Introducing the Differential Calculus: The Product Rule

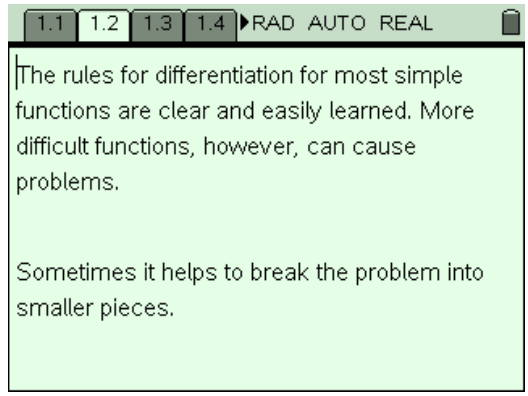
ID: XXXX

Name ______________________

Class ______________________

In this activity, we explore ways to differentiate harder functions. The focus here is on functions which can be expressed as a product of two simpler functions.

Open the file *CalcActXX_Product_Rule_EN.tns* on your handheld and follow along with your teacher to work through the activity. Use this document as a reference and to record your answers.



The Problem

While we have developed rules for taking derivatives of standard function forms, we need to be able to work with more difficult functions as well. Using a computer algebra system (CAS) like TI-Nspire CAS, we can make and check conjectures regarding these harder functions.

For example, discuss and then give your answer to the derivative of the function $x^2 + x^3$.

In general, we might say that *the derivative of a sum equals the sum of the derivatives*.

How might we **prove** which a statement? We could **verify** that it is true by trying examples, but to prove means something more. Fortunately, we have a tool which might help here – *differentiation by first principles*. Using TI-Nspire CAS, you can even use the program **FirstPrinciples(function)** to check your step-by-step working and final result (to see the final result, just type **result**!)

What about *differences* of functions? What about *products* of functions?

Can you make general statements about these function types?

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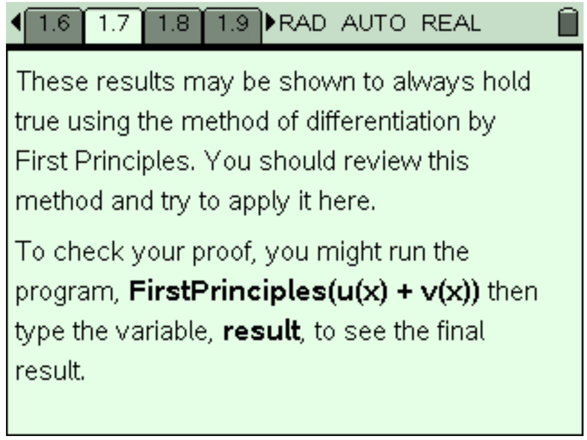
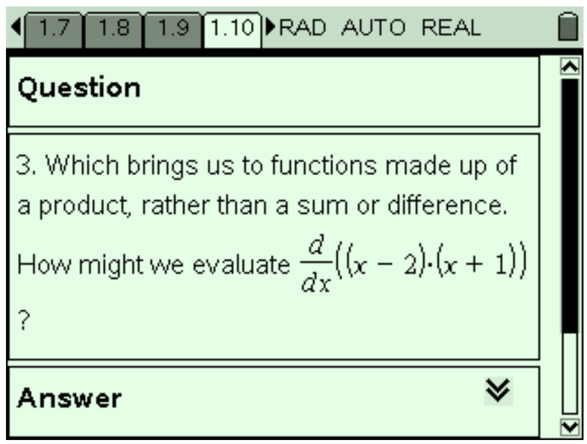
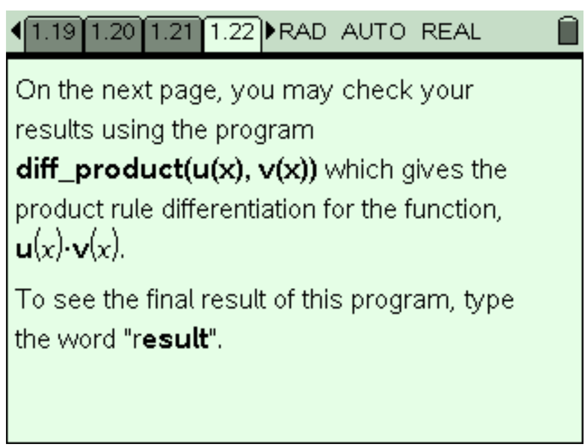
EXERCISES

1. How might you **prove** that the derivative of a sum is equal to the sum of the derivatives?
2. Does the same hold for differences?
3. What about products? See the function shown opposite: how might this be evaluated?
4. Find the derivatives of (x – 2) and (x + 1) and multiply these together.
5. Now expand (x – 2)(x + 1) and take the derivative of the parts.
6. It appears that the derivative of a product is NOT equal to the product of the derivatives. How might we find a rule for the derivative of a function of the form *u(x)* * *v(x)*? (Once again, try using first principles to evaluate the result, and check your answer using the **FirstPrinciples(u(x)*v(x))** program).
7. Try now with examples such as
   - (i) $2x*(x^2 – 4)$
   - (ii) Sin(x)*cos(x)
   - (iii) x^2*ln(x)
   - (iv) $(x – 2)^2*(x + 1)$
   - (v) 2x * sin(x)







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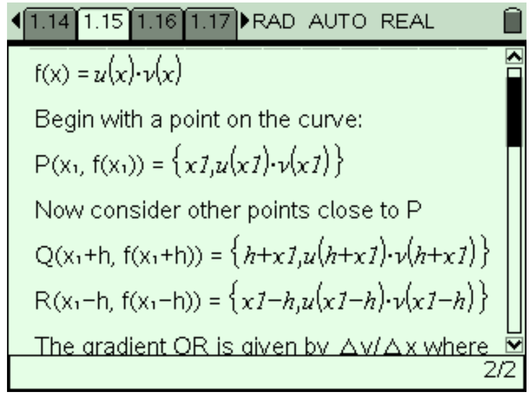
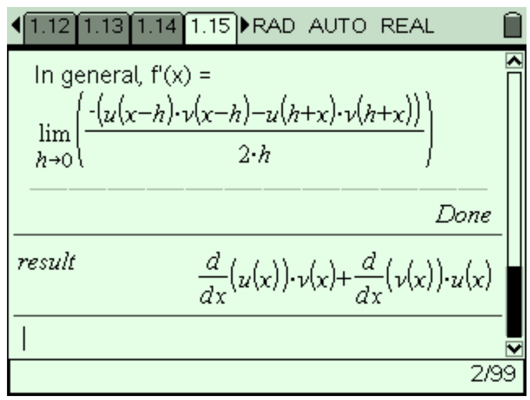
SUGGESTED SOLUTIONS

1. A proof may be constructed using differentiation from First Principles.



2. Yes, the same applies for differences.
3. We could expand the product and differentiate the parts.
4. The derivatives of **(x – 2)** and **(x + 1)** are both equal to 1, so the product is 1.
5. **(x – 2)(x + 1) = x^2 – x – 2** and so the derivative is **2x – 1**.



6. ***d*(u*v) = u*d(v) + v*d(u)**
7. (i) *d*(□**2x** * **(x^2-4)**□) by Product Rule

   d(u*v) =*d*(u(x))*v(x)+*d*(v(x))*u(x)

   u(x) = □**2*x***

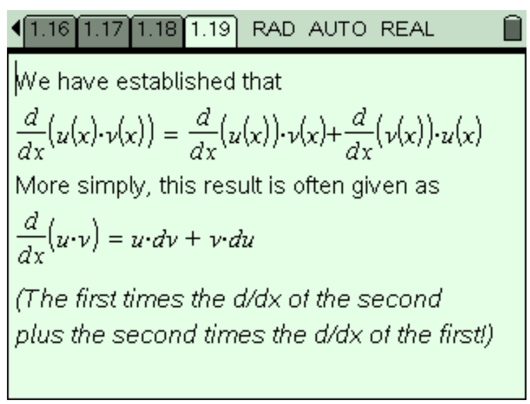
   d(u) = □**2**

   v(x) = □ **x^2- 4**

   d(v) = □**2*x***

   d(u*v) = **(2 *x*□)*(□2 *x* □) + (x^2- 4□)*(□2□)**

   result: 6x^2 – 8



(ii) *d*(**sin(*x*)*cos(*x*)**) by Product Rule

   u(x) = □**sin(*x*)**

   d (u) = □**cos(*x*)**

   v(x) = □**cos(*x*)**

   d (v) = □**−sin(*x*)**

   d (u*v) = □**sin(*x*)**□*****□**−sin(*x*)**□
   +□**cos(*x*)**□*****□**cos(*x*)**

   Result: $\cos^2(x) - \sin^2(x) = \cos(2x)$

(iii) $d\ (\boldsymbol{x}^2 * \ln(\boldsymbol{x}))$ by Product Rule

u(x) = $\boldsymbol{x}^2$

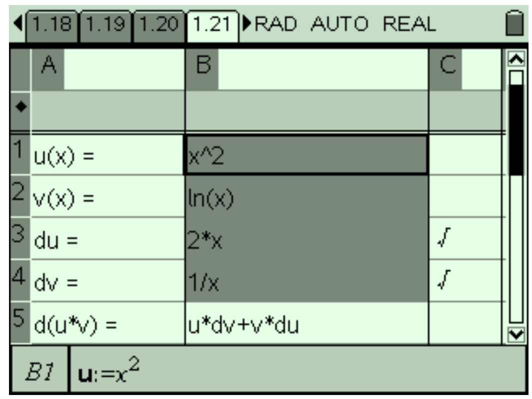
d (u) = **2*x***

v(x) = **ln(*x*)**

d (v) = **1/*x***

d (u*v) = $\boldsymbol{x^2*(1/x) + \ln(x)*2x}$

result: *x* + 2*xln(*x*)**



(iv) $d\ ((\boldsymbol{x} - 2)^2 * (\boldsymbol{x} + 1))$ by Product Rule

u(x) = $(\boldsymbol{x} - 2)^2$

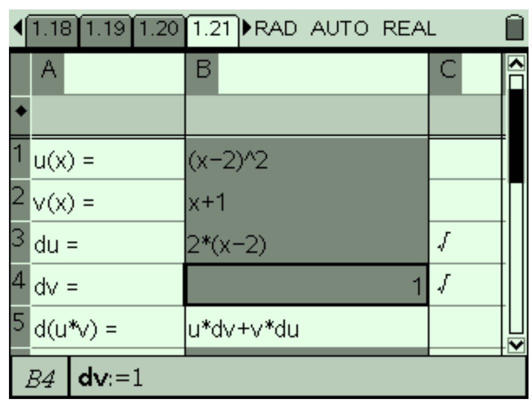
d (u) = **2(*x* - 2)**

v(x) = ***x* + 1**

d (v) = **1**

d (u*v) = $\boldsymbol{((x - 2)^2)*(1) + (x + 1)*(2*(x-2))}$

Result: 3*x(*x* – 2)**



(v) (2*x * sin(x)) by Product Rule

u(x) = 2*x

d (u) = 2

v(x) = sin(x)

d (v) = cos(x)

d (u*v) = (2*x)*(cos(x)) + (sin(x))*(2)

Result: 2*x* cos(*x*) + 2 sin(*x*)