Five
Laying out the Pieces: An Analytical Overview

In its early stages, the development of a grounded theory is analogous to the piecing together of a complex jigsaw puzzle. If the task is to be approached systematically, then it will begin with laying out the pieces, allowing them to be identified and initially sorted. In grounded theory analysis, this stage is called *open coding*, and involves the classification of the research records using codes or categories, largely arising from the data. These categories are akin to the pieces of the puzzle.

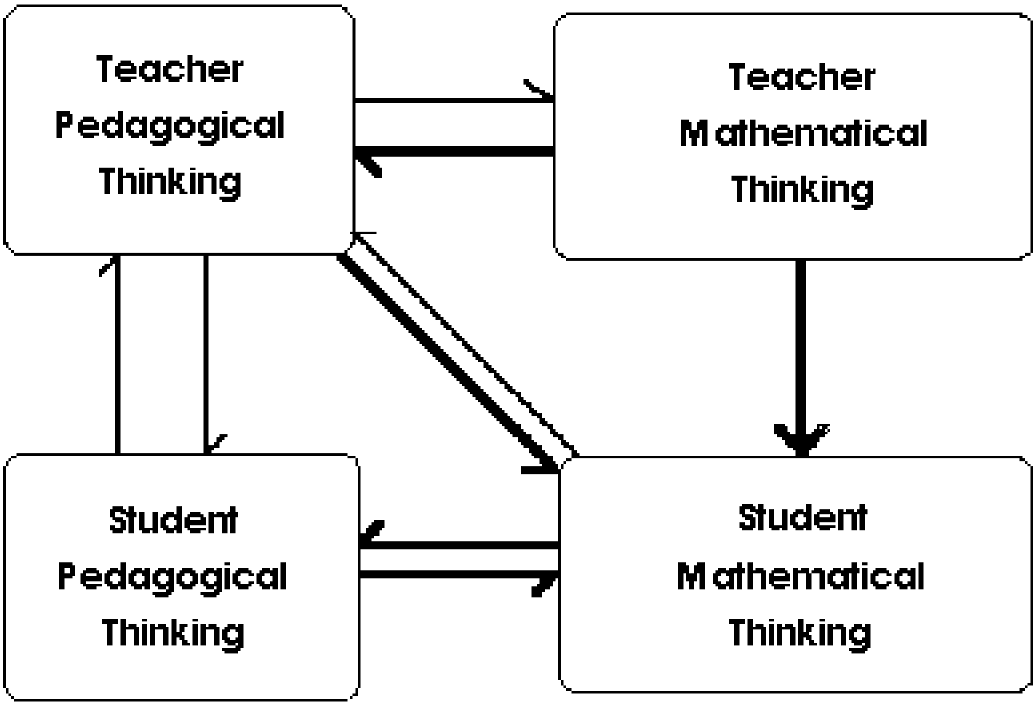
After laying them out, each piece must be studied in terms of its features, such as colour and shape. This relates to the second stage of analysis, *axial coding*, by which the individual categories are examined in terms of their properties and dimensions. Later, they will be sorted and placed in relationship to the other pieces, and the building of the grounded theory commences in earnest. Imagine now that having layed out, examined, sorted and eventually placed the individual pieces to form a coherent whole, that this whole becomes just a piece in a larger puzzle. This analogy gives some indication of the true nature of a grounded theory analysis, since it will rarely occur on a single level of complexity. Rather, it will spiral outwards as the components are examined, sorted and their nature teased out, and then they become parts of a larger puzzle which will eventually provide a rich, dense and descriptive theory of the phenomenon under consideration.

The purpose of this chapter (and the three following chapters) is to provide an analytical overview of the major categories which arose from the early “fracturing” of the data. It encompasses the two initial phases of analysis, open and axial codings. We are at the first stage of the process of building the jigsaw puzzle which will become a grounded theory of mathematical software use and by which this teacher and others might better learn to use these new tools. Prior to detailing the coding categories, however, it is relevant at this point to outline certain fundamental assumptions and perspectives which the researcher initially brought to the study, and which clearly influence the analysis which follows. Most particularly, these relate to beliefs and perceptions regarding mathematical and pedagogical thinking (the subjects of Chapters Six and Seven) and the role of mathematical software tools in the processes of mathematics teaching and learning (Chapter Eight).

An Interactive Model

Both teachers and their students engage in pedagogical and mathematical thinking, producing four identifiable domains when applied to learning situations. In the context of the present study of the use of advanced mathematical software by teachers and students, it is the *links* between these four domains of thinking which are seen to be of primary concern, providing an element of interactivity which is perceived as central to describing the processes of teaching and learning.

Figure 5.1: An Interactive Model

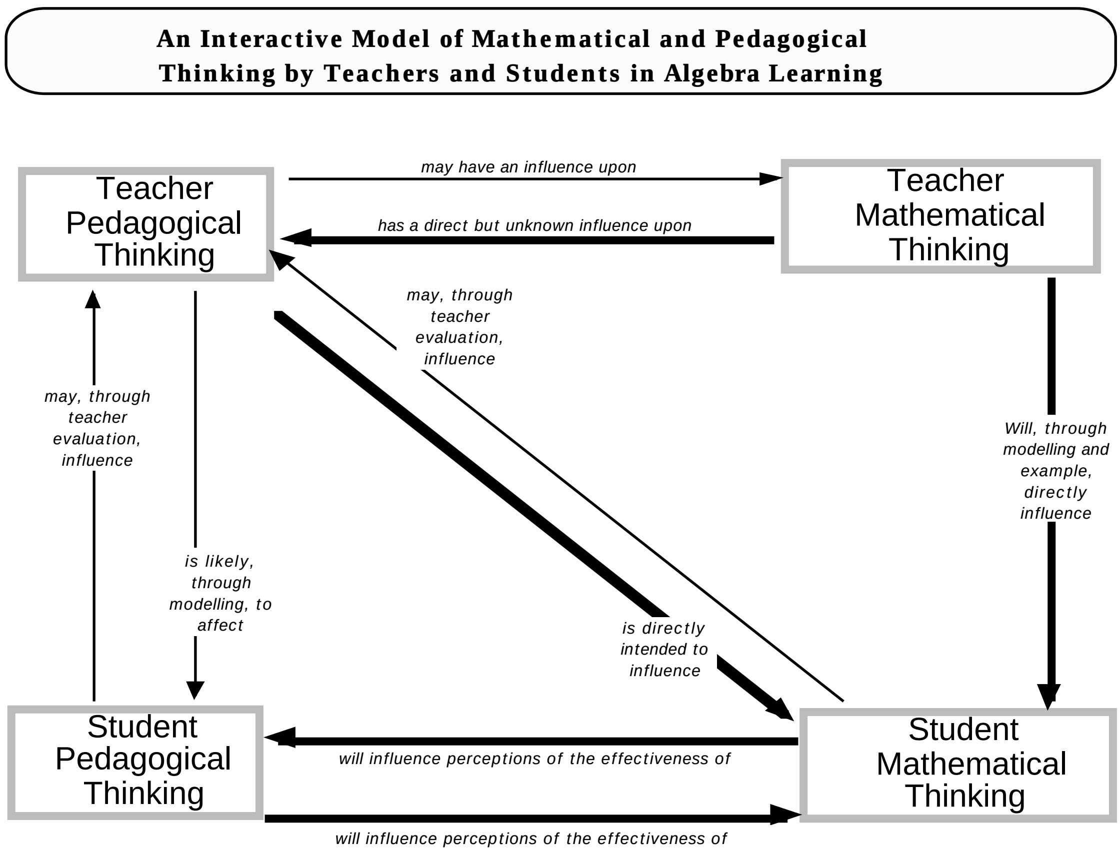


Those links shown in heavy print indicate traditional sources of focus in mathematics learning. Those in lighter print have been less emphasised and often less recognised. Each is considered below.

In the mathematical teaching/learning process, the central link is that between **teacher pedagogical thinking** and **student mathematical thinking**. This link is critical, since the primary purpose of teachers' pedagogical thinking is to directly influence the mathematical thinking of their students. Also of primary importance, **teacher mathematical thinking** is likely, through modelling and example, to indirectly influence **student mathematical thinking**. In the same way, **teacher pedagogical thinking** may, through modelling and example, influence **student pedagogical thinking** (particularly those features of the instructional environment which students associate with more and less effective learning).

The links from students to teacher are perhaps less clear, dependent as they are upon the awareness by the teacher of the students' thinking, and the willingness to respond to it. Thus, through reflection and evaluation by teachers, **student mathematical thinking** may be expected to influence the **pedagogical thinking** of their teachers. To a less clear extent, student judgements of effective method and instruction may also influence the pedagogical thinking of their teachers.

Figure 5.2: An Interactive Model of Mathematical and Pedagogical Thinking by Teachers and Students



The links between teacher mathematical thinking and pedagogical thinking have been the focus for considerable research activity (such as that by Even, 1990, 1993, Leinhardt, Zaslavsky and Stein, 1990, and Stein, Baxter and Leinhardt, 1990, in the area of “functions” alone) since Shulman’s call in 1986 to recognise the “missing paradigm” of teacher knowledge, “subject content knowledge”. At present, however, the link is still problematic, and deserving of continued study. Perhaps even more interesting may be links between the pedagogical thinking of teachers and their mathematical thinking. Teachers in general appear not to “think like” mathematicians; they think like mathematics teachers. In what ways does this affect their understanding and practice of mathematics? In the present study, it is proposed to make such links explicit through studies of the responses of teacher education students who have completed the mathematics content part of their course, but not yet studied aspects of pedagogy. Such responses (involving preferred images and levels of understanding of algebraic concepts, perceptions of effective instruction and the roles of teachers in algebra learning) will be compared and contrasted with those of secondary students.

Finally, the distinction between the pedagogical and mathematical thinking of students in schools is less clear than that for their teachers, since students are unlikely to have perceptions of mathematics beyond their classroom experiences. The two domains are certainly linked closely; although the link is unclear, each is likely to influence perceptions of the effectiveness of the other.

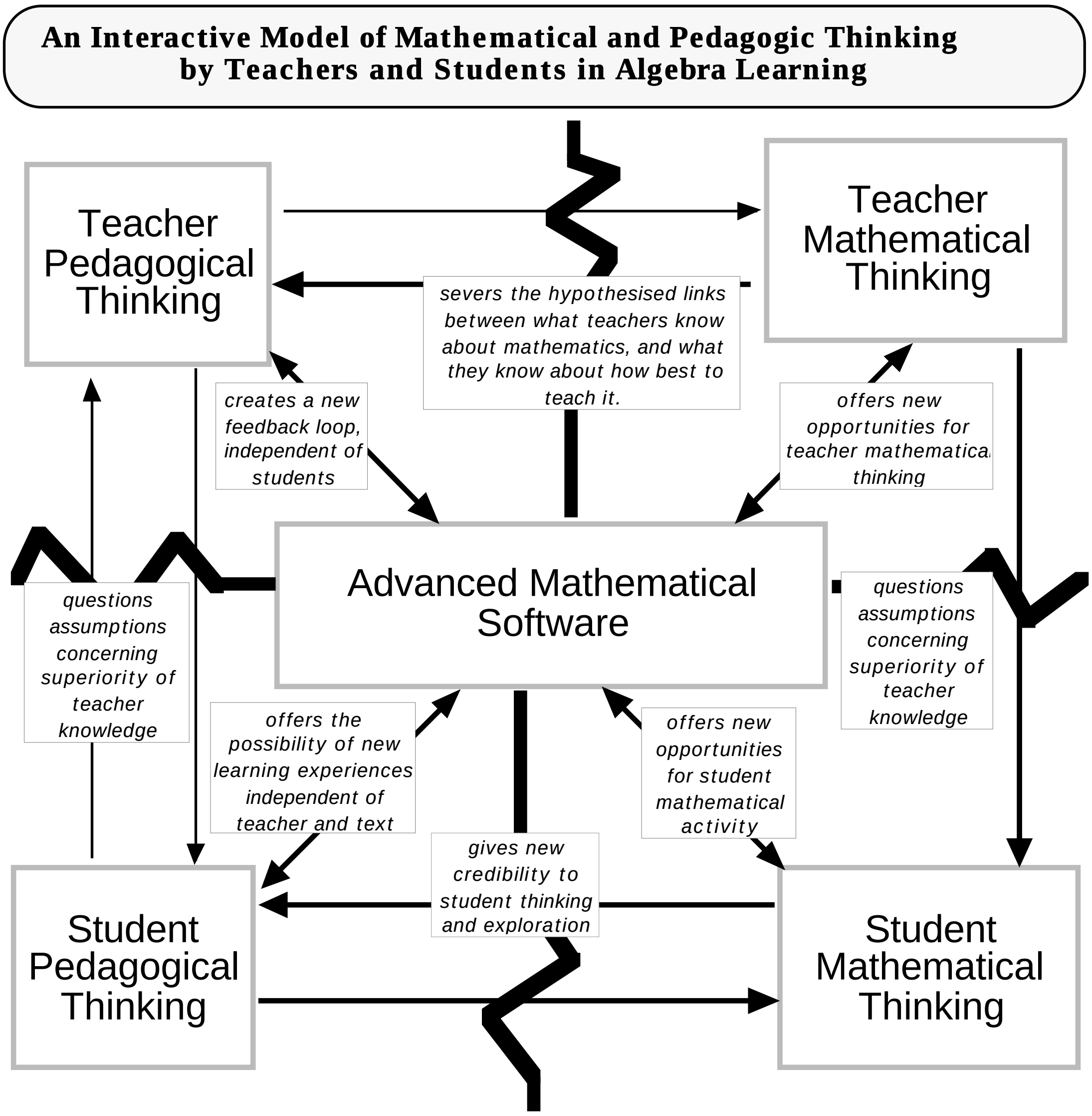
In seeking to describe and make explicit the use of mathematical software by individuals learning algebra, and its consequent effects upon their thinking, the research inquiry then must focus upon these four domains and the interactive links between them.

The Influence of Mathematical Software

In many respects, the classroom presence of computer algebra tools and other open-ended mathematical software, implies a *mediating role* between teacher and students, and between the four domains of thinking described above. It is possible to identify several possible and likely influences of such a role, and these determine the research focus which drives the present study (Figure 5.3).

The presence of mathematical software is likely to create a new feedback loop for teacher reflection and evaluation, with the software as its source. This is additional to the traditional feedback loop from students. It will also provide a new source of feedback for students, in addition to the traditional feedback offered by the teacher. It should also provide a new reference point - additional to the teacher - regarding both mathematical and pedagogical thinking. Since in many ways, teacher and students become co-learners through the use of such tools, this use is likely to question assumptions concerning the superiority of teacher knowledge of both domains. The use of computer technology in general, and mathematical software in particular, critically confronts current content and methods for the teaching and learning of mathematics.

Figure 5.3: Some Possible Effects of Advanced Mathematical Software



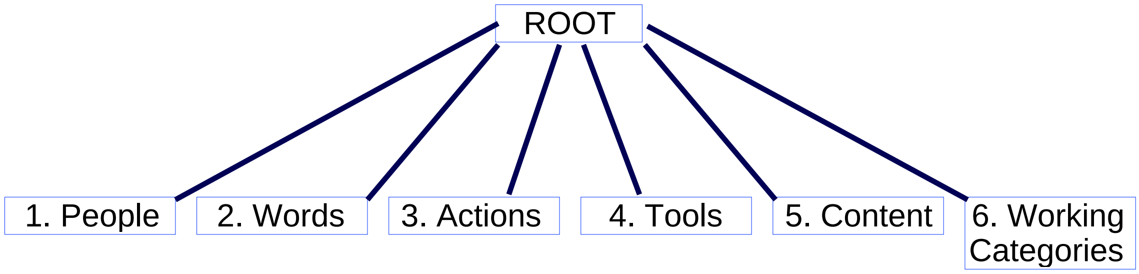
The use of mathematical software provides new opportunities for teachers to engage in mathematical thinking as distinct from pedagogical content thinking about their discipline. It is likely, too, to strengthen links between mathematical and pedagogical thinking by students: The use of mathematical software provides new opportunities for students to engage in mathematical thinking which is independent of teacher and text, and so gives new value to mathematical creativity and investigation. It may serve to provide new validity for student mathematical conjecture, and to give new credibility to student thinking about mathematics, no longer wholly subject to the teacher.

Having made public these perceptions and beliefs, it is now possible to approach the analysis of the data with some awareness of the researcher's position in constructing the initial index system. This initial overview will be structured around the three central concerns of the project - mathematics, pedagogy and computer use, each analysed in terms of the components which are outlined below.

Categories of the First Level

The qualitative analysis software program, *NUD•IST*, encourages the sorting of conceptual categories into logical tree structures as they arise from the data. Each category occupies what is termed a **node** in the tree which forms the **index system.** Beginning with a "root" node, all others are placed in relationships which will initially reflect the cognitive organisation of the researcher (Figure 5.4). Later, these nodes will be moved and reorganised to better reflect the structure of the data and the unfolding nature of the grounded theory. At this stage, however, it suffices to examine these categories within the descriptive and largely superficial relationships ascribed to them by the researcher, and so to better understand the perspective and biases which he brings to the analysis. As mentioned previously, this process of "bracketing" is essential if the theory is to develop from the data and to be more than a mirror of the researcher's own views.

Figure 5.4: Categories of the first level



Five of the six first level categories displayed in Figure 5.4 reflect the initial analysis of the project into what were perceived as its critical analytical components (category 6, *Working Categories*, provides a functional repository for new categories as they arose from the data, allowing their placement in relation to the others to be delayed until later in the process. These would generally be categories which did not fit readily into the existing structure).

Figure 5.5: Category 1: People

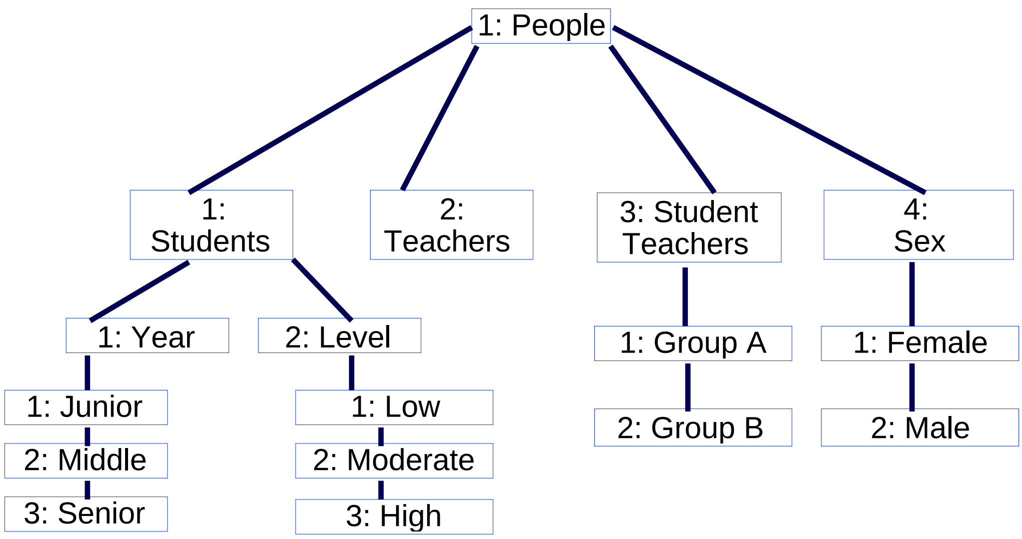


Figure 5.6: Tools and Content Categories

4. Tools

4.1 MathPalette

1. HyperGraph
2. Table of Values
3. Concrete
    1. expressions
    2. equations

4.2 Computer Algebra

1. Theorist
2. MathMaster 2.21
3. Calculus T/L II
4. CoCoA
5. Derive

4.3 xFunctions 2.2
4.4 Spreadsheet
4.5 Calculator
4.6 Graphic Calculus
4.7 RURCI

5. Content

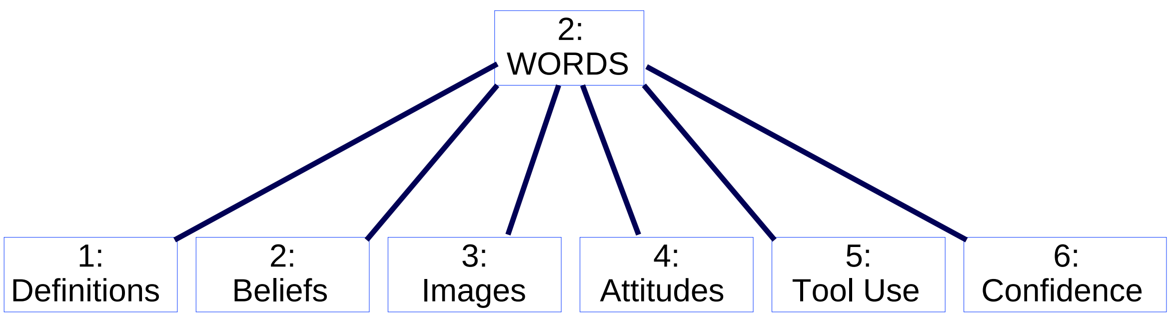
1. Beginning algebra
2. Equations
3. Graphs
4. Coordinate geometry
5. Curve sketching
6. Completing the square
7. Inverse functions
8. Composite functions
9. Continued fractions
10. Polynomials
11. Calculus
12. Problems
13. Review of Skills
    1. Elem, Algebra
    2. Elem. Trig.
    3. Basic Skills
    4. Beginning algebra
    5. Equations
    6. Curves & functions
    7. General algebra
    8. Senior algebra
    9. Stress Test
14. Guess My Rule
15. Chaos

The categories *People, Tools* and *Content* are purely descriptive in nature, reflecting aspects of the physical components which make up the study (Figures 5.5 and 5.6). These have been discussed in previous chapters. (Note that the tool "Graphic Calculus" refers to an IBM-based package by David Tall, *A Graphic Approach to the Calculus*, which was used early in the gathering of data, and *RURCI* stands for *Are you Ready for Calculus?*, a series of review quizzes compiled for IBM computers by David Lovelock, which were later adapted into the *HyperCard* format of this study.)

The two remaining categories, *Words* and *Actions*, represent major units of analysis for the data, since they are perceived as the means by which *thinking* is made explicit on the parts of the participants. The ongoing development of these groupings occupied the primary focus in the early coding, both open and axial, and the description of each that follows provides the greater part of the overview for this chapter. These two nodes capture in detail the nature of the interactions which later provide the basis for the relationships which comprise the grounded theory. In particular, the category, **Words**, provides the basis for the analysis of the three core categories - thinking about algebra, about pedagogy and about computers, which occupy the next three chapters, and so lay the groundwork for the subsequent theory generating process. Consequently, it is examined here in some detail.

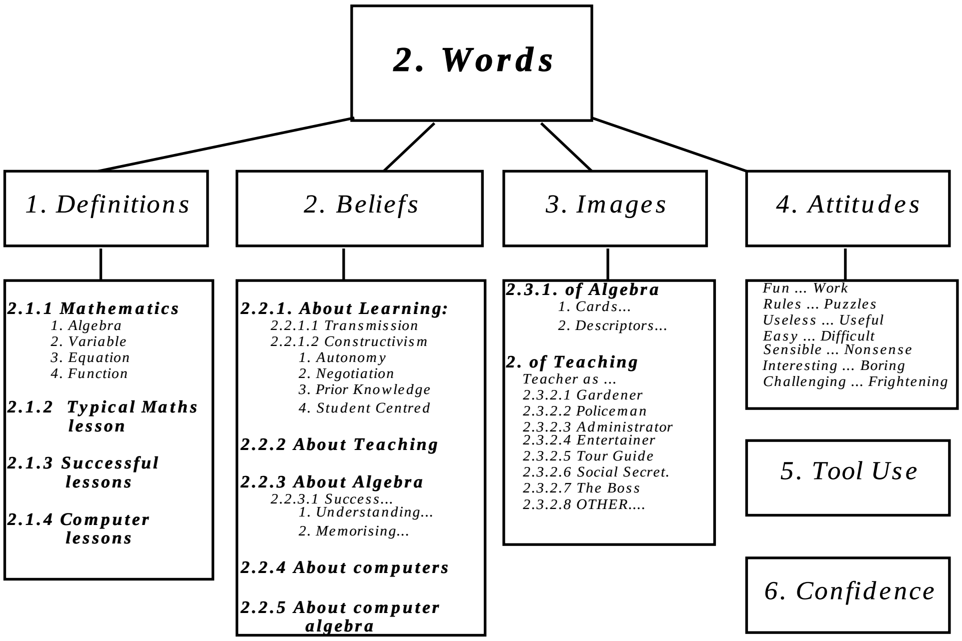
A Study of Words

Figure 5.7: Category 2: Words



The initial conception of the category involved three children, **Definitions, Beliefs** and **Images. Definitions** are seen as objective attempts to describe concepts, as opposed to **beliefs**, which may be expected to contain a subjective element, as the object is described *in relation to* the person giving the description. Within a constructivist framework there can be no definitive boundary between the two conceptions, as all knowledge is constructed from personal meanings - every "definition" represents a statement of personal belief. However, for the purposes of this study, a distinction between the two is useful, and to this end an objective/subjective distinction should suffice. The focus question, *"How would you describe algebra and the way you best learn it?"* might be considered to give rise both to a definition (*"Algebra is..."*) and a statement of belief (*"I best learn algebra by..."*). The first is seen to exist independently of the speaker, while the second is defined in terms of the relationship with the speaker, and usually involves an explicit value judgement (for example, "How do you *best* learn algebra?"). Although beliefs may be "messy constructs" to examine subjectively, "few would argue that the beliefs teachers hold influence their perceptions and judgements, which, in turn, affect their behaviour" (Pajares, 1992, p. 307). Nor may this influence be restricted to teachers. Lying at the intersection of the cognitive and affective domains, "people's conceptions of mathematics shape the ways that they engage in mathematical activities" (Schoenfeld, 1989, p. 338). The beliefs individuals hold regarding mathematics, algebra, computers, learning and teaching must be considered critical issues in the present context.

Figure 5.8: Sub-categories of WORDS



If beliefs may be distinguished from definitions by their subjectivity and their inclusion of a value dimension, then what of images? Vinner defines a "concept definition" as "a verbal definition that accurately explains a concept in a non-circular way" (Vinner, 1983, p. 293). While this very "definition" is itself circular (defining a definition as a type of definition), it offers three critical elements - a definition may be considered to be *verbal*, it *explains* something and it should be *non-circular*. It is probably the first of these which most clearly distinguishes concept *definition* from concept *image*, which Vinner describes in terms of one's *mental picture* and the set of associated properties called up by the concept. In SOLO terms, the concept image is ikonic: visual, intuitive and global, while the definition is more likely to be associated with concrete-symbolic thinking - rational, sequential and verbal. Van Hiele's distinction between *visual* and *descriptive* levels of thinking applies equally.

These three analytical units, definition, belief and image, provide the primary basis for analysis of the research data (Figure 5.8). Taken together, they allow rich and detailed depiction of elements of thinking related to both mathematics and pedagogy within a tool-rich algebraic learning context. As the analysis of the data unfolded, however, three additional categories arose, each of which reflected important considerations which did not fit easily within the early codes. Statements relating to attitudes, tool use and confidence were considered particularly significant in the study, and were positioned accordingly.

A Study of Actions

As illustrated in Figure 5.9, actions were categorised in terms of the three central concerns of the study - mathematical, pedagogical and computer-related actions. The sub-categories for each will be examined in detail in Chapter 8.

Figure 5.9: Categories of Action