The Graphics Calculator in Mathematics Education: A Critical Review of Recent Research

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ABSTRACT

The graphics calculator, sometimes referred to as the "super calculator," has sparked great interest among mathematics educators. Considered by many to be a tool which has the potential to revolutionise mathematics education, a significant amount of research has been conducted into its effectiveness as a tool for instruction and learning within precalculus and calculus courses, specifically in the study of functions, graphing and modelling. Some results suggest that these devices (a) can facilitate the learning of functions and graphing concepts and the development of spatial visualisation skills; (b) promote mathematical investigation and exploration: and (c) encourage a shift in emphasis from algebraic manipulation and proof to graphical investigation and examination of the relationship between graphical, algebraic and geometric representations. Other studies, however, indicate that there is still a need for manipulative techniques in the learning of function and graphing concepts, that the use of graphics calculators may not facilitate the learning of particular precalculus topics, and that some "deskilling" may occur, especially among males. It is the contention of this paper, however, that much of the research in this new and important field fails to provide clear guidance or even to adequately inform debate regarding the role of graphics calculators in mathematics teaching and learning. By failing to adequately distinguish the role of the tool from that of the instructional process, many studies reviewed could be more appropriately classified as "programme evaluations" than as research on the graphics calculator per se. Further, claims regarding the effectiveness of the graphics tool for learning frequently fail to recognise that judgments of effectiveness result directly from existing assumptions regarding both assessment practice and student "achievement."

INTRODUCTION

Since the advent of graphics calculators (sometimes referred to as "graphing calculators" or even "super calculators") in the mid-1980s there has been growing interest in their apparent potential to facilitate and enrich the teaching and learning of mathematics in schools and tertiary institutions. Many authors believe that graphics calculators have the potential to revolutionise mathematics education, both in the way it is taught and the content and emphases of curricula (Barling, Johnston, & Jones, 1989: Burrill, 1992; Groves, 1991; Hackett & Kissane, 1993; Kissane, 1993: Leary, 1991: Leary & Clarke., 1993: Lee, 1993; Weal, 1992). Consequently, there has been much enthusiastic comment concerning the capabilities, potential and implications of this new technology. This paper examines the published comments of both researchers and practitioners with regard to this new and increasingly significant field of mathematics education practice, with particular regard to the research literature of the past eight years. Although the first graphics calculators appeared in l985, it was not until 1990 that research dissertations began to appear which related directly to their use as tools for teaching and learning. This review canvassed the Dissertations Abstract International records for the period from 1990 to 1995, the major refereed journals related to mathematics education, relevant journals on both educational research and educational computing, in addition to other publications cited in these sources. Key words used for searching were "graphic(s) calculator" and "graphing calculator”. While not claiming to be exhaustive, the review is extensive and every effort was made to provide a balanced and representative overview of the literature for the period specified. (Several studies have been omitted from this review either due to inaccessibility or because they appeared not to add further to the discussion. A number of these which are cited briefly in Dunham's 199la and 1993 reviews on graphing technology (Dunham, 1991b, & 1993) appear to reach conclusions similar to the studies reviewed here.)

This year marks the end of the first decade of the use of graphics calculators in mathematics education. As this use becomes increasingly widespread (and, indeed, moves are afoot to legislate such use in senior school classrooms), it is most timely to reflect upon the nature and potential of this new technology. The role of research in any fledgling field of enquiry must be twofold: both to inform discussion and debate, and to guide practice. This paper examines the extent to which the experience of the last ten years illuminates our understanding and use of these powerful tools. Certainly, one might expect that, after so much enthusiastic rhetoric and so many studies specifically intended to explore the effectiveness and limitations of the graphics calculator as a tool for the teaching and learning of mathematics, we might enter the second decade of their use well-prepared. In particular, one might expect quite clear answers to two critical questions associated with the use of graphics calculators in mathematics education:
(1) *In what ways can graphics calculators be used to maximise learning and achievement,*

and
(2) *What teaching practices and what types of learning environments best complement their use in order to bring about maximum benefits for students?*

Sadly. the answers offered by research to these questions at the end of this first decade remain elusive and conflicting. As assumptions concerning the role and effectiveness of graphics calculators assume almost the status of dogma amongst educators and policy-makers alike, it is vitally important that the research basis for these assumptions be examined critically.

THE NATURE OF THE TECHNOLOGY

The "first generation" of graphics calculators provided all the facilities of a scientific calculator as well as capabilities for data analysis, linear algebra, programming and, as the name implies, the graphing of functions. Within secondary school mathematics courses and many courses at tertiary level, it appears to be this graphics component which is the most frequently used facility of the calculator. With this, the user is able to represent
cartesian, polar and parametric equations graphically on a small screen, allowing visual inspection of a graph's features by “zooming in" or "zooming” out to particular regions.

Relatively quickly. additional features were added to the functionality of graphics calculators, while retaining the same relative cost structure and consequent availability for schools and tertiary institutions. These defined what might be termed "second generation" graphics calculators, and included numerical approximation options for equation solving derivatives and definite integrals, in addition to tables of values and improved graphing facilities. By providing immediate access for students to the results of extended mathematical processes, such devices appear to challenge traditional approaches to mathematics instruction. and even to confront existing assumptions regarding the ways in which mathematics may be best learned. This challenge is further heightened by the recent release of hand-held devices which offer all the functionality of advanced mathematical software for desktop computers, including symbolic manipulation and dynamic geometry facilities. While these more powerful features appear certain to become increasingly available and affordable, it is the basic "graphics calculator which remains the standard in use at this time, and provides the focus for the review which follows. Indeed, researchers appear slow to recognise these extended capabilities which make graphics calculators far more than simply “calculators which can draw graphs." There appears at this time a conspicuous dearth of studies, at high school level in particular, which directly address the full implications of student access to these "hand-held computers," even though extended capabilities (including symbolic manipulation or "computer algebra") have been available in various forms since the earliest days of graphics calculators in the mid-1980s. (An exception is the Australian case study described by Arnold (1993) in which senior high school students were provided with access to calculators offering graphíng, solving, calculus and general symbolic manipulation capabilities.)

Adoption and use of graphics calculators within high schools and tertiary institutions in developed countries appears increasingly widespread. In Australia, recent legislation permitting their use in high stake external examinations in Victoria and Western Australia serves as a precursor for widespread adoption (in the senior school, at least), whích other states seem likely to follow. Consequently, there exists a large amount of material written for the purpose of explaining the facilities which set it apart from the scientific calculator and giving examples of ways these facilities can be used: cartesian and polar graphing (Clarke & Clarke, 1991: Clarke & Leary, 1993: Leary. 1991): data entry and analysis (Cowling & Llewelyn, 1994: Jones 1993, 1994: Sullivan, 1990): linear programming and matrix calculations (Tobin, 1991; Weal, 1992): general programming (Humble, 1992): equation solving (Day, 1993; Walton & Wines, 1993): numerical integration (Kissane, 1993); and for teaching "line of best fit" (Rubenstein, 1992). Clearly. while research may be relatively silent on the extended capabilities of graphics calculators, practitioners are not. A number of authors specifically address the role of the graphics calculator in the teaching and learning of various mathematical concepts: for example, Clarke and Leary (1994) and Barling (1991) discuss applications to calculus teaching at high school and college levels; Barnes (1994), Brown (1994), Groves (1992) and Kanold (1992) demonstrate modelling activities and exploration of real data using the graphics calculator: Day (1993), Paasonen (1993), Andrews (1992) and Borenson (1990) outline ways that the graphics calculator can be used to teach various concepts concerning the relationship between functions and graphs: Vonder Embse (1992) illustrates how its multi-line display facilitates problem-solving: and Barnes (1994) and Brown (1994) look at interesting ways of investigating properties of parabolas. The work of Demana and Waits from the University of Ohio in publicising the potential of these tools for high school and college teaching and learning has been a significant feature of this field internationally since the earliest days of the use of these devices (Demana and Waits, 1992; Waits and Demana, 1988, 1989a, 1989b, 1992).

The graphics calculator is believed to be a tool which opens up new ways to approach many problems and encourages students to experiment and investigate (Groves, 1991; Leary & Clarke, 1993; Piston, 1992). allowing a shift in emphasis from algebraic manipulation and proof to graphical investigation (Day, 1993; Demana & Waits, 1992). Others report that through its use they have been able to extend the range of student activities and problem-solving components of mathematics courses (Clarke & Clarke, 1991: Tobin, 1991), and to introduce to students at various levels of schooling mathematical concepts and topics not previously accessible to them due to computational difficulty (Greenes & Rigol, 1992).

The graphics calculator is thought by some to have significant advantages over the computer as a tool in the learning of mathematics (Hackett & Kissane, 1993; Jones, 1991; Leary, 1991). The potential of this technology for widespread use, made feasible by its portability and, to some extent, its price (Groves, 1990; Jones, 1991), has major implications for the mathematics curriculum. Revision of mathematics curricula is believed to be urgently required (Dick, 1992) with different emphases in a number of topics (Kissane, 1993), and possibly the re-ordering of complete topics (Burrill, 1992). Questions are also being asked about the mathematics that should be taught (Burrill, 1992; Groves, 1990), the relative merits of various teaching methods associated with graphics calculator use, and the possible effects of its use on current assessment strategies (Dick, 1992; Greenes & Rigol, 1992; Payne, 1992). Further, Dick (1992) and Arnold (1993) explore some implications for student attitudes and learning of access to personal technology, as opposed to desktop computers, which appear not to engender the same sense of ownership and acceptance as hand-held devices.

A small number of authors (Andrews, 1992; Day, 1993; Dion, 1990; Glidden, 1992; Hodges & Kissane, 1993) also address problems associated with the use and misuse of graphics calculators, and calculator-related mathematical misconceptions (Demana & Waits, 1988), discussing implications of these in the use of graphics calculators in learning and instruction.

RESULTS

The majority of research examined for this review appears to focus upon two major areas of concern:
(1) Testing the effects of the use of graphics calculators within specific areas of mathematical study, and
(2) making judgments regarding the effectiveness of such use.

In the studies which follow, both effects and effectiveness are commonly related to either "student learning" or "student achievement," critical terms which are surprisingly poorly defined in many cases. It is common for authors to equate these terms with "performance on assessment tasks." These tasks appeared to be frequently identified with traditional skill-based formal testing procedures, and claims regarding "effectiveness" must be judged carefully in the light of such assumptions. In particular, studies which make claims regarding the effects of graphics calculator use must carefully distinguish between the tool and the context in which it is used, while those that purport to judge effectiveness must make explicit their assumptions concerning both the method and focus of assessment procedures. These issues will be returned to throughout the review which follows.

This review considers studies within the following contexts:
(1) Precalculus studies
(2) Calculus studies
(3) Function concepts
(4) Graphing concepts
(5) Modelling
(6) Spatial visualisation studies
(7) Use of the graphics calculator in examinations
(8) Errors related to graphics calculator use
(9) Gender differences
(10) Student attitudes toward mathematics as related to calculator use
(11) Student attitudes toward the graphics calculator as a tool for teaching and learning
(12) Issues of pedagogy.

While these categories are far from being mutually exclusive, they provide a logical and relevant organisation of the large amount of material surveyed which is consistent with the internal structure of many of the studies themselves.

1. Precalculus Studies

Within the precalculus domain, several studies suggest that the use of graphics calculators in teaching and learning is beneficial in terms of students' level of understanding and achievement in elementary algebra, and also in their development of spatial skills in this area of study. On the other hand, others have indicated that the nature of their impact is not clear and have highlighted some areas of study and types of understandings for which the use of graphics calculators does not appear to facilitate learning.

Thomasson (1993), for instance, compared levels of achievement of college students who were exposed to different modes of instruction involving various types of graphics calculator use in an elementary algebra course: “Total use,” which refers to use in instruction, demonstration, and by students within and without the classroom, including use in examinations; “partial use,” referring to use in instruction, demonstration, and by students during class only; and “no calculator use” by instructor or student. She found that students in the “total calculator use” group performed better, but not significantly better, in post-tests which measured students’ performance in elementary algebra. Shoaf-Grubbs (1993), however, in her examination into the effect of the graphics calculator upon students’ levels of understanding in elementary college algebra, concluded quite definitely that its use did indeed aid students in the learning of algebra concepts. Her study involved 37 females from an all-women’s liberal arts college, divided into an experimental class and a control class. The method and content of instruction was identical for both the experimental and control classes, with the only difference between the groups being the use of the graphics calculator. Statistical analysis of pre- and post-test results revealed that the majority of students in the experimental group achieved significantly better than control group students in test results. This study appears exemplary (and quite unusual) in controlling carefully for the effect of teacher and instructional programme. It may be contrasted with Thomasson's study which, because of less rigorous research design, demonstrated only that the amount of use of graphics calculators is likely to be less important an influence upon student learning than how they are used.

Dividing precalculus knowledge into three subcategories - procedural and relational knowledge, and transfer of knowledge - Tolias (1993) found that the effect of graphics calculator use varied in the development of the different types of knowledge. Her investigation was conducted over a two-semester period and involved four classes of students studying algebraic and graphical procedures for solving equations and inequalities. The experimental classes were taught using a resequenced precalculus curriculum with an emphasis on functions and their graphs. Their programme was designed specifically with the intention of facilitating the integration of algebraic and graphics procedures. Control subjects were taught using a more traditional precalculus outline. The college's first year examination in mathematics served as a measure to adjust for initial group differences. Analysis of a post-test designed to measure procedural and relational knowledge and transfer of knowledge, produced some interesting results: while no significant differences were found between the experimental and control groups concerning procedural knowledge, Tolias reported significant differences which favoured the experimental group in relational knowledge and transfer of knowledge. Further, subjects in the experimental group who chose algebraic procedures to solve items which tested relational knowledge were found to perform better on these items than students in the control group. Again, however, the effects of the calculators cannot be distinguished from that of the instructional programme.

Similarly, Army's (1992) investigation into the effect of introducing the graphics calculator into a college course in trigonometry failed to find any evidence of achievement gains associated with its use in this study but, as part of the study, a new approach to the teaching of trigonometry was introduced which had as its focus the linking of mathematical content to real-world applications. Since the effectiveness of this approach was untested at the time of this study, it is unclear how much effect this had on the results. Hence the presence of this added variable tends to reduce the significance of Army's findings in relation to the effect of the use of graphics calculators within the study of trigonometry. Analyses of final examination results and student interviews dealing with problem-solving strategies resulted in some interesting observations by Army: students had learned that many problems involving algebraic equations could be solved both algebraically and graphically, and further, students frequently chose to solve such problems graphically. Once again, it is impossible to determine whether these results are attributable to either the graphics calculator or the instructional programme.

Hall (1993) reported that the graphics calculator made no statistically significant impact on precalculus students' achievement in the study of trigonometric functions over a three- week period. Her study involved a comparison of the performance on pre- and post-tests of precalculus high school students who used graphics calculators in the study of a three- week unit on trigonometric functions, with that of students who did not use calculators in this course of study. Hall used four treatment classes which consisted of 112 students, control classes containing a total 98 students, and four teachers from separate schools who were trained in the use of the graphics calculator. Each teacher taught an experimental class using graphics calculators and a control class without the use of these.

Hall also noted that her data indicated that the use of the graphics calculator in precalculus trigonometry may even be detrimental to students' achievement in trigonometry. However, the brevity of this study must be viewed as a limitation to the generalisation of her results, as must the problem of student preparation for the use of the graphics calculators. This is an unfortunate limitation in a study which appears in other ways carefully designed.

Dunham (1993) produced very positive results from a field test of a technology-based precalculus course, concerning the impact of the graphics calculator on "calculus readiness." She reported that 55 schools which taught this course attained significantly higher means on a "calculus readiness" test than did the 22 control schools, with students from the experimental schools who did not achieve calculus placement on the pre-test, achieving post-test calculus placement level scores at nearly twice the rate of those in traditional classes. Whether these results may be attributed to the graphics calculators, to aspects of course design and implementation, or even to the obvious novelty effect remains unclear.

It is possible that graphics calculator use may enhance achievement in precalculus courses. Nevertheless, much further research is required before any definite conclusions can be drawn concerning its value as a tool for learning in precalculus studies in mathematics. It is worthy of note, for example, that neither Hall nor Army was able to address the question of the effects of students' unfamiliarity with the facilities of the graphics calculator. Since all of the subjects who had used graphics calculators in the two studies had been introduced to them for the purpose of the study, one needs to ask whether the results found by these researchers would remain true if students had been conversant with the tool prior to the studies. Further, was the level of problem-solving in Army's study a cause of difficulties that may not arise in a unit on trigonometry which does not have a problem-solving focus? If the conclusions reached were not attributable to factors such as these, are there any intrinsic elements of trigonometric concepts, or the type of learning required, which make the study of trigonometry incompatible with graphing technology? Further research in this area must attempt to isolate factors such as these in order to identify elements attributable to the use of graphics calculators which have an impact on the learning of precalculus topics.

2. Calculus Achievement

During the 1980s in the U.S.A., much attention was directed to reforming the secondary school and university undergraduate calculus curricula, with many remedies being suggested for alleged problems (Palmiter, 1988). Central to these recommendations was the de-emphasising of computations to allow for more emphasis to be placed upon the concepts involved in calculus. As a consequence of this discussion, researchers began to investigate the effects of using computer technology and the potential role of this technology asa tool for teaching and learning within calculus curricula. Most of these studies involved computer algebra systems.

Computer algebra systems (or symbolic manipulators) are computer systems for the exact solution of computations involving calculus, limits and series, equations, and systems of equations. Commonly, such systems include a graphics component and a variety of other facilities, which may include tables of values, animation of graphs and 3-dimensional plotting. Used in conjunction with other calculus facilities within the various computer algebra systems examined, graphing technology appeared to contribute positively to students' understanding of calculus concepts (Arnold, 1993; Hawker, 1986; Heid, 1988; Palmiter, 1991). With the growing availability of graphics calculators with an effective computer algebra component, studies involving symbol manipulation in schools and tertiary mathematics have become newly relevant to discussions of graphics calculator use.

Three researchers, however, Upshaw (1994), Ellison (1994) and Estes (1990), who have specifically investigated the effects of graphing technology within the study of calculus, have also included as at least part of their studies, the effects of the use of graphics calculators in this capacity. Indeed, Upshaw concentrated wholly upon examining the effects of graphics calculator use in the study of the indefinite integral and the Fundamental Theorem of Calculus based on graph-exploration. Her research, involving a comparison between the effects of this method and a traditional method of instruction which did not involve any use of graphics calculators, failed to uncover any benefits in using the graphics calculator as a tool in these studies.

Using two multiple-choice instruments which consisted of routine, non-routine, graphical, and symbolic problems, Upshaw reported that the traditionally taught classes scored significantly higher on graphical problems in the study of the indefinite integral. No significant effect was found regarding the study of the Fundamental Theorem of Calculus for either the traditionally taught classes or the classes taught using graphics calculators. Whether this should be interpreted as a failure of the graphics calculators or a result of both instructional method and assessment procedure remains unclear. As is the case with all studies which involve the use of post-tests to compare groups of students, the validity of these results is highly dependent upon the selection of test items; that is, how these compare with what was taught to each of the groups.

On the other hand, Ellison (1994) and Estes (1990) report that the combined use of computer and calculator graphics has positive effects with regard to calculus learning. Studying the conceptual and procedural achievement of applied calculus students, Estes found that students who used calculator and computer technologies as an integral part of their studies performed significantly better on test items designed to test conceptual development in calculus than students taught by traditional means without the use of these tools. No statistically significant difference between the two groups of students was found on test items which were included to test students' understanding of the procedures used to solve calculus problems. Ellison, in contrast, looked specifically at the development of students' concept images of the derivative when the Texas Instruments TI-81 graphics calculator and the computer software A Graphic Approach to the Calculus (Tall, 1990) were integrated into the study of calculus and analytic geometry. The focus within these two units of study was upon the development of multiple representations of calculus concepts and improving the connections between these. The separate case reports of ten subjects and a multicase analysis of data, collected from three one-hour tasked-based interviews with each subject and a variety of test and class data, indicated that the majority of subjects constructed concept images that included most of the components which were identified as central to a strong concept image of the derivative. All subjects were found to have also gained proficiency with symbolic differentiation. At the same time, at the end of the study, a number of Ellison's subjects still had only a partially-formed understanding of the connections between derivatives, functions and graphs. It remains impossible to determine the precise role of the technologies in these observed developments, nor the influence of the instructional programme.

The growing interest in graphical approaches to calculus instruction as means to improve conceptual understanding accompanies and reinforces the movement away from manipulative skill-based approaches previously associated with calculus learning. When coupled with the increasing functionality of hand-held mathematics technology, the role of the graphics calculator in the teaching and learning of calculus assumes critical significance, and the demands for clear and informative research become evermore pressing.

3. Function Concepts

Studies by Devantier (1993), Alexander (1993), Martinez-Cruz (1993) and Rich (1991) deal directly with the impact of the graphics calculator on learning regarding functions and graphs. Not surprisingly at this point, their findings tend to be mixed. Some evidence suggests that use of the graphics calculator aids the development of a more global understanding of the features of functions, encouraging enhanced conceptual images of functions and understanding of the relationship between functions and graphs. Other evidence appears to demonstrate that its use can result in incomplete understandings of function concepts.

Devantier (1993) and Alexander (1993) report nothing but positive results in regard to graphics calculator use. Devantier was concerned primarily with the impact that the use of the graphics calculator had on precalculus students' understanding of the relationship between functions and their graphs. This was a comparative study of seven pre-calculus classes in mid-western United States, involving whole classes of experimental and control students. A test instrument designed specifically to examine understanding regarding the relationship between functions and graphs was administered as both a pre- and post-test. Analysis of results indicated that students with experience in using the graphics calculator scored significantly higher than students with no calculator experience.

Alexander, in contrast, investigated the success of a "technology-assisted instructional module" of study for college algebra. One of the aims in the development of this module was to enhance the instruction of functions using concrete visualisation in an attempt to raise the achievement levels of college students in this area of study. To this end, both computers and graphics calculators were used by the experimental group; the graphics calculator being used specifically as a visual aid in the study of functions. Sixty-eight students were equally divided between experimental and control groups. Tests designed to measure basic algebra knowledge and precollege and college algebra knowledge with a focus on functions, were used as both pre- and post-tests. Results indicated that the experimental group had a significantly better understanding of function concepts than the control group. These results appear to be strengthened by the fact that the testing instruments were not designed specifically for the study, and so were not designed specifically for the testing of technology-assisted instruction. It would probably be fair to say, then, that the test questions were those typically used within instruction which does not include extensive use of visual aids, and in particular, the investigation of functions using graphic means; in other words, those which were used for the teaching of the control group.

Rich (1991), however, failed to find a strong relationship between graphics calculator use and the development of conceptual knowledge of functions. Rich's purpose was to examine the ways in which the learning of students in precalculus courses, regarding function concepts and other related concepts, was affected by the use of the graphics calculator. Two classes were taught using a precalculus textbook designed for use with a Casio FX- 7000G graphics calculator, while three comparison classes were taught without the aid of graphics calculators. Following treatment for a full year, a "conventional" algebra post-test was given. On the basis of qualitative and quantitative analyses, Rich found no evidence that increases in achievement could be attributed to calculator use. Further, he found that students who had used graphics calculators tended to be weaker than students from the comparison classes on paper-and-pencil procedures for finding a slope and verifying trigonometric identities. Nevertheless, there was some indication that students taught using a graphics calculator did have a better understanding of the relationship between an algebraic function and its graph; comprehended that problems in algebra can be solved graphically as well as through algebraic manipulation; and tended to do more conjecturing and generalising. As with the results of Alexander's study above, one wonders whether the results of the post-test would have been substantially different if a portion of the questions within the test had been devised specifically for the study of algebra using graphical rather than conventional, or algebraic, means, and for the testing of conceptual understanding rather than procedural skill.

A short study by Chandler in 1993, at a suburban high school in Houston, U.S.A., to gather information on the mathematics achievement of high school students who use a graphics calculator in their mathematics studies, gives a little more insight into its uses in teaching function concepts. Her area of study was the transformation of functions and her subjects were the members of nine precalculus which were taught by three teachers. Five of these classes formed the experimental group, with the other four being the control group. Following the administration of a pre-test which tested knowledge of precalculus mathematics, the two groups were instructed on the transformation of functions, the experimental group using graphics calculators throughout the unit of study. On a post-test which assessed achievement on the transformation of parabolas, trigonometric functions and functions with no simple algebraic formula, the experimental group scored significantly higher than the control group, leading Chandler to conclude from this data that graphic visualisation of concepts and problems contributed to increased understanding and achievement in these areas of study.

In an attempt to learn more about the effect of graphing technology on precalculus students' concept images and concept definition of functions, the development of procedural and conceptual knowledge of functions, students' application of functions, and on the stages of development involved in gaining an understanding of the function concept, Martinez-Cruz (1993) followed the progress of eight high school students over a period of nine months. The use of various types of graphing technology, including graphics calculators, was an integral part of the precalculus course the eight students were studying so that they were thoroughly conversant with the graphing technology that they were required to use throughout the study. He reported that over this period these students did not develop a complete understanding of functions, with students demonstrating no consideration of the domain and range of functions, being unable to see the relationship between algebraic and graphical representations of a function, and tending not to search for the existence of irregularities. Students were able to demonstrate procedural understanding, but did not exhibit a consistent conception of functions, with one student perceiving functions as graphs, six students viewing them as equations, and one as a combination of equations and unique correspondence.

The addition of a control group may have added substantially to Martinez-Cruz's findings. Nevertheless, the fact that his subjects were students who had used graphing technology extensively prior to the study, tends to add weight to his conclusions. It is important to note, however, that Rich's (1991) findings, which resulted from a study of lengthy duration, contradict those of Martinez-Cruz, suggesting instead that as a result of graphics calculator use students demonstrated a clear understanding of the behaviour of functions as it relates to various aspects of the algebraic form. Interestingly, Rich's subjects, unlike those of Martinez-Cruz, had no prior experience with graphing technology in instruction. These findings, taken in conjunction with the conclusions of both Alexander (1993) and Chandler (1993), that concrete visualisation aids understanding and achievement in the area of functions, tend to indicate that there is much merit in the use of graphics calculators within the learning of functions. Once again, inconsistencies in the results are as likely to result from differences in instructional approaches and assessment procedures as from the calculators themselves.

4. Graphing Concepts

There appears to be general agreement among researchers that students' understanding of graphing concepts is markedly enhanced by the use of graphics calculators. This is especially the case concerning understanding of the connection between functions and graphs. There are indications, however, that there remain some aspects of graphs of functions that are not well-learned through types of instruction which are based on the use of the graphics calculator.

Rich (1991) noted, for example, in a study involving precalculus students, that subjects who used graphics calculators were better able to understand the connections between graphs and their respective functions and gave more consideration to the behaviour of graphs of functions such as their domain, asymptotic behaviour and end behaviour, than did students who did not use graphics calculators. The same study, however, failed to find that the use of the graphics calculator resulted in improvements in these students' learning of function concepts.

On the other hand, Shoaf-Grubbs (1993) found, in her study of female college students studying elementary algebra, that students' understanding of graphing concepts was clearly enhanced through the use of the graphics calculator. Similarly, studies by Boers- van Oosterum and Beckmann (1990, 1991, both cited in Dunham, 1993) appeared to illustrate that students who use graphics calculators, in comparison with those who do not, were better able to read and interpret graphs and obtain more information from them.

Ruthven (1990) was specifically interested in the influence of the graphics calculator on students' achievement and methods used in translating functions from graphic to symbolic forms. To investigate this he compared the test performance of upper-secondary advanced- level mathematics students who were thoroughly conversant with the use of the graphics calculator as a tool in mathematics, with that of students of similar background without regular access to graphing technology. The test consisted of two types of graphic items, twelve in all, drawn from topic areas central to British advanced-level courses and for which the use of graphs is normal practice: those which required an algebraic description of a cartesian graph ("symbolisation items") and items requiring the extraction of information from a graph which described some type of physical phenomenon (referred to as "interpretation items"). To avoid conferring a direct advantage upon the graphics calculator users, items were chosen so as to include only those for which there was no automatic graphics calculator procedure.

A sample consisting of 87 students in parallel classes from four schools was chosen, 47 of these making up the "project group" classes; that is to say, classes in which students were conversant with the facilities of the graphics calculator. The results clearly indicated superior performance by the calculator users over the comparison group on the symbolisation items, but superior performance by the comparison group on interpretation items. It was also found that calculator users tended to use the calculators' graphing facility to repeatedly modify a symbolic expression as a method of solution in preference to an "analytic-construction" approach or a "numeric-trial" approach. (The latter approach refers to formulating a symbolic conjecture, trialling this using a small number of coordinates and modifying, repeatedly if necessary. An "analytic-construction" approach, on the other hand, refers to altering a graph by placing some value before the variable(s) in its function.) Since this study does not attempt to investigate which of these three approaches, if any, demonstrates a more complete understanding of graphing concepts, it offers little to support claims regarding the relative effectiveness of graphics calculators over other approaches. It does, however, quite clearly inform their use, and suggests implications for versatile instructional approaches which may capitalise upon the strengths of a graphical approach.

In one of the few studies which carefully distinguished the role of the tool from that of the instructional process, Asp, Dowsey and Stacey (1993) also reported positive results with regard to graphics calculator use within a unit of work on linear and quadratic graphing with six Year 10 classes. Their study involved a comparison of the effects of the use of graphics calculators and the ANUGraph software package (Smythe and Ward, 1990) within this unit of study. Three of the classes were taught by two teachers using ANUGraph, while for the other three classes graphics calculators were used in instruction by the same two teachers and also by the students. The emphasis in both treatments was on the interpretation of graphs relating to real situations. Students' learning was found to have improved in most of the topics and statistically significant improvements (though smaller than expected) were observed for both treatment groups in interpreting graphs (with intersections, maximum or minimum values, etc.) and matching graph shape and algebraic form. In reading and plotting points and in drawing graphs, the calculator groups demonstrated greater improvement than computer groups. Nevertheless, Asp, Dowsey and Stacey added that data also showed that practising these concepts by hand was still essential. It is also worthy of note that post-test results and interviews with twelve of the students indicated that although students could demonstrate that they were able to use the graphing technology to which they had access, those using calculators experienced more difficulty than computer users in gaining facility in using the tool.

Conclusions reached by Giamati (1991) appear to illustrate that, in the study of graphing concepts, much care is still required in how and when the graphics calculator should be used. In her study, which investigated the effect of its use on students' understanding of variations in a family of functions and the transformations of the corresponding graphs, Giamati found that the learning of some graphing concepts appears not to be facilitated by the use of the graphics calculator. The study involved 126 students, of which 85% were African-American, for a five-and-a-half week unit on advanced graphing techniques. The focus was upon stretches and shrinks, reflections, translations, and forming reciprocals of functions. Graphics calculators were used to enable students to observe and analyse the effects of parameter changes on graphs of functions and relations. Analysis of open-ended mathematical questions, assessment scales by which responses were coded, interviews and anecdotal information, showed that at the conclusion of the unit the control group was better able to sketch functions, exhibit understanding of translations, stretches and shrinks, and describe parameter variations than were students who had access to graphics calculators. Possibly even more importantly, students who had partial or poor understanding of the relationship between graphs and equations were found to be "cognitively distracted" by also having to learn how to use the calculator's graphing facility. In addition, her data suggested, as did that of Asp, Dowsey, and Stacey (1993), that physically constructing tables for values of functions by hand was essential to the development of students' understanding of the relationship between graphs and equations.

Giamati (1991) concluded that use of the graphics calculator had not aided students' understanding of stretches, shrinks or translations. Students who were seen to have benefited from graphics calculator use were those who had begun the unit of work with a firm understanding of the relationship between graphs and equations. She noted, however, that unfamiliarity with certain characteristics of the calculator may have accounted, to some extent, for its lack of effectiveness, adding that this factor may have also affected initial student achievement.

Studies by Vasquez (1991) and Steele (1993) give further indications that the use of graphics calculators may not always have positive effects on the development of graphing skills for some groups of students. Vazquez investigated Year 8 students' acquisition of the skill of graphing linear functions. This study, involving 57 students, from two intact eighth grade classes, used a pre-test and a post-test which tested this skill, a card rotation test involving the rotation of shapes, and homework assignments. Steele also noted, upon the introduction of graphics calculators to Year 11 classes at a private boys' school in Victoria, Australia, a reluctance among students to investigate a complete graph of a function displayed on the calculator's screen, and a "blind acceptance" of function shapes which were unreasonable. These students had used the TI-81 graphics calculator intermittently throughout the year, chiefly for graphing and solving equations.

It is clear, then, from the findings of Giamati (1991), Vazquez (1991) and Steele (1993), that continual recourse to graphic displays to observe and examine graphs does not necessarily lead to enhanced understanding of how various algebraic components of functions are related to graphs. What were the aspects of these learning situations, then, which contributed to students' weak grasp of the particular graphing concepts focused upon in the respective courses of study? What of the role of the teacher? Were any of these aspects related to the use of graphics calculators? Or, on the other hand, did the use of this tool help to minimize the effects of weaknesses in these learning situations?

Asp, Dowsey and Stacey (1993) and Giamati (1991) stress that their evidence suggests that there is still a very definite need for the construction of tables of function values and argue from this that pencil-and-paper techniques still have a place in the learning of graphing concepts when graphics calculators are used as an integral part of the course of study. This suggestion appears to be further strengthened by Dinkheller's (1994) finding that precalculus students with whom an emphasis was placed on working with tables of values in the study of graphs of functions, scored significantly higher on a large number of test items than the control group for which this skill was not emphasised. It remains to be seen, however, whether the table of values facilities now offered by many graphics calculators constitute an effective replacement for the pencil-and-paper approaches assumed in these studies.

5. Modelling

Of the studies reviewed, only Alexander (1993) investigated the effect of the use of graphics calculators on the acquisition of mathematical modelling skills. This involved using a mathematical modelling component built into the technology-assisted instruction module described previously (refer to the section in this paper entitled "Function Concepts'). Analysis of the results of a mathematical modelling achievement test, used as a post-test, suggested that ready access to the graphs of functions, made possible through the use of graphics calculators, resulted in students gaining a significantly better understanding of modelling real-world problems when compared with others who did not have access to this technology.

As modelling real-world problems is very difficult for the majority of students (Sellke, Behr, & Voelker, 1991), these results are encouraging. Nevertheless, the findings of one study are not sufficient evidence on which to base changes to mathematics curricula in this area of study. Studies of longer duration with students of different ages, engaged in different levels of study, are needed to be able to draw definite conclusions concerning the potential for the graphics calculator as a tool for mathematical modelling. Such studies must delineate clearly between the effects of the tool use and those which result from the learning environment, including the instructional programme and the effects of the teacher.

6. Spatial Visualisation Ability

For all their limitations, there is significant consistency among the studies reviewed concerning the potential of the graphics calculator in facilitating the development of spatial visualisation skills. Results of Shoaf-Grubbs' (1992) study on the effect of the graphics calculator on levels of understanding and visual thinking of female students in an elementary college algebra course, for example, indicated that use of the graphics calculator enhanced the development of both general and specific spatial visualisation skills. Upon statistical analysis of pre- and post-test results, she found that the mean test gains were significant for the majority of students in the experimental group regarding both general and specific spatial skills. Shoaf-Grubbs noted that no such "positive momentum" in this type of development was evident in the control group.

The results of two other studies, Vazquez (1991) and Ruthven (1990), also indicated that use of graphics calculators can enhance students' development of spatial visualization skills. Vazquez reported that Year 8 students who were taught using a programme of study into which graphics calculator use was integrated, demonstrated statistically significant development in these skills in a unit on linear functions, and Ruthven found that there was a notable correlation between graphics calculator use and spatial visualisation skills in senior secondary students, especially among females.

In the studies by Shoaf-Grubbs (1993) and Ruthven (1990), statistically significant gains in test results accompanied students' development of spatial visualisation skills. Since several studies have found that development in these skills is positively correlated with measures of mathematics performance (McGee, 1979; Smith, 1964; Tartre, 1990) graphics calculators may also aid spatio-mathematical skills.

7. Use of the Graphics Calculator in Examinations

Of those reviewed, only one group of studies has been completed which attempts to discover whether use of the graphics calculator within a course of mathematical study has an impact upon examination performance. Boers and Jones (1992, 1992a, 1992b, 1993) investigated the examination performance of students of a semester-length course on introductory calculus for which graphics calculators had been used in instruction and learning. Upon analysis of 37 examination scripts, randomly chosen from the scripts of 274 students who had scored 15 marks or more out of 60 on the paper, Boers and Jones (1992) reported that “most students had difficulty integrating algebraically derived information with information they gained from the graphics calculator” (p. 392) especially with regard to discontinuities. One student, for example, labelled two correct algebraically derived stationary points of the function $\mathbf{y = x^{1-4}e^{-x}}$ on the graph of a function which has no stationary points, namely $\mathbf{y = x^{1.4}e^{x}}$, after mistakenly entering this latter function instead of $\mathbf{y = x^{1.4}e^{-x}}$.

Boers and Jones (1992, 1992a, 1993) suggested that graphics calculators are under- utilised in examinations. Their data (1992b, 1993) also suggested that males may be disadvantaged by the introduction of the graphics calculator in calculus examinations based on traditional techniques since, overall, females significantly outperformed males, this being the first examination for which this had happened in this course. Boers and Jones (1992a, 1992b) commented that allowing students to use graphics calculators in examinations tended to generate more information about students’ understanding of concepts than examinations in which calculator use is not allowed. For this reason they felt that allowing the use of graphics calculators in mathematics examinations following their use by students in the particular courses of study was worthwhile. Clearly, issues related to assessment involving the use of graphics calculators are critically important within the context of their increasing availability and use, and yet again this is an area of research which is significantly under-represented at present.

8. Errors Related to Graphics Calculator Use

Little research has been conducted on this subject, with only one study of those reviewed specifically concerned with calculator-associated misconceptions. Using an instructional environment based upon graphics calculator use, Tuska (1993) analysed the multiple-choice mid-term examination results of approximately 1000 students in an attempt to determine which errors, if any, were directly related to graphics calculator use. She identified eight such misconceptions. These errors were seen to fall into four distinct categories, three of which demonstrated incomplete understanding concerning the domain of a function, end behaviour and asymptotic behaviour of functions, and the solution of inequalities. The fourth was the apparent belief that every number is rational.

Errors related to the domain of functions arose from an apparent belief that a function must be defined at every point in every interval, and that the domain must necessarily be a subset of the range. Students’ misconceptions regarding the behaviour of functions stemmed directly from viewing functions within the window of a graphics calculator. These students believed that the “large viewing window” displayed enough of the graph of a function to be able to always determine the function’s behaviour at end points, and that a function always has a vertical asymptote at points at which it is not defined. Misconceptions regarding the solution of inequalities involved the idea that solving an inequality meant either “Find the zeros” or “Find the intersections or cutting points,” and that to solve an inequation graphically it is necessary only to find where the intersections of the two graphs appear to occur, leaving no perceived need to substitute values of x into the respective functions to ensure that an intersection does indeed occur at that point on the graphs. As part of the same study, Tuska (1993) also attempted to discover whether it is possible to overcome such calculator-associated misconceptions by the use of examples and nonexamples. To do this, three worksheets were developed, on inequalities, vertical asymptotes, and end behaviour. Following field testing with 130 students, the worksheets were used in interviews with four of these students. However, only some of their misconceptions were overcome using this method for remediation. While the clarity and usefulness of these findings may be appealing, there remain some questions concerning the ability to deduce such detailed conclusions concerning student beliefs and misconceptions from multiple-choice responses. Such conclusions would be more consistently associated with student interview data, to which a sample of only four offers little weight. As a consequence, the results remain unconvincing, and must be treated with caution.

Some of the misconceptions described above do coincide with Steele’s (1993) comments on the ready acceptance by students of whatever was displayed on the window of the graphics calculator (refer to the section in this paper entitled “Graphing Concepts”). This has also been noted with students’ use of scientific calculators (Bobis, 1991; Schoen, 1987), and highlights the need to ensure that students’ conceptual understanding of subject matter taught using these tools is strong. Nevertheless, many studies cited in this review suggest that students who use graphics calculators in their mathematical studies may develop a strong conceptual base of understanding, and that in some circumstances their level of understanding is significantly stronger than students who do not use graphics calculators. To ensure that the use of this tool benefits students’ learning it remains important to identify factors associated with graphics calculator use which can inhibit or enhance students’ development of conceptual understanding.

9. Gender Differences

A number of studies which investigate the impact of graphics calculator use on students’ understandings and achievement in specific areas of mathematics have also sought to determine whether, in these areas, there are differences between males and females in understanding and achievement which could be related to graphics calculator use.

A well-known study in this area is that by Dunham (1991a), whose research was primarily concerned with gender differences. Her aim was to determine whether there is evidence of any gender differences in mathematical confidence and performance which relate directly to the use of the graphics calculator. The confidence and performance of 213 college students on specific algebraic and graphical tasks were examined during the first and last weeks of a ten-week precalculus course taught with the aid of the graphics calculator. Confidence and performance scores were collected using a 24-item Mathematical Confidence scale developed by Dunham, with items being divided into visual and algebraic subscales. The results of the pre- and post-tests were analysed statistically. She reported a strong correlation between sex and type of problem on both tests. On the post-test both sexes improved significantly in both performance and confidence on the visual and algebraic scales. Initially, no difference between males and females was found in performance on visual items, but males exhibited more confidence than females on these items on the pre- test. On the post-test, however, males demonstrated superior performance on visual items while for females, significantly higher positive correlations were found between task-specific confidence and performance. Once again, however, poor design fails to allow discrimination between effects of the use of the graphics calculators and effects which may be attributable to other factors related to instruction and learning environment.

More useful information was derived from interview data. For information concerning shifts in technology-user patterns, eight high- and eight low-confidence subjects were interviewed. Dunham (1991a) found that low-confidence females relied more heavily on graphics calculators and used algebraic approaches less than any other group, while high- confidence females were more likely than any other group to choose an algebraic approach and less likely to use a graph to solve a problem. Low-confidence males also tended to use the graphics calculator more than other males but did not rely upon them to the extent to which low-confidence females relied upon them. High-confidence males used graphing and algebraic methods of solution almost equally, and were the most likely group to mix methods in a single solution. Dunham noted that a “surprising number” of males and females, especially high-confidence females, felt that they relied too much on “easy” calculator solutions and would gain more benefit from learning algebraic techniques.

Ruthven’s (1990) study of the influence of graphics calculator use on the mathematical performance of upper secondary school students revealed a rather surprising result in relation to differences between males and females. He found that performance of upper secondary female students who used graphics calculators was clearly superior to their male counterparts on items which required visual-spatial abilities. This result is exceptional in view of substantial evidence that males at upper secondary level tend to give superior performance on tasks which require these abilities (Bishop, 1983; Burden, & Coulson, 1981; Fennema, 1979). The fact that the test performance of Ruthven’s control group revealed superior performance by males, a result that agrees with the conclusions of this evidence, tends to strengthen his findings.

Ruthven attributes the exceptional experimental-group result to the extent of exposure his subjects had previously experienced to symbolised graphic images through the regular use of a graphics calculator. He maintains that since it is also well documented that upper secondary females tend to display more anxiety and less confidence than their male counterparts under conditions of uncertainty (Fennema, 1979), this regular use is likely to result in improved performance by female students. It is therefore interesting to note that Vazquez (1991) reported no apparent differences between males and females in mathematics achievement regarding spatial visualisation skills related to the graphing of linear functions. In the same study he did, however, report statistically significant gains in these skills for all students who received the graphics calculator enhanced instruction.

In distinguishing the studies of Ruthven and Vazquez, it is notable that only Ruthven's experimental subjects regularly used graphics calculators prior to his study. This was not the case with Vazquez' subjects. An important consideration in examining the differences between males' and females' spatial visualisation abilities must also involve determining their relative performance on various types of spatial visualisation items. Research indicates that visual-spatial tasks may be categorised into several types. Burden and Coulson (1981), for example, group such tasks into six distinct categories. Ruthven andVazquez, however, have each used only one type in their tests. Items included in Ruthven's testing instrument involved describing and interpreting curves which represented graphs of functions and natural phenomena, while Vazquez was concerned with testing students' ability concerning the rotation of shapes. Comparisons between the conclusions reached by Ruthven and Vazquez and the results of research which directly investigates males' and females' spatial visualisation skills on tasks closely related to those of Ruthven and Vazquez (for example, Burden, & Coulson, 1981; Fennema, 1979; Fennema, 1975) confirm, however, that the conclusions of these two researchers is atypical.

In examining the strategies that male and female college students had used in an examination in which use of graphics calculators was permitted, Boers & Jones (1993) found that more males than females were successful on questions that required the integration of graphical and algebraic information, with slightly more females preferring the algebraic strategy of solving a problem when there was an alternative method of solution. In comparison, slightly more males preferred graphical strategies. Boers and Jones (1993) also found that females significantly outperformed males in the examination, with females' better performances being related to more success on algebraic questions. In view of these findings it is worthy of note that Boers and Jones (1991, 1992b) reported that female students had more difficulties with the introduction of the graphics calculator than males. It is also notable that even when females exhibited confidence in their mathematical studies and in their use of graphics calculators, males tended to use the tool more flexibly (Boers & Jones, 1992b). Nevertheless, this result seems to be reasonable when considered in relation to Dunham's (1991a) report that high-confidence females' were found to be reluctant to rely on graphics calculators and preferred using algebraic approaches.

To determine whether low-confidence students' dependence on graphics calculators is to their advantage it would be worthwhile to compare their levels of achievement before and after the introduction of the graphics calculator. Is their reliance upon the graphics calculator accompanied by higher levels of understanding and achievement than would otherwise be the case?

Differences in males' and females' attitudes towards the use of the graphics calculator in mathematics have also been found. These are discussed in the section entitled "Student Attitudes Toward the Graphics Calculator as a Tool for Teaching and Learning."

10. Student Attitudes Toward Mathematics as Related to Calculator Use

Although Alexander (1993) and Thomasson (1993) report no statistically significant change in the overall group attitude towards mathematics for students studying college algebra, a number of researchers do note some degree of positive change in attitudes towards mathematics following the introduction of graphics calculators into mathematics units of study and courses. These changes in attitude appear to be a response to changes in modes of instruction and teaching style which have accompanied graphics calculator use.

Army (1992), for example, reported that students had an enhanced image of mathematics as a useful study, following the trialling of an approach which linked mathematical content to real-world applications in a college course in trigonometry into which incorporated the use of graphics calculators. Students perceived the value of applications in learning mathematical concepts and in realistic problem-solving situations.

In a study involving a low-achieving class of Year 11 students within a suburban school near Lisbon, Portugal, da Ponte (1993) investigated specifically students' views and attitudes concerning the classroom use of graphics calculators. The students used graphics calculators regularly for a full academic year in mathematics classes and for the completion of homework assignments, as well as using them freely in tests. The class teacher had previously completed a full-year inservice programme focusing on the use of technology in teaching, and had already trialed, with a Year 11 class in the year previous to this study, many of the activities designed specifically for graphics calculator use which were used in the study.

Data on students' views and attitudes was collected by means of classroom observations, interviews with teacher and students, and a questionnaire designed by the class teacher. da Ponte (1993) reported that students' attitudes become more favourable towards the study of mathematics as a result of the treatment. He concluded, however, that this change in attitude was a direct result of the change in teaching approach which accompanied the introduction of graphics calculators, rather than the use of the calculators per se, since changes in instruction tended to be innovative, involving more exploration as well as more student activity.

Tolias (1993) and Estes (1990) also linked college students' changes in attitudes towards their studies in mathematics to alterations in mode of instruction. The applied calculus students involved in Estes' study indicated that they liked the interaction between algebraic, graphic and tabular viewpoints and would have liked to learn college algebra using this approach. Tolias claims that students' positive change in attitudes regarding graphs and graphical procedures were a direct result of the mode of instruction and the introduction of the graphics calculator. They felt that the decision to introduce this type of instruction had facilitated the students' understanding of the relationship between algebraic and graphical methods for solving equations and inequalities.

11. Students' Attitudes Toward the Graphics Calculator as a Tool for Teaching and Learning

Students' attitudes have been found to be generally very positive toward the graphics calculator as a tool for teaching and learning mathematics. By means of a survey which was completed by students in April and again in September, 1991, a majority of applied science students at Swinburne University of Technology, Victoria, Australia, indicated that they felt more positively towards their study of mathematics as a result of the introduction of the TI-81 graphics calculator into the first year calculus course in which they were enrolled (Boers, & Jones, 1992a, 1992b). Also as a result of this survey, it was found that students tended to feel that the introduction of this tool had caused them to adopt mathematical behaviour which is considered to aid learning: specifically, using more exploration in the solution of problems, using graphs as an aid to solving problems, and using the graphics calculator to check algebraic solutions.

In the same study, as a further means of obtaining feedback from the students, a "brainstorm" session was held. During this session, students were encouraged to discuss both the positive and negative aspects of learning mathematics with the TI-81. As a result, students formed a list of 15 benefits that they considered important. The five benefits that the students felt were most important were: the ease of sketching and obtaining information from graphs; being able to check quickly the correctness of derivatives, integrals and solutions; being able to understand and interpret graphs and derivatives more easily; the ease of calculation and checking procedures regarding difficult formulae; and that the use of the tool increased confidence and enthusiasm. Five concerns about learning mathematics with the graphics calculator were also raised. The concern which was thought to be most important by the students was the possibility of becoming dependent upon the tool, since they had noticed in themselves a tendency to rely solely upon the use of it:

Dunham (1991a) also reported that college students involved in a precalculus course into which graphics calculators were fully integrated in instruction and learning, viewed graphics calculators as “fast, efficient, reliable tools.” Nevertheless, in the same study she also noted that many students, especially females, were concerned about relying on the calculator, feeling that it was important to learn and be able to use algebraic techniques as means of solution as well. This finding appears to agree with that of Boers and Jones (1992a, 1992b), who pointed out that although college females’ high level of examination performance, in comparison with males, seemed to be associated with graphics calculator use, they were less favourably inclined than males to the introduction of this technology. Of course, if the nature of the curriculum demands proficiency in algebraic skill, then such concerns may be seen as justifiable and the issue of assessment demands again becomes significant.

Findings of Dinkheller (1994) appear to be at variance to those of Boers and Jones on the same issue, however. Dinkheller reported that undergraduate females experienced no more anxiety than their male counterparts upon having to learn how to use this technology. There were, in fact, no significant differences found on either the pre- or post-tests designed to measure anxiety. As might have been expected, Dinkheller found a significant reduction in anxiety among both males and females from the beginning of the course to the end, but, again, no differences between males and females were found. Nevertheless, other factors may account for these differences; for example, level of learning, and nature of course of study (Boers and Jones’ subjects were Applied Science students studying calculus while Dinkheller’s study involved students in a precalculus course).

Most of the applied calculus students involved in Estes’ (1990) study were concerned only that they understood how to use the graphics calculator before having to use it in mathematics courses. Apart from this concern, they firmly believed that the technology was helpful to their learning. These students had also been exposed to computer technology as an instructional tool during the same course of study, and it is interesting to note that they indicated a preference for using graphics calculators over computer demonstrations. Improvements in attitude toward the use of graphics calculators were also found by Vazquez (1991), who reported an investigation in which students were taught a unit on linear functions with the aid of this technology.

12. Students have persistently voiced one main concern regarding graphics calculator use; specifically the potential of the calculator use to lead to deskilling (Dunham, 1991a; Boers & Jones, 1992a, 1992b; Ruthven, 1992, de Ponte, 1993). Boers and Jones’ (1992b) findings confirm that this can indeed occur, and it seems to be especially likely with male students despite the fact that it was principally the female students who were concerned about this. Boers and Jones (1992b) also noted that students with weak mathematical backgrounds reported mixed reactions to the use of graphics calculators, with none regarding the graphics calculator as a major factor in helping them to bridge the gap between secondary and tertiary mathematics.

Issues of Pedagogy

As pointed out previously, the relationship of the use of the graphics calculator to issues of pedagogy is of central concern in interpreting the results of research. The tool itself, like any tool, is meaningless in isolation from its use, and such use must involve a learning context with its myriad of associated (and frequently uncontrolled) variables. Those studies which purported to investigate curricular issues (especially the learning of calculus and aspects of precalculus studies reviewed above), have been seen within this review to be, instead, studies of pedagogy, as the use of the tool remained intertwined with the effect of the learning context. Of more benefit, then, may be those studies which directly attempt to address the issues of graphics calculator use within particular learning environments.

Studies examined for this review were found to focus upon either the effects of different teaching styles on graphics calculator usage, or the effectiveness of graphics calculators when they are integrated with various teaching approaches. Emese (1993), for example, investigated the effects of graphics calculator use when integrated with a guided discovery style of teaching (a lecture/discussion instructional technique) within a university differential calculus course. She compared this with the same approach without the use of graphics calculators, and also with traditional instruction. In the two variations of the guided discovery approach, worksheets were used to guide discovery. These consisted of sequences of questions and problems which were designed to lead students to discover new concepts and an understanding of relationships or techniques. A pre-test/post-test design was used, the post-test being the final examination for the course. Upon analysis of pre- and post-test results, Emese found no significant differences in student achievement between students participating in the three types of instruction. However, students who had been taught using the guided discovery style of instruction with the aid of graphics calculators indicated on a questionnaire that they were in favour of this style of learning.

Jost (1992), on the other hand, examined teacher beliefs, practice and curriculum requirements relating to the introduction of the graphics calculator in the calculus classroom, through comparative case studies. Information was collected through in-depth interviews with teachers who were involved in implementing a new curriculum which included the use of graphics calculators in teaching calculus. This was supplemented with fieldnotes from classroom observations, workshops and seminars. The data suggested that certain teaching styles are more compatible with graphics calculator use than others. Jost found that teachers who tended to employ interactive or inquiry-oriented methodologies used the calculators during instruction more than teachers who used other teaching approaches. She also reported that there was a correlation between specific perceptions concerning the use of graphics calculators and specific teaching approaches. Teachers who perceived the graphics calculator as a computational tool tended to stress content-oriented goals and viewed learning as listening. Teachers who saw it as an instructional tool had student-centred goals and discipline goals, interactive inquiry-driven teaching styles and student-centred views on learning. It seems that an important concern regarding this research design, as with that of Emese (1993), is whether it is possible to determine the extent of learning from three very different teaching approaches by using the one style of test.

A study by Strait (1993) also compared the effectiveness of two teaching strategies which included use of the graphics calculator. These were deductive and inductive approaches used in teaching a unit on functions and analytic geometry as part of a college algebra course. The deductive approach involved a sequence of “rule, examples, practice” while the inductive teaching strategy involved a “example, rule, practice” sequence. Two classes, with a total of 50 students, participated in the study. Results of a pre-and post-test were analysed to compare students’ procedural skills, conceptual understanding and factual knowledge. Strait found that there were no significant differences between the two groups of students in procedural skill development or in their conceptual understanding, but that students taught using the deductive teaching strategy demonstrated higher levels of factual knowledge. However, in the absence of a study to determine whether similar results are obtained using these strategies without graphics calculator use, it is unclear whether this has any influence on the relative effectiveness of either strategy. These results may very well be a direct reflection of teaching strategy only. Moreover, as with the studies by Emese

(1993) and Jost (1992), it may be of value to conduct this type of study using three post- tests, each designed specifically to test one teaching approach, as testing instruments for all subjects, instead of having just the one post-test. It may well be that students achieve better results on a test which reflects the teaching approach to which they have been exposed; or, that there is one of the three types of test on which subjects from all three treatments do better.

Army (1992) found that use of the graphics calculator in instruction enables more rapid illustration of the basic properties of graphs of trigonometric functions. Rich (1991) noted, too, that teachers in her study who taught a precalculus college algebra course with the aid of graphics calculators, asked more higher-order questions, used examples differently, and stressed the importance of graphs and approximation in problem-solving to a greater extent than teachers who taught the same course without the use of graphics calculators. Graphics calculator use was also found to encourage students to do more conjecturing and generalising. (Rich, 1991), and Army (1992) found that an increase in student interaction resulted from the use of this tool. Thus it would seem to be the case that using graphics calculators in the teaching and learning of mathematics is conducive to the adoption of student-centred teaching approaches.

In addition, it appears that a teaching approach which combines the use of graphics calculators with real-world applications tends to result in a shift in emphasis from algebraic manipulation and proof to graphical investigation and linking algebraic and geometric representations of problems (Army, 1992). This type of shift is evident in most of the studies cited in this review, even when real-world problem-solving is not a major focus.

The studies cited in this section have not reported any improvement in test performance associated with the adoption of different styles of teaching. Although Rich (1991) found evidence of a positive effect in students' learning of graphing concepts, no overall achievement gains were noted. Similarly, upon the implementation of an approach which emphasised real-world problem-solving in a college trigonometry course, Army's study (1992) on the effects of graphics calculator use within this course (reported in detail in the section in this paper entitled "Precalculus Studies") also concluded with no gains in achievement having been noted.

Nevertheless, since most of the studies in which the effects of graphics calculator use were investigated involved changes in pedagogy and some shift in emphasis, as noted above, and a number of these studies also provided evidence of significant improvements in learning, it would seem that variations to traditional teaching methods can positively affect learning. As yet, however, only Jost (1992) and Emese (1993) have attempted to investigate the relative effectiveness of various teaching approaches, with no significant differences being noted in their comparisons concerning positive learning effects. Further investigation into the effect of different teaching approaches, and aspects of these, used in conjunction with graphics calculators, is required to isolate factors concerning pedagogy which are associated with significant improvements in conceptual understanding and achievement in specific areas of mathematics.

It is also clear that positive changes in attitude associated with the pedagogical changes which accompany the use of graphics calculators do not necessarily result in significant improvement in learning. This is interesting to note since much emphasis has been placed on the importance of motivation to students' learning (Raffini, 1993; Reynolds, & Walberg, 1992).

It is evident, however, that the graphics calculator may serve as a catalyst, making possible the implementation of programs of study in mathematics which enable exploration and discovery. It would be worthwhile to further examine ways in which approaches which emphasise exploration and discovery can result in improvements in learning since graphics calculator use lends itself to these approaches, and it is clear from the evidence cited within this review that such a partnership is frequently associated with significant gains in conceptual understanding and achievement in mathematics.

CONCLUDING REMARKS AND SUGGESTIONS FOR FURTHER RESEARCH

The current state of research into the use and effects of graphics calculators, then, remains inconclusive. Few studies distinguish carefully between the use of the tool and the context of that use. Claims regarding the relative effectiveness of the tool are frequently based upon assessment procedures which equate “student learning” and “achievement” with performance upon traditional tests, and fail decisively to account for important influences upon attitudes and conceptual understanding. However, not all is lost. Certain studies, through careful design, stand out like beacons in informing our knowledge in this significant area. Even amongst the remainder, it is possible to identify recurring and consistent themes associated with the use of graphics calculators which may serve to guide our practice in the coming decade and, perhaps, beyond.

At both the precalculus and calculus levels, research results have consistently indicated that the use of graphics calculators can be associated with significant gains in students’ understanding of function and graphing concepts; in reading and interpreting graphical information; and, to some extent, in understanding the relationship between functions and their graphs, and algebraic and graphical methods for solving equations and inequalities. Graphics calculators have also been found to aid in the development of spatial visualisation skills and students’ understanding of mathematical modelling concepts. In order that students may gain the benefits of these findings, there is an urgent necessity to discover aspects of graphics calculator usage which best facilitate learning in each of these areas.

It is still unclear, however, whether the graphics calculator is an effective tool for developing understanding of transformations of functions, precalculus and calculus computational knowledge, and for helping students to integrate algebraically and graphically derived knowledge of functions and graphs. Also, we need to ask whether the shift in emphasis from algebraic manipulation and proof to graphical investigation, which tends to occur with the use of graphics calculators (Army, 1992), is desirable. The findings of longitudinal studies into this issue could have major implications for future education, especially as graphing and other computer technology becomes more accessible to educational institutions and students.

The results of a number of studies reviewed here suggest that its use may create difficulties for some students, especially those with weak conceptual links between algebraic and graphical knowledge. Evidence also suggests that, even though students’ attitudes are generally positive to the use of graphics calculators, a concern still remains, especially among females, that their use may result in some “deskilling.” This is a concern substantiated, to some extent, by Boers & Jones’ study (Boers & Jones, 1992b).

Investigation into the “deskilling” potential of the graphics calculator is vitally important, and consideration should be given to the question of whether students with differing mathematical ability should be taught differently, with different usage of these calculators. The effect of the use of graphics calculators on the nature of testing has also not been explored as yet. It seems that differences in instruction which are associated with graphics calculator use would call for test items, and possibly test emphases, which may vary considerably from those of traditional tests.

These issues raise many questions. For example, what are reliable ways of measuring the effects of the graphics calculator? What methods allow valid comparison between technology and non-technology users? What knowledge and understanding should be most highly valued, i.e. which are ultimately most useful in facilitating problem-solving skills and for further learning in mathematics and other related areas of study? Does the use of the graphics calculator enhance the development of problem-solving skills? If so, in what ways? Also, what kinds of achievement instruments should be used in research which seeks to compare the effectiveness of two or more methods or teaching approaches, or classes where graphics calculators are used and classes where they are not used? If these instruments were developed prior to the era of graphics calculators, or even just prior to the introduction of graphics calculators into a course of study which is used as a foundation for research, then they could very well be biased in favour of traditional methods and approaches. For example, when working with the graphics calculator, considerable attention needs to be given to working out appropriate scales on x and y axes. Also, there is significant emphasis upon the connections between symbolic and graphical representations, as well as the inspection of graphical solutions by “zooming in.” Are these skills reflected in the tests used?

An important issue raised by the introduction of the graphics calculator as a tool in mathematics education concerns the potential effects upon what is currently taught. Is there a need for the resequencing of topic items or entire topics? In addition, are changes in emphases within mathematics curricula needed, such as more concentration on graphing, analytic geometry, and real-world problem solving? It seems that the use of graphics calculators enables more exploration within situations which require complex computational manipulation, and it certainly allows wider choice of examples and provides the opportunity to ask interesting new questions (Barling, 1993). Arnold (1995) notes that, for teachers, a significant aspect of learning to use the new tools involves learning to ask new questions, as traditional questions frequently become trivial within the new technological context. How might this affect the content and emphases of topics such as calculus, precalculus algebra and statistics? With regard to this last area of study, Hackett and Kissane (1993) comment that the advent of the graphics calculator brings with it a wealth of new possibilities for the statistics curriculum, making the examination of teaching approaches, current emphases, examples and activities used in the study of statistics eminently desirable.

Demana, Schoen, and Waits (in Biehler, 1994) also emphasise the need for the rewriting of textbooks, asserting that current textbooks are deficient in tasks which are appropriate and often even essential in mathematical studies which incorporate the use of graphing technology. For example, an understanding of the effects of changes in scale is crucial to the successful use of graphing technology, and students need to understand that shape is an artefact of the scale used (Dunham & Osborne, 1991, Goldberg, 1991). Traditionally, textbooks contain examples of graphs for which scale is not important. Nevertheless, use of the graphics calculator and other graphing technology allows the investigation of graphs that are not as “nice”; those, for example, which have stationary points and asymptotes located outside the default screen of a graphics calculator, such as most cubic and quartic polynomials, rational functions, and trigonometric functions. With many of these graphs, symmetric scaling is impractical for investigation purposes, making it crucial that students have a thorough understanding of scale, especially in relation to the uses and effects of different scales on the x and y axes. However, most, if not all, examples of graphs given incurrent texts have symmetric scales.

These factors highlight the need to provide exercises and activities appropriate to instruction enhanced by graphing technology. The identification by Tuska (1993) of a number of misconceptions directly related to graphics calculator use, and others highlighted by Dunham and Osborne (1991), such as confusion of the idea of a geometric transformation with a scale change, further demonstrates this. is justified when a variety of computer packages are already available to individuals and schools which provide all the functions of graphics calculators and more besides (Small, & Hosack, 1991). In addition, these computer packages are generally devised so as to require much less memory work than does the successful operation of the graphics calculator. To use the graphics calculator effectively, one must learn the key sequences necessary to access the various graphics calculator functions. A partial knowledge of these tends to result in error. Added to this, various limitations of the tool can lead to error and possibly the development of misconceptions. For instance, the fact that the screen of a graphics calculator is small means that the default screen can generate incomplete or confusing graphs so that it may be necessary to view several screens to obtain a complete picture of the graph. In addition, singularities do not always appear on a graph, and in evaluating the limit of some expressions by graphing the function, “round-off errors” for sufficiently large or small values of x, lead to incorrect conclusions (Dion, 1990). Such issues become moot as the distinctions between desktop computer and graphics calculator blur. As the technology advances, early problems associated with the use of graphics calculators (and, indeed, desktop computer packages) are minimised, as the tools become increasingly affordable and appropriate for use as tools for learning.

The main advantages of graphics calculators over desktop computers remain as portability and price. Graphics calculators can be easily transported, and the price of computers and the lack of available space within classrooms prohibit their use within regular mathematics classes, requiring the use, instead, of computer laboratories. In Australia, at present, there are generally only one or two of these within any secondary school, making daily access to computers impossible (Zammit, 1992). As a consequence, as more teachers become interested in using them with their students, it becomes increasingly difficult to obtain regular access. This also seems to be the case elsewhere (ten Brummelhuis & Plomp, 1994). This situation is clearly unsatisfactory. At present, then, desktop computer technology remains a less practical alternative to the use of the graphics calculator as a tool in the teaching and learning of mathematics.

It is one thing, however, to use a tool, but quite another to use it effectively. Teachers who know little or nothing of graphics calculators will most likely be reluctant to use them, and certainly will not be able to utilize their full potential. For this reason, teacher education in the use of this technology is of paramount importance. Perhaps this will be the determining factor in whether the graphics calculator is the means by which new ways to teach mathematics and to extend the mathematical experiences of students are opened up, or whether it is relegated to being just another of the many mathematical tools available for use. The potential, however, is clear. Approaches to teaching and learning which emphasise problem solving, exploration and within which students actively construct and negotiate meaning for the mathematics they encounter, find in this new technology a natural and mathematically powerful partner.

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