Introducing the Differential Calculus From First Principles

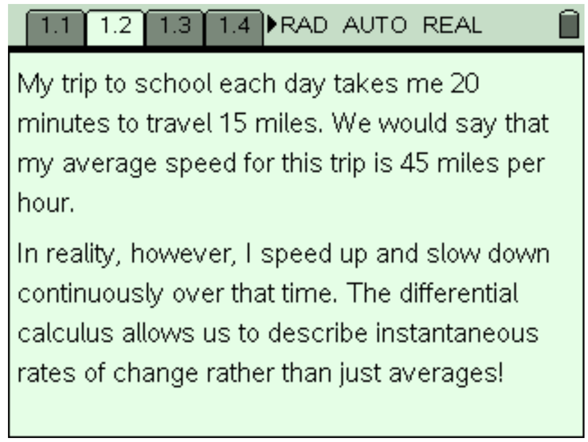
ID: XXXX

Name ____________________

Class ____________________

In this activity, we begin with the concept of gradient between two points, which lie on the graph of a function. We learn that for many functions (like my trip to school) the gradient (or rate of change) actually changes constantly and we can calculate this rate at every point of my journey, not just at start and finish.

Open the file *Alg1ActXX_Sine_Rule_EN.tns* on your handheld and follow along with your teacher to work through the activity. Use this document as a reference and to record your answers.



The Problem

My trip to school each day takes me 20 minutes to travel 15 miles. We would say that my average speed for this trip is 45 miles per hour. In reality, however, I speed up and slow down continuously over that time. The differential calculus allows us to describe instantaneous rates of change rather than just averages!

In this activity, we begin with the concept of gradient between two points, which lie on the graph of a function. We learn that for many functions (like my trip to school) the gradient (or rate of change) actually changes constantly and we can calculate this rate at every point of my journey, not just at start and finish.

Begin with a function f1(x) describing my journey and a point P (x1, f1(x1)) which lies on the graph of the function, y = f1(x).

Now picture another point, Q, which also lies on the graph of y = f1(x), close to P. In fact, suppose the distance from P to Q along the x-axis is h units, where h is a small amount.

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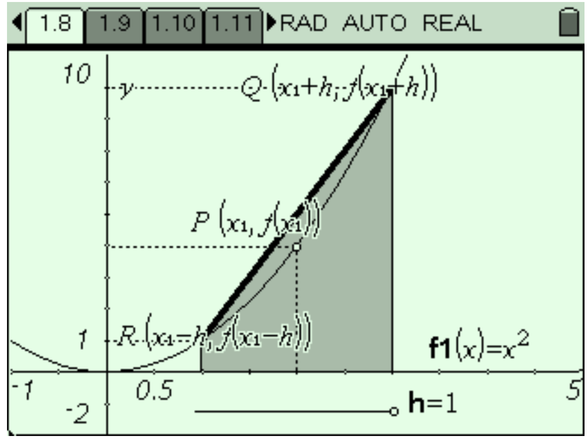
EXERCISES

1. Give the coordinates of the point Q and R in terms of x1 and the function f1(x).



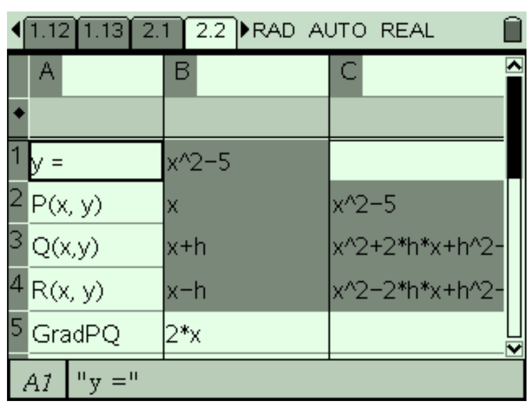
We are interested in the slope or gradient of the segment QR for different functions, for different points along these functions, and for different values of h.

2. What do you observe about the slope of QR for linear and non-linear functions?

3. How would you calculate the gradient of the segment QR for Q(x2, y2) and R(x3, y3)?
4. Now express this gradient formula using the relationships y2 = f(x1+h), x3 = x1-h, and y3 = f(x1-h).
5. Use the dynamic algebra page as a template to show your step-by-step solutions for the functions 3x^(2), sin(x), √(x) and ln(x).
6. Can you find any functions for which this process does NOT work?

SUGGESTED SOLUTIONS

1. Q (x1+h, f(x1+h)), R(x1-h, f(x1-h))
2. The slope remains constant for linear functions, but changes constantly for all others.
3. Gradient QR = (y3-y2)/(x3-x2)



4. Gradient QR = (f(x1+h)-f(x-h)/2h
5. d(3x^(2)) = lim(((3(x+h)^(2)-3(x-h)^(2))/2h,h,0) = 6*x

   d(sin(x)) = lim(((sin(x+h)-sin(x-h))/(2h)),h,0) = cos(x)

   d(√(x)) = lim(((√(x+h)-√(x))/(h)),h,0) = 1/(2*√(x))

   d(ln(x)) = lim(((ln(x+h)-ln(x))/(h)),h,0) = 1/x
6. Functions which are not continous at all points (e.g. ln(x) for x <=0) and not smooth (such as abs(x) at x = 0) are not differentiable.

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