.*

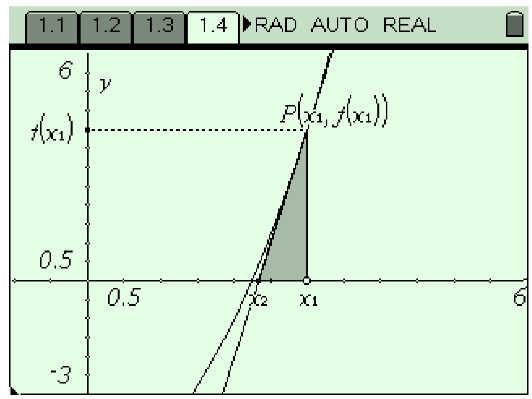
TI-Nspire™ Applications

Graphs & Geometry, Notes, Lists & Spreadsheet, Calculator, Programming.

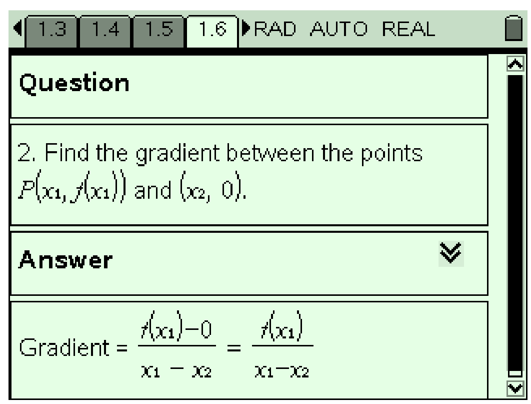
Step 1: Begin with discussion concerning approximation methods in today's computer age. Many students will be under the impression that the majority of functions are well-behaved and have exact solutions which can be found. They need to appreciate that computers are ideally suited to repetitive mindless work – exactly the sort associated with iterative processes.



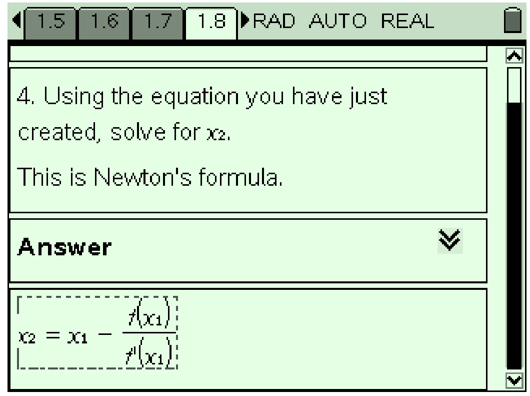
Step 2: A graphical approach is likely to be most meaningful for students – they should be given the opportunity to verify for themselves by dragging the point x1 on the axis that the place where the tangent meets the axis is likely to be closer to the zero than the initial guess. Some may also observe that this is not always the case. This will be the basis for further investigation at the end of the activity.



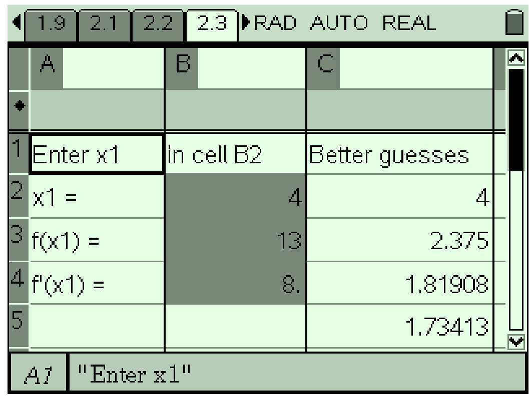
Step 3: Beginning with a review of concepts of gradient and the idea of tangent as gradient at a point on the curve, students begin to develop the algebraic basis for this result.



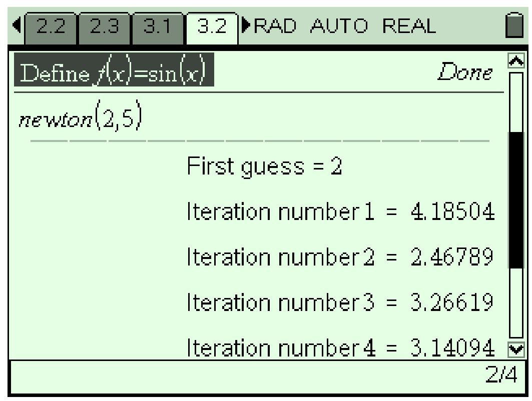
Step 4: Equating the derivative formula with that for the gradient of the tangent should lead readily to the expression of Newton's formula, as shown. Discussion should follow regarding the iterative nature of this formula, since this may be a new technique for some students.



Step 5: The use of graph leading into a spreadsheet analysis of the process can be very powerful here. Students enter and view the graph for a function, and then use the spreadsheet to enter their first guess, and observe how it generates a sequence of values approaching a limiting value. They should note that the better the initial guess, the faster the sequence converges.

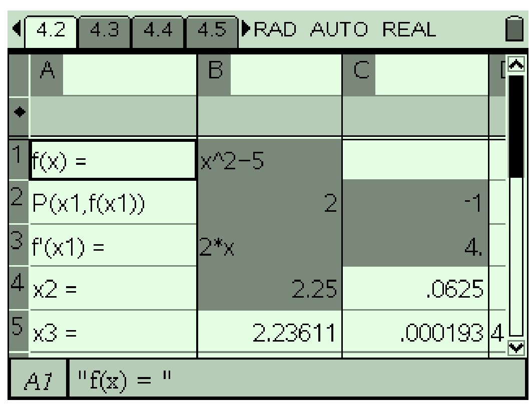


Step 6 This iterative understanding may be further developed using programming – first encourage students to develop their own program step-by-step outline and, if appropriate, to attempt to develop a program themselves which will generate at least individual steps. They may study the program **newton(guess, iterations)** at an appropriate time, and use it to further investigate this method of approximation.



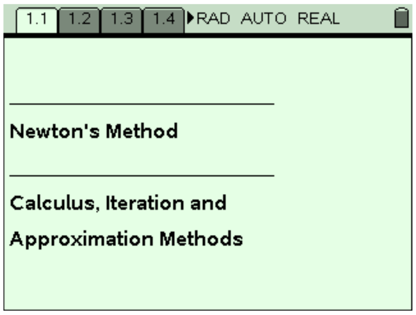
CAS EXTENSION

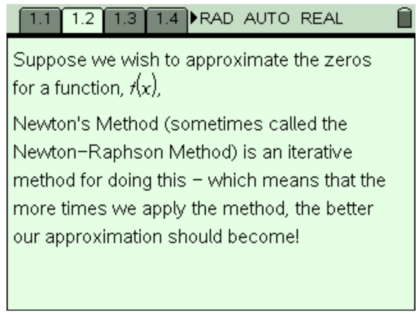
The more powerful algebraic tools available within TI-Nspire CAS make possible even more opportunities for investigation, using spreadsheet, functions and programming. Students should be challenged to use any or all available tools to develop a report with a main focus upon the limitations of this method – **when and why does it fail?**

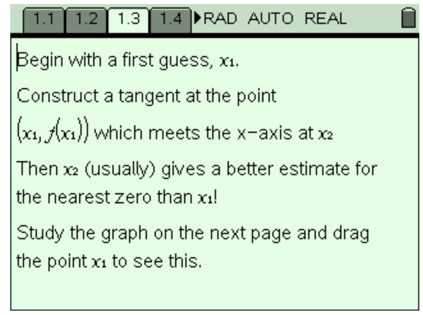


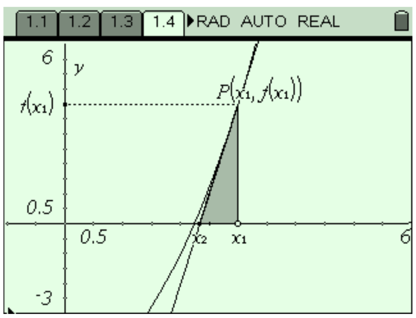
Exploring Newton's Method – ID: XXXX

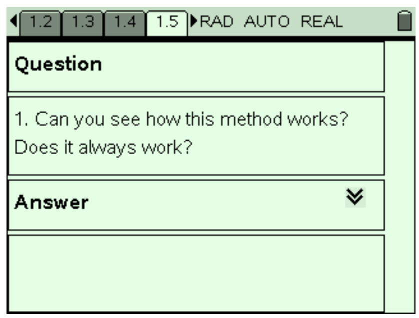
(Student)TI-Nspire File: *CalcActXX_Newtons_Method_EN.tns*

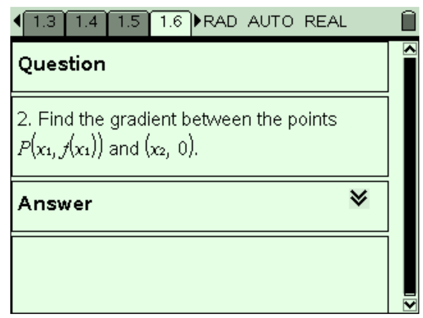


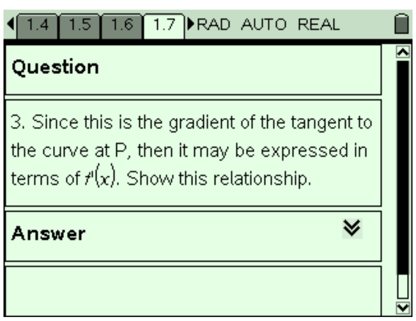


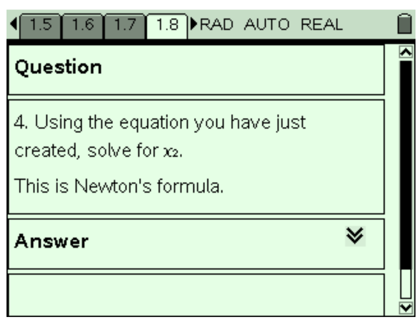


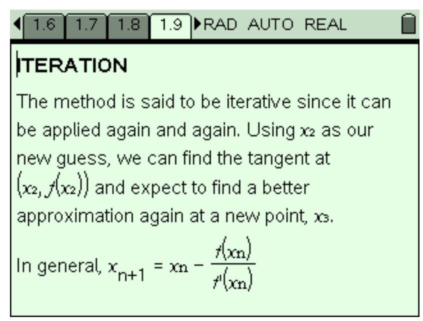


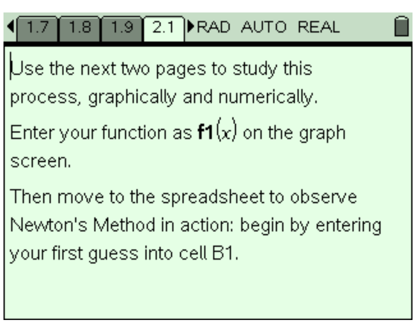


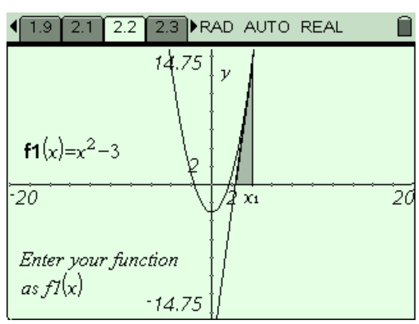


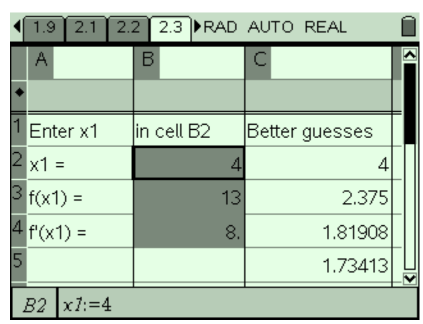


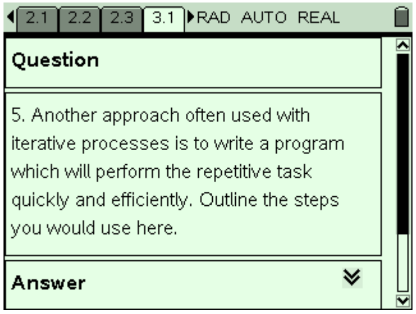


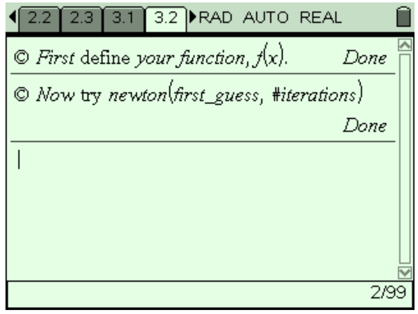


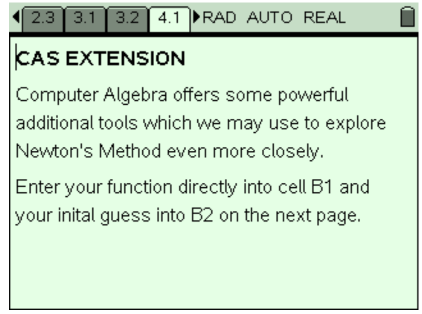


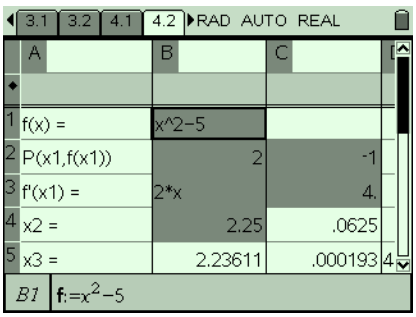


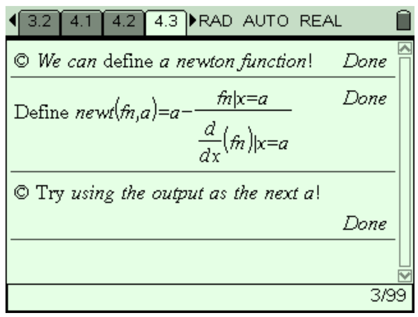


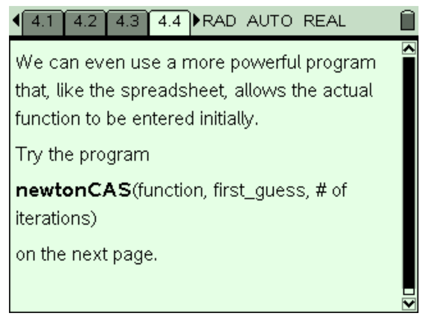

2.1 2.2 2.3 3.1 RAD AUTO REAL
Question
5. Another approach often used with iterative processes is to write a program which will perform the repetitive task quickly and efficiently. Outline the steps you would use here.
Answer

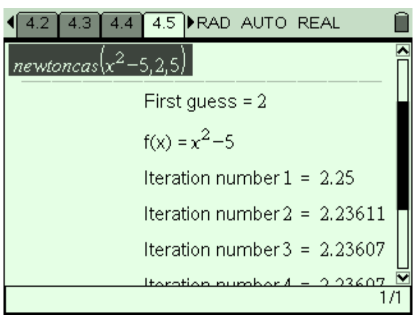

2.2 2.3 3.1 3.2 RAD AUTO REAL
© First define your function, f(x). Done
© Now try newton(first_guess, #iterations) Done
2/99

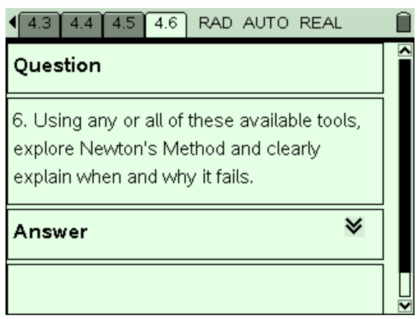

2.3 3.1 3.2 4.1 RAD AUTO REAL
CAS EXTENSION
Computer Algebra offers some powerful additional tools which we may use to explore Newton's Method even more closely.
Enter your function directly into cell B1 and your inital guess into B2 on the next page.


3.1 3.2 4.1 4.2 RAD AUTO REAL
A B C
1 f(x) = x^2−5
2 P(x1,f(x1)) 2 -1
3 f'(x1) = 2*x 4.
4 x2 = 2.25 .0625
5 x3 = 2.23611 .000193
B1 f:=x²−5


3.2 4.1 4.2 4.3 RAD AUTO REAL
© We can define a newton function! Done
Define newt(fn,a)=a−fn|x=a / d/dx(fn)|x=a Done
© Try using the output as the next a! Done
3/99


4.1 4.2 4.3 4.4 RAD AUTO REAL
We can even use a more powerful program that, like the spreadsheet, allows the actual function to be entered initially.
Try the program
newtonCAS(function, first_guess, # of iterations)
on the next page.


4.2 4.3 4.4 4.5 RAD AUTO REAL
newtoncas(x²−5,2,5)
First guess = 2
f(x) = x²−5
Iteration number 1 = 2.25
Iteration number 2 = 2.23611
Iteration number 3 = 2.23607
1/1


4.3 4.4 4.5 4.6 RAD AUTO REAL
Question
6. Using any or all of these available tools, explore Newton's Method and clearly explain when and why it fails.
Answer