Introducing the Differential Calculus
The Product Rule

ID: XXXX

Time required
45 minutes

Activity Overview

In this activity, we explore ways to differentiate some more difficult functions. The focus here is on functions which can be expressed as a product of two simpler functions. The approach taken here is largely symbolic and makes full use of the computer algebra facilities of TI-Nspire CAS. Prepared programs and algebraic spreadsheets are also utilized for skill development and consolidation.

Concepts

- *Product rule for differentiation, differentiation from first principles.*

Teacher Preparation

This investigation offers opportunities for review and consolidation of key concepts related to the methods of first principles and differentiation of simple functions. Opportunities are provided for skill development and practise of the method of taking derivatives of products. As such, care should be taken to provide ample time for ALL students to engage actively with the requirements of the task, allowing some who may have missed aspects of earlier work the opportunity to build new and deeper understanding.

- *This activity can serve to consolidate earlier work on first principles. It offers a suitable introduction to derivatives of more difficult functions.*
- *Begin by reviewing the method of differentiation of first principles, both graphically and symbolically, and methods of differentiation of the standard function forms.*
- *The screenshots on pages X–X (top) demonstrate expected student results.* *Refer to the screenshots on page X (bottom) for a preview of the student .tns file.*
- ***To download the .tns file, go to http://education.ti.com/exchange and enter "XXXX" in the search box.***

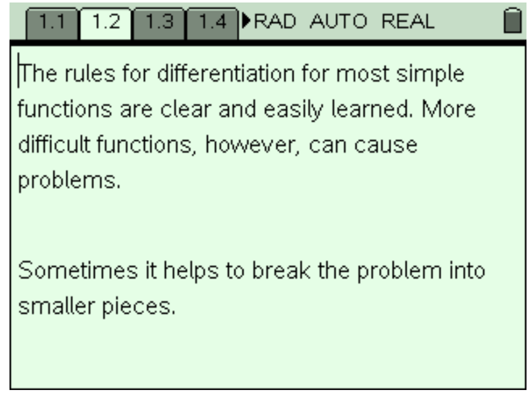
Classroom Management

- *This activity is intended to be* ***teacher led.*** *You should seat your students in pairs so they can work cooperatively on their handhelds. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds, although the majority of the ideas and concepts are only presented in* ***this*** *document; be sure to cover all the material necessary for students' total comprehension.*
- *Students can either record their answers on the handheld or you may wish to have the class record their answers on a separate sheet of paper.*

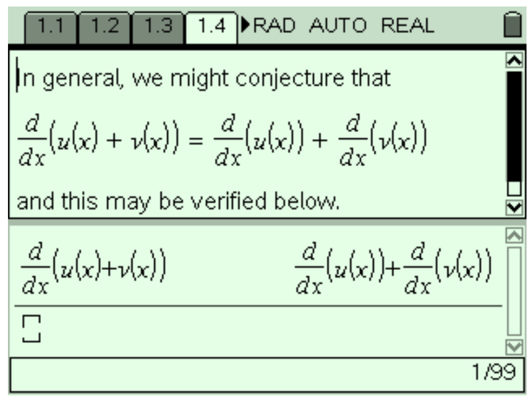
TI-Nspire™ Applications

Calculator, Notes,Lists & Spreadsheet and Programming.

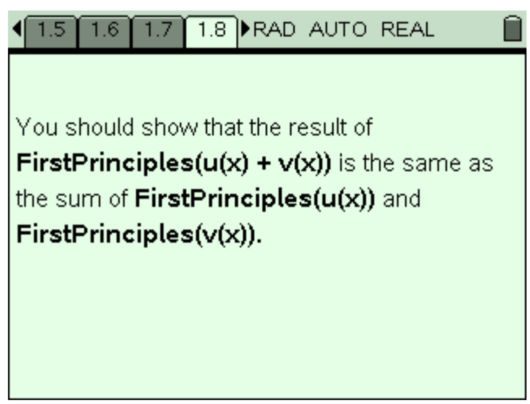
Step 1: Begin with discussion and review of both first principles methods and of the derivatives of standard function forms. Ensure that students are comfortable with these and then challenge them to consider more difficult forms – initially sums, differences and products of functions.



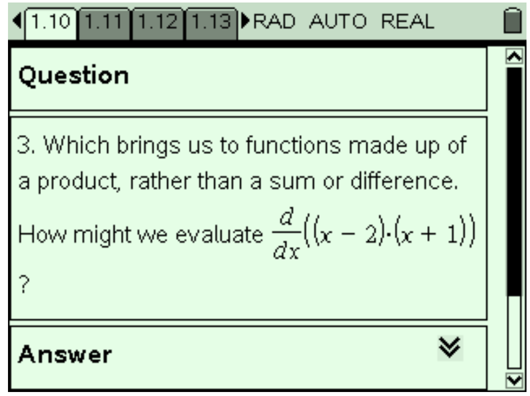
Step 2: After trying some simple examples of sums of functions and their derivatives, students should be asked to conjecture general rules for sums, differences and, finally, products of functions. They may use the CAS (if available) to verify their conjectures as shown.



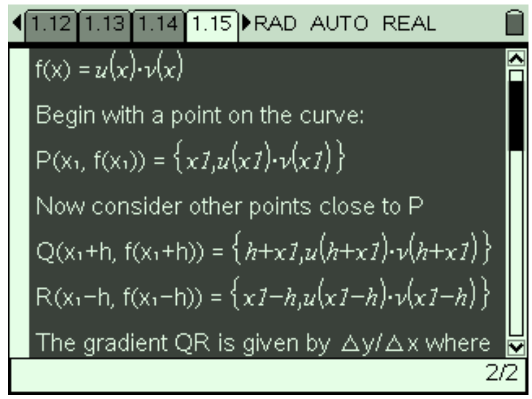
Step 3: It is one thing to conjecture and verify that conjecture, but quite another to prove such a conjecture. This may be done using the method of First Principles (as shown) and then, if CAS facilities are available, checked using the program **FirstPrinciples(u(x)+v(x)):result**, which shows the step-by-step working and the final result. This offers a valuable guide for students to check their own solutions and a model to help them construct better solutions.

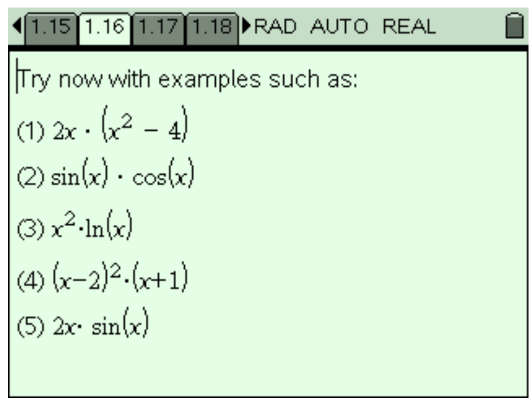


Step 4: Applying these methods to products of functions offers a further challenge: students should discuss ways in which they might check their product conjecture using the given function. Clearly, they can compute derivatives of the expanded form, and of the individual factors and see whether the derivative of a product equals the product of the derivatives (it does not!)

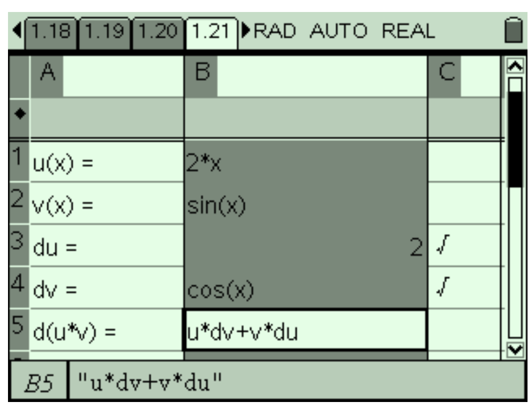


Step 5: Again, using the **FirstPrinciples(u(x)*v(x))** program, students may be assisted to work towards a verifiable method which reveals the product rule. By typing the variable **result** they have the product rule revealed to them.



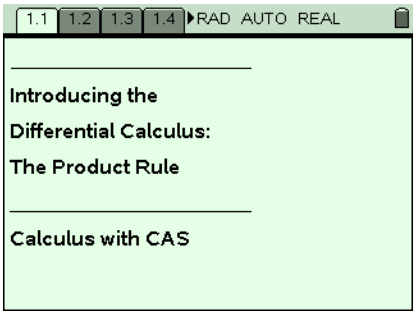


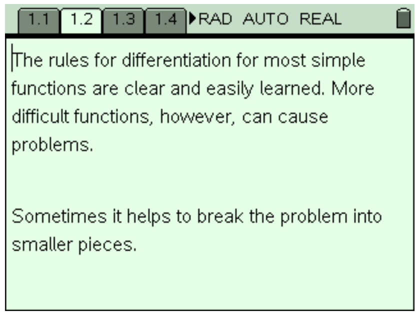
Step 6 An **algebraic spreadsheet** is available to support students in working through the process: students supply each step, which is checked for algebraic equivalence. Again, a prepared program may be used to help students in applying and consolidating their learning of this new rule. The **diff_product(fn1, fn2)** program requires students to express a given function as the product of two algebraic factors, and then it displays a step-by-step application of the product rule. Students are asked to simplify the final step, and then check their work by typing **result**.

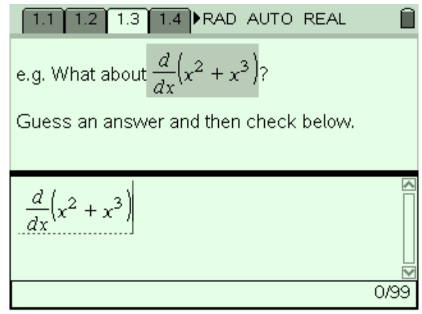


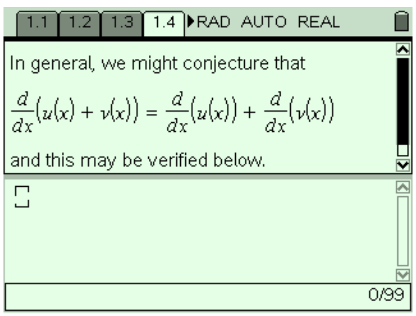
Introducing the Differential Calculus: The Product Rule– ID: XXXX

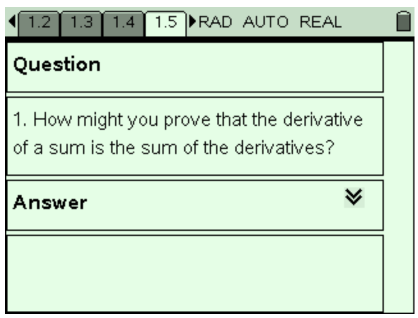
(Student)TI-Nspire File: *CalcActXX_Product_Rule_EN.tns*

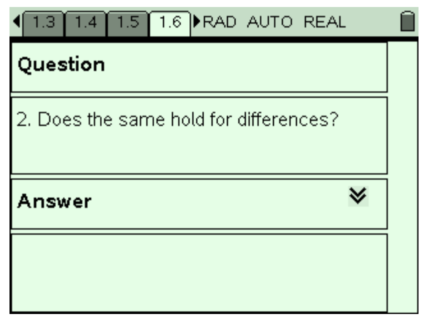


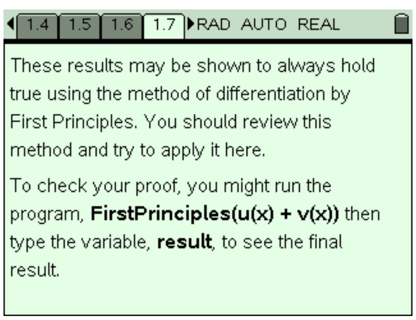


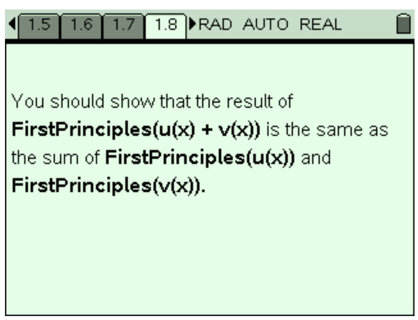


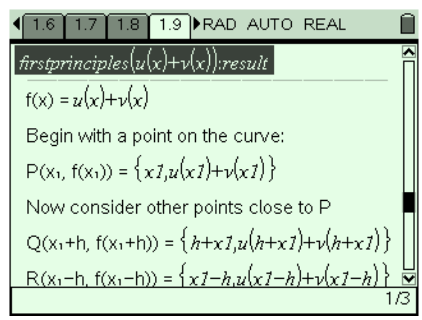


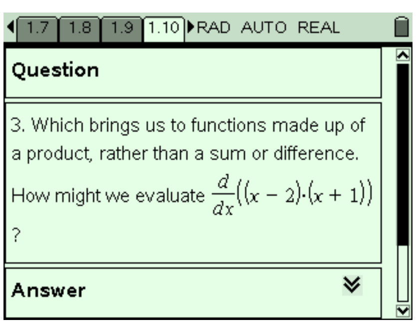


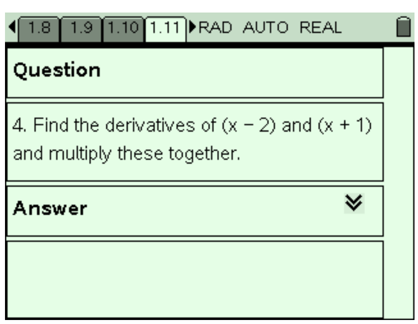


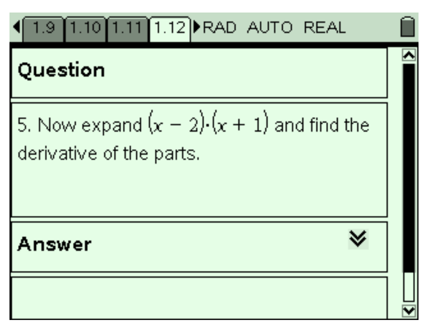












It appears that the derivative of a product is NOT equal to the product of the derivatives.

How might we find a rule for the derivative of a product function of the form $u(x) \cdot v(x)$?

Question

6. Once again, try using First Principles to evaluate this result, and check your answer using the **FirstPrinciples(u(x)*v(x))** program.

Answer

In general, f'(x) =

$$\lim_{h \to 0}\left(\frac{-(u(x-h)\cdot v(x-h)-u(h+x)\cdot v(h+x))}{2\cdot h}\right)$$

Done

result $\frac{d}{dx}(u(x))\cdot v(x)+\frac{d}{dx}(v(x))\cdot u(x)$

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While the program gave us the result we desired, it did NOT actually provide all the steps needed to understand that result.

The last line of the working gives f'(x)=

$$\lim_{h \to 0}\left(\frac{-(\mathbf{u}(x-h)\cdot \mathbf{v}(x-h)-\mathbf{u}(h+x)\cdot \mathbf{v}(h+x))}{2\cdot h}\right)$$

But how do we get to the final result?

One method for simplifying this limit is to add zero to the numerator – but a special version of zero: Clearly,

$\mathbf{u}(x-h)\cdot\mathbf{v}(x+h) - \mathbf{u}(x-h)\cdot\mathbf{v}(x+h) = 0$

Adding this to the numerator gives a result which can be factored.

$$\frac{(\mathbf{u}(x+h)\cdot\mathbf{v}(x+h)+[\mathbf{u}(x-h)\cdot\mathbf{v}(x+h) - \mathbf{u}(x-h)\cdot\mathbf{v}}{2\cdot h}$$

Question

7. Break this expression into two halves and factor each. Then take the limit as h approaches 0 and give the result.

Answer

One half =

Try now with examples such as:

(1) $2x \cdot (x^2 - 4)$

(2) $\sin(x) \cdot \cos(x)$

(3) $x^2 \cdot \ln(x)$

(4) $(x-2)^2 \cdot (x+1)$

(5) $2x \cdot \sin(x)$

Use the spreadsheet on the next page to work through these exercises.

Enter the two factors of your product function into cells B1 and B2 ("u" and "v").

Enter the derivatives of these functions into cells B3 and B4 ("du" and "dv").

Finally enter the product rule into B6.

Each of your steps will be checked.

	A	B	C
1	u(x) =	?	
2	v(x) =	?	
3	du =	?	X
4	dv =	?	X
5	d(u*v) =	u*dv+v*du	

B1 u:="?"

On the next page, you may check your results using the program **diff_product(u(x), v(x))** which gives the product rule differentiation for the function, $\mathbf{u}(x)\cdot\mathbf{v}(x)$.

To see the final result of this program, type the word "**result**".

d(v) = cos(x)

d(u*v) = (2·x)*(cos(x)) + (sin(x))*(2)

Try to evaluate this then check your answer by typing <result>.

Done

result $2\cdot x\cdot\cos(x)+2\cdot\sin(x)$

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We have established that

$$\frac{d}{dx}(\mathbf{u}(x)\cdot\mathbf{v}(x)) = \frac{d}{dx}(\mathbf{u}(x))\cdot\mathbf{v}(x)+\frac{d}{dx}(\mathbf{v}(x))\cdot\mathbf{u}(x$$

More simply, this result is often given as

$$\frac{d}{dx}(\mathbf{u}\cdot\mathbf{v}) = \mathbf{u}\cdot\mathbf{dv} + \mathbf{v}\cdot\mathbf{du}$$

(The first times the d/dx of the second plus the second times the d/dx of the first!,