# An Action Research Evaluation of a Computer Enhanced Senior Secondary Mathematics Curriculum

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Stephen Arnold
B.Sc.,Dip.Ed.,M.A.

(School Of Learning Studies)
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# Abstract

At a time when schools face increasing pressures of accountability in both the economic and educational senses, there is a growing need for teachers to assume the role of evaluators of their own work. This study demonstrates that such a role can serve to fulfil both the specific demands of the teacher and school in assessing the worth of a programme or innovation, and also the wider demands of the educational community, providing practical research data and results which may contribute to the improvement of teaching practice. The study investigates the use of hand-held calculators capable of graphics, calculus and symbolic manipulation as a means of enhancing the teaching of Mathematics at the Senior Secondary level. The results indicate that such tools can bring about improvements in concept understanding, attitudes towards the subject and confidence in students' abilities in this regard.

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# Chapter One

# Introduction

*"An attempt is needed to integrate action and evaluation into a unified research model. The key is the role of teacher as researcher. It is not enough that teachers' work be studied : they need to study it themselves."*

[Stenhouse, 1975; 141, 143]

Fifteen years later, the challenge of Lawrence Stenhouse to teachers to adopt a research mentality towards their work - in his words, an "extended professionalism" - has lost none of its original vigour, nor its relevance to the continued advancement of education. Nonetheless, teachers at all levels may well be having change thrust upon them rather than attempting to anticipate its direction. Innovations are occurring in schools everywhere, but the knowledge of teachers in general concerning theories of curriculum, and their skills in evaluating the impact and effectiveness of these innovations, are open to question. The role of case studies in this context is an important one, since it is an opportunity for teachers to observe their colleagues, their successes and failures, and apply this experience to their own situation.

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## 1.1 Evaluation or Research?

The distinction between Curriculum Evaluation and Curriculum Research has become somewhat contentious in recent years. While writers such as MacDonald [1976] in the seventies had little trouble telling them apart - *"Evaluation is distinguished from educational research by its political dimension"* [in Fraser, 1986; 101] - this was in the main part because of the large scale nature of the majority of the evaluation projects of that period. On a smaller scale, such distinctions tend to become arbitrary. The growth of Action Research methods, the more widespread embrace of ethnographic and other qualitative approaches to curriculum research, and the development of the role of the teacher as curriculum developer and evaluator have all had the effect of blurring the lines which have divided the two fields in the past.

Writers such as Stenhouse [1975] and, more recently, Scriven [1983] have, in fact, embraced the notion that each should be seen as, not only complementary, but as necessary counterpoints to the other. Scriven's paper, **"Evaluation as a Paradigm for Educational Research"** [1983], although tending towards the theoretical rather than the practical, clearly seeks to lay the groundwork for a position in which one of the purposes of Evaluation should be seen as contributing to the improvement of teaching and the advancement of educational practice. As early as 1967, Robert Stake foreshadowed the notions of teacher-as-evaluator and teacher-as-researcher when he described the purpose of evaluation as a means for the teacher *". . . to understand better his own own teaching and to contribute more to the science of teaching."* [Stake, 1967; 523] Stake went on to outline a model

which he termed the "Countenance Model" of evaluation and which, with some adjustments, has formed the basis for the evaluation which follows. It is felt to be in sympathy with the philosophy of qualitative research methodology, which looks for the answers to educational questions to be found in the observation of classroom practice.

Such a stand raises questions as to the very nature of research itself. Is research to be defined by its practice or by its outcomes? As an evaluation, the following project utilises methods and generates data which are little different from those of more traditional research activities - are we justified in describing the endeavour as "research" as well as "evaluation"? It would appear that practice does little to distinguish the two activities. It is more likely that the difference is to be found in the outcomes, in the sense that research generates theory which may be used to inform and guide future practice in other situations. The use to which the results are put is surely the distinguishing feature - evaluation serving short-term goals as determined by the relevant audience(s), research seeking to serve longer-term goals, in terms of providing a body of theory by which practice may be understood and directed. In seeking to attend to both these goals, then, the current study may justifiably lay claim to both titles, sharing as it does the concerns of both research and evaluation.

Thus, it is expected that internal evaluations such as this will help teachers:

(i) to judge the worth of specific curriculum innovations with which they are involved, and

(ii) to better understand what they are doing beyond the immediate situation, helping them to build concepts and relationships which will inform future practice.

## 1.2 The Teacher as Evaluator

At a time in Australian education when schools and the teachers within them are becoming increasingly accountable in both the economic and social senses (as documented in the publications of the Australian Curriculum Studies Association, *Curriculum Concerns* and *Curriculum Perspectives* , over the past decade), two critical issues are perceived :

(i) research must continue to be devoted to the development of effective models by which the activities of classroom evaluation and research may be carried out by willing teachers (Davis, 1980; Hughes, Russell and McConachy, 1981; Skilbeck, 1984), and

(ii) the means by which teachers may be encouraged to accept the notion that such roles are an essential part of their responsibility must be explored. (Walker, 1988; McTaggert, 1989; Bezzina and Chesterton, 1989; Kirk, 1989)

While the Curriculum Development Centre (through their **"Teachers as Evaluators"** Project, Hughes, 1979; Davis, 1980) has been devoting considerable effort towards the former for the past decade, the evidence of experience at school level suggests strongly that their approach to the latter problem may have been less than successful. As noted by Russell [in Skilbeck, 1984; 245] much of the effect of the large-scale evaluation projects of the seventies has been ". . *the mystifying of*

*evaluation for the classroom practitioner. For teachers evaluation appeared to be something that somebody else did."*

The evaluation which follows, then, is an attempt to use Stake's (1967) Countenance model as the framework for an evaluation of a curriculum innovation in the Senior Secondary Mathematics course, in which computer technology is used to enhance the teaching and learning of selected topics. The research methodology is naturalistic in design, and makes use of the Action Research Model [Kemmis, 1982, 1985] as a means of gathering data which is systematic, collaborative and self-evaluative. These two models support and complement each other in the present study - action research techniques being applied in order to study the processes of the innovation in a cyclic fashion and so enhancing the formative component of the evaluation.

The inquiry seeks to describe and interpret the effects of this *". . small-scale intervention in the functioning of the real world."* [Cohen and Manion, 1985; 208] The evaluator is very much part of the picture which he seeks to evaluate, and so ethnographic techniques are used in an attempt to interpret and make meaningful the situation in which the evaluation occurs and to seek to understand the effects of the intervention upon the participants. In this way it is sought to faithfully represent the situation of the evaluation, and so to judge the effectiveness or otherwise of the implementation under discussion.

Stake's *Countenance Model* is best represented by a matrix arrangement which illustrates the major components required for the evaluation and the ways in which they relate to each other, as follows:

| | Description Matrix | | Judgement Matrix | |
|---|---|---|---|---|
| | INTENTS | OBSERVATIONS | STANDARDS | JUDGEMENTS |
| ANTECEDENTS | | | | |
| TRANSACTIONS | | | | |
| OUTCOMES | | | | |

Table 1.1 Stake's Countenance Model of Evaluation (1967)

It is proposed in this evaluation that a cyclic relationship be set up at the Transactional level between Intents and Observations, as suggested by the Action Research Model, and that the situation be studied through a cycle of successive changes, rather than evaluating on the basis of a "still-life", a single Transaction.

Further the evaluation is to be collaborative and democratic, as suggested by the Action Research tradition (Kemmis *et al*, 1985), involving the students of the sample group at each stage of the process in the roles of co-researchers and participants, rather than the more traditional role of subjects. The sample group here consists of the thirteen students (six females, seven males) studying Three Unit Mathematics in Year 12 at Saint Joseph's Regional High School, Albion Park in 1989. The study occupied the 1989 school year.

## 1.3 Rationale

In approaching any evaluation, it is necessary to identify some area or issue which is seen to be problematic. The Aim of the Three Unit course

in Mathematics for the New South Wales Higher School Certificate as stated is to give students a

*". . thorough understanding of, and competence in, aspects of mathematics including many which are applicable to the real world."*

[B.S.S.S., 1982; 1]

Specific objectives of the course are :

(a) to give an understanding of important mathematical ideas such as variable, function, limit, etc., and to introduce students to mathematical techniques which are relevant to the real world.

(b) to understand the need to prove results, to appreciate the role of deductive reasoning in establishing such proofs, and to develop the ability to construct such proofs.

(c) to enhance those mathematical skills required for further studies in mathematics, the physical and the technological sciences.

Assessment for the award of the Higher School Certificate is prescribed in terms of two components, labelled "A" and "B". The former tests the student's knowledge, understanding and skills in each content area listed in the syllabus. It is this component which forms the basis for the Higher School Certificate examination. The latter is primarily concerned with the student's reasoning, interpretative, explanatory and communicative abilities. Component B is allotted to schools to assess in a variety of ways which are intended to be broader and more flexible than those possible in a formal examination. It is the discrepancy between the success of many students on the former type of task (indicative of a mastery of manipulative skills) and their

difficulties on the latter which suggests that a problem exists and is worthy of investigation.

This discrepancy has been observed in the students at Saint Joseph's over the four years since the introduction of this school-based assessment component in New South Wales, and appears from discussion with teachers at other schools to be generally widespread. A strong criticism which followed the introduction of this scheme in 1986 concerned the lack of preparation of teachers who were expected to implement these new assessment procedures. The departure from traditional types of mathematics assessment tasks saw many teachers forced to recognise aspects of the learning process which (at least in Mathematics) had all too often been overlooked. The Cockroft Report (1980) had heralded a new view of Mathematics as **process** rather than content; this view was recognised by the new assessment procedures, but teachers themselves seemed slow and often reluctant to vary from the familiar approaches which had been hallowed by tradition.

In the growing body of literature concerning the enhancement of the teaching of mathematics using computer technology [e.g. Arnold, 1989; Clayton et al, 1988a and 1988b; Heid, 1988] there appears to be general agreement that such enhancement can help to bring about both process and outcome changes in Mathematics instruction. The former is observed in the creation of a classroom environment which encourages group work and discussion among peers, and where students are encouraged to investigate problems and explore concepts. The potential outcome changes include greater understanding of concepts and relationships, improved ability to communicate results and to

analyse their own reasoning, and improvements in attitude towards the subject and its applications. [Arnold, 1989; 26]

The primary focus of the study which follows lies in the introduction to the Senior Mathematics classroom of hand-held calculators capable of both symbolic manipulation and graphical representation of algebraic expressions. Such tools combine many of the advantages of computer technology described in the various references above :

- They allow students immediate feedback from quite complex questions involving algebra, calculus and curve sketching. This is not only satisfying and motivating for the students, it also allows for the exploration of topics and ideas which would be too time-consuming or difficult without such aids.

- Real-life situations may be more effectively explored using such aids, since such applications usually involve calculations too tedious and difficult to do manually.

- The concepts associated with some of the more difficult ideas of functions and calculus may be readily investigated, again freed from the manipulative constraints which otherwise frequently form a barrier to understanding.

- Since they are most often used in small groups, such calculators encourage discussion and verbalisation of concepts, as well as co-operation and group approaches to problem solving.

Such tools thus have the potential to be a means by which the enhancement of the curriculum described above may be achieved and so help to close the gap between the "ideal" and the "real" in the teaching and learning of Mathematics at this level. It is suggested that the appropriate use of such technology can help to bring about significant gains in the understanding by students of mathematical concepts and their applications to the real world, as well as improvements in communication skills and attitudes towards problem solving in Mathematics.

## 1.4 Conclusion

It is the purpose of the following project to explore the effects of the use of computer technology as an aid to the teaching of Mathematics in the senior years of secondary education. It is proposed that such enhancement may be effective in the following ways :

- Providing deeper understanding of the primary concepts and relationships between these concepts as opposed to skills of manipulation.
- Encouraging a positive attitude towards Mathematics, and the non-trivial role it is likely to play in the future lives and careers of students at this level.
- Contributing to an understanding of the nature of the subject and the breadth of its applications in society (particularly the relationship between Mathematics and computer technology).
- Increasing the ability of students to communicate answers effectively, to describe and analyse their own reasoning and

problem-solving processes, and to initiate investigations which are self-motivated

In particular, the research which follows arises as a response to three focus questions :

- How may I implement a particular curriculum innovation?
- How may I judge the effectiveness of this implementation?
- What may be generalised from such an evaluation concerning both the use of computer technology in the teaching of mathematics, and the roles of the classroom teacher as evaluator and researcher?

The Review of the Literature which follows attends to the three critical perspectives which define this project and are implicit in these focus questions, namely :

(1) Computer Technology and the Mathematics Curriculum

(2) Classroom teachers as evaluators of their own work, and

(3) Teachers as Researchers.

# Chapter Two

# Review of the Literature

## 2.1 Computer Mathematics and the Curriculum

The subject of the use of computer technology in the Mathematics classroom has been a popular one in the various Mathematics Education publications over the past decade both in Australia and overseas [Hansen and Zweng (1984), Hirsch (1985), Howson and Kahane (1986), Wilcox (1986), Hunting (1987) and Czernezkyj (1988)] with the emphasis changing over what may be viewed as three main stages over that period. The early thrust focused upon the use of computer programming in one of the high level languages such as FORTRAN, BASIC or, later, LOGO, as a tool for the teaching of Mathematics [see Wilcox (1986)]. With the advent of higher quality commercial software, the emphasis moved to this arena, although it was the open-ended software of the last ten years which has most stimulated writers - LOGO [Papert, 1980] led the way and opened the doors for business applications software such as word processors, spreadsheets and databases to be applied in educational settings. Of these, spreadsheets (especially those with graphics capabilities) imply perhaps the most promise in the field of Mathematics education, and

have been the subject of numerous books and articles suggesting ways in which they can be used effectively [Balla (1988), Briggs (1988), Czernezkyj (1988), Fifoot (1988) and Ryan (1987)].

Recent forms of open-ended software developed specifically for mathematical purposes have tended to fall into two main divisions :

(i) *Graph Plotters* [Atkins (1988), Barnes (1988), Clayton et al (1988a, 1988b), Czernezkyj (1988), Fifoot (1988) and Giles (1988)], and

(ii) *Symbolic Manipulation Software*, such as muMATH and REDUCE [Belward (1988), Heid (1988), Waits and Demana (1988), Ryan (1987a and 1987b), Hunting (1987), Usiskin (1985), Fey (1985), Ralston (1985), and Fey and Heid (1984)].

While the former are probably the easier to assimilate directly into the existing Senior mathematics classroom and syllabus, it is undoubtedly the latter which have caused the greater excitement in terms of their potential to bring about significant and long term change to the traditional secondary Mathematics curriculum. Much of the work of the American National Council of Teachers of Mathematics (N.C.T.M.) in the middle of the decade was centred on this theme, reflected in two Yearbooks - **Computers in Mathematics Education** [Hanson et al (1984)] and **The Secondary School Mathematics Curriculum** [Hirsch (1985)]. A similar message comes through from the 1986 Kuwait symposium of the International Commission on Mathematics Instruction (I.C.M.I.) [Howson and Kahane (1986)].

Combining the major capabilities of both of the above types of open-ended software are hand-held calculators such as the Hewlett-Packard HP-28C, which provide the major focus for the "computer" enhancement of the senior Mathematics course outlined in the current study. Available in Australia only since late 1987, such mathematical tools have already aroused considerable interest both here and overseas [Arnold (1989), Belward (1988), Waits and Demana (1988), Wheal (1988) and Ryan (1987a, 1987b)]. References in this regard tend to be largely descriptive in nature; the current project appears to provide the first systematic research into the implementation of such devices into the Mathematics classroom in this country.

References such as those by Heid (1988) on the resequencing of skills and concepts with the use of computer enhanced teaching methods (in this case, muMATH), Clayton et al (1988a, 1988b), Atkins (1988) and Giles (1988) on the use of graph plotters, and Simpson (1988) on the use of structured and open-ended worksheets with computers in Mathematics teaching, have all proven useful in providing a guide as to how the HP-28C calculators can best be utilised in the senior Mathematics classroom. A further central point concerns the importance of learning to estimate in the process of concept development. As noted in the preface to the N.C.T.M. 1986 Yearbook, **Estimation and Mental Computation**,

> *" . . . an important by-product of learning to estimate is better conceptual understanding and, conversely, concepts must be understood in order to acquire the flexible set of processes and decision-making rules needed by the proficient estimator."*
>
> [Schoen and Zweng, 1986; vii]

These principles are as applicable to function and curve sketching skills as to numerical estimation.

The articles by Waits and Demana (1988), Steen (1986), Ralston (1985), Usiskin (1985) and Fey and Heid (1984) have served to provide the curricular overview needed to see these innovations in perspective in terms of both their immediate and their potential impact upon the nature and direction of changes to the Secondary Mathematics curriculum. Such writers herald long term changes, not only to the way mathematics may be taught using such technology, but to perceptions about the subject itself. Certainly it is anticipated that, at senior levels, the increased use of such technological aids will force a shift away from the current emphasis upon skills of algebraic manipulation and towards improved concept development and understanding. Parallels may be drawn with the effect upon the primary school mathematics curriculum brought about by the widespread introduction of hand-held calculators in the seventies and early eighties.

## 2.2 How to Evaluate ?

The primary purpose of the following project is the evaluation of the implementation of the curriculum innovation outlined above. As an internal evaluator, it was possible to take advantage of the work done by the Curriculum Development Centre over the past decade in their Teachers as Evaluators Project [Hughes et al (1979 and 1981), Davis (1980)] as well as the work of those involved more generally in school-based curriculum development [especially Evans, Ruddick and Russell in Skilbeck (1984), Dekkers et al (1984) and Price et al (1981)].

Michael Scriven, in his influential early work in the field of Curriculum Evaluation [Scriven, 1967], required the evaluator to distinguish clearly between the goals and roles of evaluation. While the essential task (the goal) of the evaluation is the gathering of information and this may in some senses be considered "value-neutral", it is true too that such information may be utilised in various ways (these being the roles of the evaluation). If we accept as a suitable goal that the task of evaluation is ". *.delineating, obtaining and providing information for judging between decision alternatives*" [Stufflebeam, 1971 in Hughes, 1981; 9] then the teacher as evaluator must yet be clear as to the role the evaluation is to play. Hughes *et al* (1981) in an overview of the approaches of the various State Education Departments in Australia towards this question note that

> "*Policy statements identify two major purposes of evaluation : To improve curriculum offerings and to provide information for accountability.*" [Hughes et al, 1981, 12]

While these two features are undoubtedly key elements in this delineation of the roles of curriculum evaluation, it is necessary to add a third : that of contributing to Educational Research. Stenhouse (1975) saw Evaluation as the linchpin of his proposed Research Model of Curriculum Development (explored further below) while more recently, Scriven has attempted to formalise the relationship in his 1983 paper. This study builds on the idea that the processes of curriculum evaluation provide an effective research methodology.

The particular problems with which the teacher as evaluator must come to terms include those of time, bias, error and ethical considerations [after Davis, 1980]. Time is the perennial problem both for the

evaluator and those others involved, especially since small-scale projects are unlikely to attract time release; unfortunately, no simple solutions exist for this problem. Bias and error may be minimised by collaboration (inviting colleagues to assist, and even to fulfil roles of "critical friend" or "adversary") and triangulation - obtaining data from various perspectives, and identifying the concerns and priorities of all those involved. These are the means by which the teacher may test the validity and reliability of the findings of the research. [Davis, 1980; 56-59]

The other critical factors of which the internal evaluator must take account are those of confidentiality and ethics. Since the teacher must continue to work with those affected by the evaluation, and seek their co-operation in the process, it is essential that the value stance which the evaluator brings to the process, and the criteria by which worth is to be judged, be made explicit at the outset. The ownership of the data must also be addressed. [Davis, 1980; 59-63; Walker, 1988]

## 2.3 A Model for the Teacher as Evaluator

In his paper, **"The Countenance of Educational Evaluation"** [1967], Robert Stake suggested a model by which the processes of educational programmes as well as their products could be evaluated. It clearly distinguished between the activities of description and judgement, both of which were considered necessary for evaluation to occur, as well as providing a distinction between the intended programme and that which is actually observed in practice. The programme is described in terms of three factors - antecedents, transactions and outcomes - allowing the evaluator to consider not only the outcomes of

a curriculum innovation, but also the means by which it is effected and the context in which it occurs. Finally, the "Countenance Model" requires the evaluator to make public the standards by which the effectiveness of the programme is to be judged.

Taken together, these features make this model a suitable one for use by an internal evaluator, such as a teacher interested in deciding the effectiveness of an innovation or programme. As Stake cautions,

> *"To understand better his own teaching and to contribute more to the science of teaching, each educator should examine the full countenance of evaluation."*
>
> [Stake, 1967; 523]

Stake moved on later to develop a "new" model which came to be known as the "Responsive Model" of evaluation [Stake, 1975], in which the evaluator attempts to "portray" the situation naturally. Stake himself describes it by saying

> *"An educational evaluation is responsive evaluation if it orients more directly to program activities than to program intents; responds to audience requirements for information; and if the different value-perspectives present are referred to in reporting the success and failure of the program.*
>
> [Stake, 1975; 14]

The current study is close in spirit to this latter view. It seeks to derive data from antecedents and outcomes, but most importantly from transactions. In particular, it recognises a diversity of audiences which include :

- the teacher responsible, who is the primary audience for consideration in this context. As an internal evaluator, the classroom teacher seeks to study and learn from his own experience in a systematic way which will lead to improvements in his own teaching. This is the essential purpose of the evaluation; if no other audience than this one is served, then the evaluation may yet be considered worthwhile.
- the School Administration (primarily in the person of the Principal), which has made a capital investment in the programme and seeks to ensure that standards are maintained at an appropriate level in terms of examination performance;
- the students themselves (and their parents) whose concern regarding the effectiveness of the instruction they receive must be a foremost consideration throughout the evaluation. It is not ethically justifiable to place at risk the final examination results of students, no matter how worthwhile a research project may appear to the advancement of teaching.
- professional colleagues, both inside and outside the school, who will also share an interest in such an evaluation with its potential to shed light on the effective implementation of recent developments in computer technology into mathematics classrooms. Such an interest may well span both Mathematics educators and Curriculum Evaluators with an interest in classroom evaluation.

The recognition of these diverse interests allows their values to be considered in planning, carrying out and reporting on the evaluation.

In the final analysis, however, the project serves the purposes of the teacher.

A general move from the early evaluation models of the sixties (such as Stake's Countenance Model) to those of the seventies (such as his Responsive model) was a tendency away from prescriptiveness. The detailed matrix arrangement of the 1967 model was typical of the time in which models attempted to perform a "how-to" function - seeking to give practitioners enough detail to allow them to put such models immediately into practice. With the growth of professional, large-scale evaluations in the seventies, we may also observe the models becoming more theoretical, less detailed - in a sense, less prescriptive. The writers were not preaching to amateurs but to experienced professionals, many trying to change their point of view about the purposes and nature of evaluation itself. The naturalistic models of Stake (1975), Parlett and Hamilton (1972) and others were largely eclectic in terms of their actual practice, recommending a diversity of methods by which data should be obtained.

In seeking a model for the Teacher as Evaluator it must be remembered that we are again dealing with practitioners, many of whom will not have conducted any form of formal evaluation previously, and would benefit from as much detail as possible. Hence a model such as the Countenance model proves most useful, providing as it does a checklist and structure which help to ensure that the internal evaluator covers all the important bases. It is not difficult to adapt the model to make it more process-oriented and more responsive than was perhaps originally intended - the model may be extended to include a cyclic relationship between Intents and Observations using an Action Research approach

[Kemmis, 1985], which will make it a more flexible tool for formative as well as summative evaluation.

## 2.4 Classroom Evaluation and Research

The evaluation models of the sixties and at least the early seventies were, in the main, of large scale and conducted by professional evaluators. Scriven (1967) was not alone in his belief that ". . *the very idea that every school system, or every teacher, can today be regarded as capable of meaningful evaluation of his own performance is . . . absurd.*" [Scriven, 1967; 53] Nonetheless, there were those even in those early years who opposed this notion, and proclaimed the role of the teacher as central to the task of Curriculum Evaluation. Most notable of these was Lawrence Stenhouse, who advocated what he termed a Research Model of Curriculum Development, in which "*.....evaluation should lead development and be integrated with it*". [Stenhouse, 1975; 122] and the teacher's role is central. Stenhouse, however, envisaged the teacher's role as that of researcher, and the curriculum in the nature of an experiment to be tested and learned from : "*A curriculum without shortcomings has no prospect of improvement and has therefore been insufficiently ambitious.*" [Stenhouse, 1975; 125]

It is in this context that we approach another perspective of this project - that of inducing teachers to accept that Curriculum Evaluation is not " . . *something that somebody else did . .*" [Russell, 1984; 245] but is quite definitely their responsibility. It is useful here to borrow another of Stenhouse's concepts, that of extended professionalism, defined as ". . *a capacity for autonomous, professional self-*

*development through systematic self-study, through the study of the work of other teachers and through the testing of ideas by classroom research procedures.* " [Stenhouse, 1975; 144] Here in Australia this concept has been further developed in the notion of action research as proposed by Kemmis *et al* (1982, 1985) which places the teacher in the role of educational researcher and has been described as *". . an evaluation process at the individual-classroom level."* [Russell, 1984; 252] The Action Research model [Elliott (1978), Quinlan et al (1987) and, particularly, Kemmis (1982, 1985)] provides the form by which, to use Stenhouse's terms, we may *". . integrate action and evaluation into a unified research model."* [Stenhouse, 1975; 141] It provides the framework by which it is possible to answer both the evaluation and the research questions cited as focii for the study.

# Chapter Three

# Description of the Research Methodology

The decision to portray the research situation of this study using largely naturalistic means was not undertaken lightly. Like most people coming from a Mathematics background, this researcher felt far more comfortable initially with quantitative techniques which produced at least a semblance of precision and scientific rigour. It was with some regret that these were recognised as, not only all but impossible to achieve, but fundamentally inappropriate to research conducted in a classroom situation.

The traditional research goals of generalisability and replicability cannot be achieved through the observation of a relatively small number of classroom situations - complex patterns of interaction which are uniquely determined by a multitude of factors. In recognising that teaching itself carries more of the characteristics of art than of science, it is also recognised that it is not amenable to study by traditional scientific procedures. Certainly the researcher must be systematic and accurate, but the belief that the classroom situation may be controlled and manipulated in such a way as to identify specific outcomes resulting from particular interventions is, at best, doubtful and, at

worst, ethically suspect. The teacher as researcher must retain at all times a fundamental loyalty to the students being taught, and this may not be compromised by the demands of research. The teacher may do nothing which might jeopardise the effective learning of the students, and this dramatically limits the degree of experimental freedom allowed in a classroom situation.

As mentioned previously, the other primary limiting factor for the teacher as researcher lies in the *scale* of the project. The classroom teacher has access to a limited number of students in any period of time, and effective access to even less. It becomes necessary, then, to attempt to draw as much information from this limited sample as possible, while still attempting to derive from this data observations and conclusions which may be of use in other situations at other times. Statistical data on a single small sample may have little to say beyond this sample; qualitative data, on the other hand, may have much to offer. *Qualitative research* may be thought of as a process involving systematic observations and inferences about people and events in natural settings. Its basic aim is to discover recurring patterns in the data collected, and to explain the meaning and significance of these patterns. When thought of in this way, such an approach is seen to be entirely appropriate to the demands and limitations of the classroom research situation.

## 3.1 The Role of Ethnographic Techniques

In a critical and informative article entitled *"The Ethnographic Research Tradition and Mathematics Education"* [Eisenhart, 1988], Margaret Eisenhart describes with regret the fact that *"..... relatively few educational researchers have actually undertaken ethnographic research....."* [Eisenhart, 1988; 99]. Mathematics educational research is described as a "microcosm of what is occurring in the educational research community" in which the questions being posed are often susceptible to an ethnographic approach and yet *"... these researchers tend to use case-studies, in-depth interviews, or in-classroom observations without doing what educational anthropologists would call ethnographic research."* [ibid]

At the risk of further disappointing the author above, such is the case in the present study. "Pure" ethnography involves the *"holistic depiction of uncontrived group interaction over a period of time, faithfully representing participant views and meanings. . "* [Goetz and Le Compte, 1984 in Eisenhart, 1988; 99] The ethnographer seeks to interpret and understand a particular social situation -*why* it occurs as it does. Two assumptions are implicit here: first, that a "holistic" depiction of any social situation is possible, and second, that the researcher does not manipulate the situation in any way, as implied in the word "uncontrived" above. In fact, it is in this sense that ethnography and traditional educational research are sometimes presented as incompatible - they may be thought of as simply asking different questions. This study poses the question : "What is the effect upon this mathematics class of introducing graphics calculators into the learning programme?" The "pure ethnographer"

would perhaps ask : "How does this mathematics class operate in this context? Why does it operate in this way?" There would appear to be an arbitrariness at work here, since any teaching strategy may be considered to be a "contrivance". Clearly any classroom activity is open to study by ethnographic techniques, including those which are innovative.

Thus, failure perhaps to qualify as a "pure" ethnographic study does not preclude the effective use of ethnographic techniques as a means of gathering detailed and significant information concerning the processes and outcomes of interactions such as this one. Further, it is seen as possible to gather and interpret data in a way which is more sympathetic to the ethnographic research tradition than to traditional positivist modes : the recognition of the perception of reality as a social construct which varies between participants is an important perspective in seeking to describe and understand a given social situation.

If educational research is considered by some to be incompatible with ethnography, then educational evaluation should be even more so. The prospect of gathering data for the purpose of making judgements about relative worth should surely be anathema to ethnographic purists. And yet increasingly evaluations are taking advantage of ethnographic techniques. As described by Le Compte and Goetz [in Fetterman, 1984; 39] :

> *"The characteristics of ethnographic research . .contribute to providing more integrated baseline and process data and generating more comprehensive parameters for values data than do conventional evaluation designs (Guba, 1980).*

*Because ethnography uses multiple data collection strategies, it provides the flexibility needed for the variety of situations that evaluators are requested to assess."*

The advantages of using ethnographic techniques in the present study are clear - in terms of Stake's Countenance model, they provide a valuable means of furnishing data for the "Description Matrix" while providing information on attitudinal changes which might accompany such an intervention. In the final analysis, the researcher seeks to understand the effects of the curriculum innovation upon the participants, and this altogether alters the nature of the relationship between researcher and subjects; the "balance of power" is seen as fundamentally different from that of the traditional educational research experiment.

## 3.2 The Role of Action Research

This political perspective on the nature of the research is entirely appropriate in the context of Action Research, particularly as it is practised in Australia. The origins of action research in the work of Kurt Lewin in the 1940's and 1950's centred on attempts to solve social problems, with a particular emphasis upon systematic and collaborative investigation by the participants [Kemmis, 1985]. This was an empowering process for those involved, since traditional research tended to be imposed upon those affected, rather than controlled by them. When applied to educational research, this mode of investigation has found favour with teachers as practitioners of their art, whose professional questions and concerns tended to

vary from those who investigated educational concerns from outside the classroom. This is the concept explored by Scriven in his paper on *"Evaluation as a Paradigm for Educational Research"* [1983]

The present study seeks to take this political dimension one step further. Traditionally, educational action research involves the systematic and collaborative study by teachers of some educational issue which is seen to be problematic. The involvement in the process of those affected has always been a central tenet, and yet all too often the primary participants are excluded from being part of the decision-making which accompanies such research. These primary participants, of course, are the students affected by the process. When, initially, collaborators were sought to assist in gathering and interpreting the data for this evaluation, it was difficult to find many teachers who shared the interest and could spare the time to fill such a role. It was some time before the interest and enthusiasm of the students involved showed them to be ideal partners in the process. While not allowing the usual detachment required, ethnographic techniques provided a further justification for such an approach - they became the "informants", the creators and participants with regard to the social situation in question.

As mentioned previously, the cyclic nature of the action research process also enhanced the evaluation model which was proposed for this study. Rather than planning the nature and sequence of the programme implementation strictly in advance, each unit provided the basis for reflection and discussion, which could then guide the direction and format of the next unit. This flexibility was an important aspect of the implementation of the curriculum innovation in question,

and enabled greater participation than would otherwise have been possible from those involved.

## 3.3 The Sample Group

The group of students who participated in the current project consisted of the thirteen students undertaking the Three Unit Mathematics course in Year 12 in 1989. As such, they represented the most capable mathematics students from their Year cohort. Of these, six students were also undertaking the additional Four Unit course, aimed at students of particularly high ability in this subject. The breakdown by sex was as follows :

| COURSE | FEMALE | MALE | TOTAL |
|---|---|---|---|
| Three Unit | 6 | 7 | 13 |
| Four Unit | 3 | 3 | 6 |

Table 3.1 : Breakdown by sex of sample group

The relative balance of males and females within these classes is unusual, particularly at the higher levels of Mathematics studied for the Higher School Certificate, which are traditionally dominated by males. A particular policy in Mathematics at Saint Joseph's has been the encouragement of girls to attempt study at the higher levels if they have demonstrated such ability. Counselling of students at Year Ten level concerning the appropriate choice of Mathematics subject reflected this policy. In class, every attempt is made to involve the female students, especially where there is a tendency for males to

attempt to dominate. These procedures appear to have been successful in encouraging equality of participation in this group.

## 3.4 Collection of Data

In keeping with the multi-modal approach to data collection required in qualitative research, a variety of techniques have accompanied each stage of the innovation. A summary of the data collection procedures is given at the end of this chapter.

**3.4.1 Baseline Data :** It was necessary to gather information prior to the intervention being described from which it might be possible to assess its impact. Probably the most significant of these in this context was an Attitude Scale constructed for this purpose [Appendix B]. This was modelled on a five-point Likert scale, in which students were asked to rate their level of agreement or disagreement with given statements from 1 to 5 (1 - disagree strongly, 5 - agree strongly). It consisted of twenty six items divided into four sections -

(1) General attitude towards Mathematics,

(2) Knowledge of the applications of Mathematics,

(3) Attitudes towards problem solving, and

(4) Perceptions as to the place of Mathematics in their futures.

Each item was coded with a three-digit number with the first digit derived from the four question-types above. In general, statements for which the code number ends in "0" indicate a positive attitude towards Mathematics, while those ending in a "1" show a negative perception. As a means of internal validation, some cases with a common "tens" digit (e.g. 2.30 and 2.31) indicate a "matched pair", in the sense of opposite statements about the same factor . In this

way, it is possible to have some measure of internal consistency for the scale. Such items are marked with a "*" below.

The scale items were as follows (the actual question number on the student scale occurs in parentheses after each item):

**(1) General Attitudes Towards Mathematics :**

1.10 : Even though it can be hard, I get a lot of satisfaction from doing Maths (5) *

1.11 : People who like Maths are often weird. (1) *

1.20 : I enjoy learning about new and different areas of Maths. (17)

1.21 : Without Maths, school would be a better place. (7)

1.30 : Maths is easy (10) *

1.31 : It is impossible to understand Maths (20) *

1.40 : Maths is one subject I would really like to improve in. (14)

1.41 : I wish I did not have to study any Maths at all. (26)

1.50 : Maths is one of the most important subjects I study at school. (25)

1.51 : Being able to add, subtract, multiply and divide is all the Maths the average person needs. (11)

**(2) Knowledge of Applications of Mathematics :**

2.10 : Mathematics is needed in most areas of life. (6)

2.11 : I will probably never use most of the Maths I do at school. (23)

2.20 : The Maths questions I most enjoy are those which involve real-life situations. (19)

2.21 : Only Mathematicians need to study Maths. (18)

2.30 : Using Computer technology is a great way to learn Maths. (2) *

2.31 : Computers are not much use in learning Maths at higher levels. (9) *

**(3) Attitudes Towards Problem Solving :**

3.10 : Working out Maths problems is fun, like solving a puzzle. (8) *

3.11 : Doing Maths problems is a waste of time. (24) *

3.20 : I like doing problems which show unexpected relationships. (15)

3.21 : If I can't work out a problem quickly, I prefer to give up. (21)

3.30 : I never know what to expect in my Maths class. (12) *

3.31 : Maths is doing the same thing over and over again. (3) *

**(4) The Place of Mathematics in the Future :**

4.10 : I expect to have to do a lot of Maths after I leave school. (4) *

4.11 : Maths will play little part in my future career. (13) *

4.20 : I would like to continue to learn more about Maths in the future. (22) *

4.21 : I will be glad to be finished with Maths after Year 12. (16) *

Table 3.2 : Mathematics Attitude Scale

The scale was administered in the first weeks of the year to the sample group and to the corresponding group at another Catholic high school in the region (Holy Spirit College, Bellambi). For each item, the mean and standard deviation were calculated; negative items were then "adjusted" by subtracting the mean for such items from six. In this way, when the information was represented graphically, the relative heights of the graph for each item gave an immediate visual reference to the attitude towards the subject - whether the item was positive or negative, a "high" graph indicated a positive attitude towards the feature described by that item.

After completing the scale, the sample group was involved in a discussion of its effectiveness and given the opportunity to criticise

and make suggestions. They were asked questions such as "How accurately did the scale allow you to express your feelings?" and "Did you find the questions clear or ambiguous?" Some variations were suggested in the interpretation of question 1 (1.11 above) and some slight confusion was felt concerning the five-point scale - students had to keep reminding themselves of the correct interpretation. Overall, responses were positive, and students expressed satisfaction in being involved in the process.

Although it would be anticipated that such information may be very useful in the context of this evaluation, it is essential at the same time to be aware of the limitations of this data. Certainly the sample group would be expected to reflect some of the enthusiasm of their teacher who was conducting the research, moreso since they are involved as collaborators in the project; students at the other school had no such affinity - even the way in which the survey instrument was administered is likely to have affected the attitudes expressed, since there was no allegiance to the study on the part of the co-operating teachers - it may well have been an inconvenience to them. Nonetheless the results prove valuable in providing at least a "baseline" for student attitudes which can be used in comparison with corresponding data gathered at a later date. It should permit the observation of changes within the groups over the time period of the evaluation - one school year - allowing it to be used as a measure of outcome changes which may have resulted from the curriculum innovation in question. The possible significance of such changes may be measured by the use of a T-test on the means and standard deviations for each item. In allowing the responses to be anonymous, it was then not possible to relate such responses individually over the

two applications of the Attitude Scale; this precluded the use of the usual t-test, which uses the differences between responses on each item. It was thus decided to use Fischer's T-test for Unrelated Samples, which required only means and standard deviations for each item from the pre-test and the post-test (see Appendix B).

Other baseline data was gathered in the form of statistics on participation and achievement in two external examinations : the New South Wales Year Ten School Certificate Moderating examination in Mathematics (Advanced Level) and the Australian Mathematics Competition for the Years 1987 - 1989. These allowed comparison of the Mathematical abilities of the sample group with previous Year 12 groups at the same school, and with corresponding groups on a state-wide basis.

Valuable information regarding the broader school community (parents, students and teachers) had been gathered prior to this study in two unpublished surveys conducted within the school. [Arnold, 1986, 1988a and 1988b] Described in more detail in the chapter on "Antecedents" which follows, these surveys provided information on the attitudes and priorities of these various groups regarding educational purposes of the school and their roles in this process. The results were presented to these groups for validation, and implications drawn from these discussions.

It might be noted at this point that this baseline data was largely quantitative in nature. Although partly a reflection of the confidence of the researcher at this point in the project, this also reflected a deliberate attempt not to pre-empt too much of the project with the

sample group at too early a stage. It was felt that in-depth interviews concerning their attitudes towards Mathematics and computer technology, and their observations of classroom processes might overly sensitise them to these factors, making a realistic assessment of their worth more difficult.

**3.4.2 Process Data :** The majority of data gathered on the Transactional stage of the evaluation occurred by means of participant observation techniques. As classroom teacher, the researcher was in a position to observe the interactions and effects which accompanied the use of the calculators and structured worksheets which were the focus of the project. These observations were then recorded in a research diary accompanied by comments and reflections derived from the transactions. Such reflections proved a major tool in deciding subsequent steps of the process, in line with the spiral effect advocated by Action Research methods.

Process data was also provided by means of observations by teaching colleagues; in some cases they served as observers, in others they taught lessons using the calculators and worksheets and allowed the researcher to act as observer. In such cases it was considered important to minimise any threats which such colleagues may have felt under observation (the researcher as Head of Department is also supervisor to these colleagues and this can create tension for some teachers). The use of Clinical Supervision techniques proved valuable here : some years earlier a previous action research project had centred on the introduction of Clinical Supervision into the school situation, and teachers were familiar with the process. By falling into

the sequence of Pre- and Post-observation interviews with the teachers concerned, and particularly by agreeing to focus in the observation upon specific factors (rather than "general teaching ability") many of the concerns were minimised. Colleagues had, of course, the same opportunities to observe their supervisor and comment upon his lessons.

Another important tool by which information was gathered at the transactional level was a standard Evaluation Report sheet, which students were asked to complete each time they used the calculators (Appendix C). Designed for ease of use, students had merely to tick a response to each of five questions:

(1) Which FUNCTIONS did you use on the HP-28C?

(2) Did you use the HP-28C individually or in a group?

(3) Describe briefly the way in which you used the HP-28C :

* To verify your answers * To draw graphs

* To learn a new concept * As a tool for investigation

* Other . . .

(4) How effective/useful was the HP-28C for this purpose?

(5) Are you interested in using the HP-28C again in other areas?

Space was provided for further comments for each of these questions. Although the time taken to complete these sheets was never long, it was interesting to note that most students came to dislike this task intensely. Although they continued to fill them out as requested, they indicated that they found the task an inconvenience, and were thrilled if they were permitted to use the calculators at any stage without the sheets at the end. Nonetheless, many useful and informative comments were made on these sheets, which could then be followed up, usually by means of an informal "playground" interview. In such a non-

threatening situation, students were invited to expand upon their responses and to explain their feelings in more detail.

The observation of classes other than the Year 12 sample group was a further important aspect of the gathering of process data. The Year 11 Three Unit class provided a useful comparison group in which most of the same units could be trialled. Observations and interviews with teacher and students from this class yielded some valuable information which was quite distinct from that gathered from the sample group. Such differences provided considerable "food for thought" as to why certain techniques and approaches were successful in one situation and not in another.

A particularly useful technique which was used to accompany the unit entitled "Inversions" (described in the Chapter on "Transactions") involved the observer in recording the student activities throughout the lesson on a minute-by-minute basis. Armed with a sheet divided into fifty lines, each marked with a minute (e.g. 10.10, 10.11, . . ) the observer recorded the sequence of student activities using the code :
"A" for "active" (writing, discussing, questioning, calculating, graphing, . . . );
"P" for "passive" (listening to instructions and explanations), and
"U" for "unspecified" (no specific teacher instruction at that time).
Such a system allowed the observer time to record for each few minutes a summary of the type of activity taking place, and permitted then a fairly comprehensive portrayal of the lesson in this way.

On one occasion, it was also possible to act as non-participant observer, when the Year 12 class was set an assessment task which

they were free to discuss in groups. This allowed the teacher the freedom to withdraw from the usual dominant role and observe the movements and interactions within the groups, and even to "eavesdrop" and record the discussions occurring. This proved to be an effective means by which the group processes could be observed, and some record made of the reasoning processes which accompanied the problem solving task which was set. This is further described in Chapter Five.

**3.4.3 Outcome Data :** The gathering of outcome data involved the use of further techniques which attempted to capture some record of achievements in concept development and changes in attitudes which had accompanied the innovation, as well as perceptions by the students of the effectiveness of the programme and the processes by which it had been implemented.

The Attitude Scale was re-administered to the groups at both schools again at the end of the school year, and the observed changes in both groups over this period provide the basis for a number of conclusions examined in Chapter Six.

Further, at this stage it was felt to be appropriate to interview each of the students in the Year 12 sample group and attempt to draw upon their experiences throughout the year. Such ethnographic techniques as beginning with a "grand tour" question [Spradley, 1979; 223] proved useful. Opening with the question "Describe a typical maths lesson" not only began the interview with something familiar and relatively easy to describe, but also served as a reflective device, by which the informant is given cause to question things which may have been

taken for granted. Such interviews provided a wealth of information concerning the perceptions of the students which would not otherwise have been possible through quantitative means. Additionally, these interviews gave insights into the processes of the implementation, although in a summative sense, looking back upon the interactions.

The interviews themselves took place with students either individually or in pairs, whichever was preferred. After the initial "Grand Tour" question, students were asked a second such question describing a "typical lesson using the HP calculators". Finally, they were asked to classify a series of prompts such as "group work", "discussion", "understanding" as being associated more with lessons with or without the calculators. The responses were recorded by the interviewer in a transcript of the session. Students were encouraged to examine these transcripts and further discuss any points of interest.

Finally, open-ended assessment tasks provided a means by which students could demonstrate their understanding of the concepts being studied in a variety of ways. Although students knew that such tasks would be marked, they were told that they would not contribute towards their Higher School Certificate assessment. Discussion of their answers afterwards helped further to ensure the validity of their responses.

## 3.5 Summary of Data Collection Techniques

The various means by which the data was accumulated for this project may be represented in tabular form as follows :

| | ANTECEDENTS | TRANSACTIONS | OUTCOMES |
|---|---|---|---|
| (i) Logbook | • | • | • |
| (ii) Attitude Scale | • | | • |
| (iii) Surveys | • | | |
| (iv) Examination results | • | | • |
| (v) Participant Observation | | • | |
| (vi) Non-participant Observation | | • | • |
| (vii) Colleague observation | | • | |
| (viii) Pupil Evaluation Record | | • | |
| (ix) Student Interviews : | | | |
| - Informal | | • | • |
| - Formal | | | • |
| (x) Open-ended assessment tasks | | • | • |

Table 3.3 : Summary of Data Collection Techniques

# Chapter Four

# Antecedents

In Stake's *Countenance Model*, which provides the structure for this evaluation, each of the three "levels" of the intervention, antecedents, transactions and outcomes, is described in terms of that which is intended and that which is actually observed; the extent of the congruence between these is then judged by reference to certain standards which are made explicit by the evaluator. Finally, that which Stake labels *contingency* must be described between the antecedents, the transactions and the outcomes in the description matrix - logical contingency in the case of the intents, and empirical contingency across the observations. In this way Stake maintains it is possible to consider the full countenance of the evaluation.

Initially, we seek to describe in fine detail the situation as it occurred before the intervention which is to be described and judged. This may be done by reference to the three major parties involved in the innovation - the school, the teachers and the students - and by reference to the Curriculum - that which is initially in place and that which is proposed. Each is described in terms of intents, observations and the standards by which these are to be judged. At the antecedent stage we seek to describe the conditions which determined the nature

and direction of the innovation and which are considered necessary for it to be fully understood and its effectiveness judged.

## 4.1 The School

The unique advantage of the internal evaluator lies in a presumed intimate knowledge of the situation in which the study occurs. With regard to the characteristics of the antecedent situation, the evaluator occupies a privileged position and could be expected to anticipate many of the possible problems. At the same time, there is the danger of being "too close" to the object of the evaluation, making it difficult to attain an objective perspective. It may even be difficult to see the familiar school situation "as it really is", rather than as it appears to an insider. These problems are confronted commonly in ethnographic studies in which the perspective of the researcher is accorded its own (face) validity - there is no single "correct" interpretation of any social situation; rather the perception of reality is considered to be a social and individual construct, each person or group recognizing a perspective which is "true" for them. It would, of course, be wise for researchers to seek to compare and contrast their version of the reality of the situation with that of other participants and so enrich their interpretation. Such ethnographic techniques as informant interviews, and participant and non-participant observations may prove useful to the internal evaluator in this way.

In order to understand the situation in which the evaluation takes place, it is necessary to give a description which is as "fine-grained" as

possible. This initially involves describing the nature of the school and the "school climate" which characterises it.

### 4.1.1 The Nature of the School

Saint Joseph's Regional High School is a Catholic co-educational secondary school situated in the Illawarra region, south of Sydney. It caters for some 800 students from Years 7 to 12, drawing them from a large and diverse area, spanning primary, secondary and tertiary industries. The school itself is relatively new, having commenced operation with a single Year 7 group in 1982; 1987 saw the completion of the school's first full cycle of Secondary schooling, presenting the first Year 12 group for the Higher School Certificate examination. Each of the first six years, then, represented a significant growth for the school in student numbers and staff, a pattern which has only stabilised over the past three years. This last period, then, represents a time of consolidation in which the school has begun the process of looking at its purposes and nature, and attempting to evaluate these in terms of the experience of the initial phase, the perceived needs of students and the demands placed upon the school by society.

Physically, the school is set in picturesque surroundings, close to farmland and dominated by a nearby mountain escarpment as a backdrop, while overlooking Lake Illawarra and the ocean. The local area is rapidly growing in population as young families move in to housing which is more affordable than that in the cities. The school itself is notable, not only by the architectural design of the buildings which complement the natural surroundings, but by the landscaping and gardens provided by the voluntary work of the parents. There is a

sense of commitment and ownership of the school by these parents which is evident throughout the school community.

### 4.1.2 Educational Priorities

As part of an overall self-appraisal, the school in 1988 conducted a survey of parents, teachers and students in order to obtain some picture of the educational priorities perceived by each group [Arnold, 1988a] - what did the different participants see as the major purposes of the school? In the context of the current project, it is significant to note that, when asked to rank a list of educational purposes - the acquisition of knowledge, the acquisition of skills, spiritual, social, cultural and physical development - the three groups identified quite different priorities. Teachers saw skill and social development as the major purposes of schooling, parents saw it in terms of the acquisition of knowledge and skills, and students saw the primary purpose of schooling as the imparting of knowledge.

The overall picture obtained from this survey is one of a school in which parents and students share a largely utilitarian view of education - aspects such as cultural and physical development were invariably classed as of low importance. While teachers and parents saw spiritual development as important, students did not agree. The rating by the students of the acquisition of knowledge as the main function of the school suggests the possibility of conflict with teachers at classroom level, since they see their primary purpose differently - imparting skills and fostering social development. Students and, to some extent their parents, may feel dissatisfied when teachers do not attend to the teaching of "facts". The current innovation may well encounter problems in this regard, since the computer enhancement of

Mathematics teaching does not centre on the presentation of new knowledge, but rather deeper understandings and improved communication skills - features which may not be appreciated by students who "just want to get on with the task at hand". It may be anticipated that students will be frustrated by lessons centred around group work, discussion and problem solving. Students, parents and teachers will need to be convinced of the worth of an innovation which does not produce immediate returns, anticipating questions such as "If it cannot be used in the final examination, then what use is it?" The "final examination", in this case, is the New South Wales Higher School Certificate Examination in Mathematics. While presenting questions which require distinct skills in problem solving (particularly at the higher levels, Three Unit and Four Unit), the major thrust of the paper is still perceived as requiring a mastery of manipulative skills, albeit in situations often unfamiliar to the students.

### 4.1.3 The School Climate

Some valuable information regarding the school climate was gathered in two surveys of the teaching staff at Saint Joseph's, administered in 1986 (with approximately a fifty per cent response rate) and then again in 1988 (at which time all teachers responded ) [Arnold, 1988b]. Of particular relevance here are items concerning :

(i) the involvement of the Principal in classroom matters, which was seen to decrease over the two year period. It is suggested that this was concurrent with a devolution of responsibility to Heads of Department with the increasing size of the school. This is relevant in that the current project arises from an initiative suggested by the evaluator, who was a

Department Head, and demonstrates the support of the Principal for the judgement and priorities of such Executive members of the staff.

(ii) The items on Work Challenge support the view of staff as being given some measure of autonomy and being personally committed to and satisfied by their work. Such a situation would be considered a necessary antecedent for a venture such as this which may well involve teaching colleagues in various roles.

(iii) Although teachers perceived relatively high levels of collaboration with regard to sharing information and teaching techniques they also noted strongly that classroom observation by peers is a rarity. This had changed little over the two year interval 1986-1988.

This last result suggests that difficulties and resistance may well be encountered in attempting to gather information at the Transactional stage of the evaluation, there being something of a tradition of non-intrusion in other teachers' classrooms. Not only will teachers find such a presence unusual, but so also will the students and this may present problems in observing a classroom situation unbiassed by the presence of the observer.

The support assumed from the Principal in allowing the evaluation was observed to be justified, he seeing it as a worthwhile professional development activity on both a personal and Faculty level. This support was made concrete with the grant of an additional allowance to purchase six Hewlett-Packard HP-28C calculators for use in the Senior classes. Another two units were donated to the school by the Hewlett-

Packard Corporation in support of the Evaluation, and with the expectation that the results would be published as a guide to the use of such machines in high school classrooms. This satisfied the requirement that appropriate technology be available for the project, allowing their use in the target classes in groups of two or three students. Additionally, the school computer laboratory, consisting of thirteen Apple IIe computers (with Spreadsheet and Graphics software), was available when required.

It was observed then that the situation at the school was conducive in the main to an evaluation and research project. Since the evaluation was to be internal, the evaluator was in a position to draw upon intimate knowledge of the school and its workings in setting out the intended antecedents, and so a high level of congruence between these and the observed situation could be expected. The author was one of the original staff at the school when it opened in 1982 and had been extensively involved in its growth and development, both as Head of Mathematics and as a member of the school executive. His involvement with the sample group began for most as their teacher in Year 7, and then again when they entered Year 11. Both teacher and students had a comfortable and effective working relationship.

## 4.2 The Teachers

The intents with regard to the teachers involved centre upon :

(i) their willingness to participate, either directly (as observers and co-evaluators) or indirectly by offering ideas and suggestions.

(ii) their knowledge and skills with respect to the use of computer technology as an aid to the teaching of Mathematics.

(iii) their attitudes towards, not only computers and mathematics, but also as to the worth of Curriculum Evaluation and Research and their perceived role in this regard.

As mentioned above, the staff survey results would suggest the likelihood of reluctance on the part of teachers to participate in classroom observation. In practice, however, teachers indicated a willingness and enthusiasm for the project which appeared to belay these fears. Faculty colleagues not only sought more information from the researcher, but most offered to help as observers. It seems likely that teachers will be supportive of such ventures as long as they do not require too great a time commitment on their part. Such evaluation and research activities appear to be viewed as "icing on the cake" to many teachers - a nice luxury, but not necessary to the day-to-day business of teaching.

The problem of available time is probably the single most significant factor in limiting the occurrence of Evaluation and Research projects in Secondary schools. While the initiator of such a project may be convinced of its worth, it is quite a different matter to find others who share that belief to the same extent; hence, demands made upon the time of colleagues must be carefully controlled.

The attitude of teachers towards the use of computer technology and their perception of their own competence in its effective implementation in the classroom are important factors in the current project. When a number of optional lunch-time workshops were

suggested, initially to learn to use Spreadsheet software for administrative and classroom purposes, all of the Faculty teachers chose to attend and were positive about the outcomes. This would seem to indicate a willingness to learn new skills and a recognition of the relevance and importance of computer technology on the part of the Mathematics staff. Some fifty per cent of teachers since then have made some use of computers in selected lessons. This slow process of familiarisation is vital for the long-term acceptance of such innovations, and requires personal support on the part of the change-initiator if it is to last. This is consistent with the premises of Hall's Concerns-Based-Adoption-Model (C.B.A.M.) for the implementation of change [Bents and Howey (1981); Marsh (1988)]

## 4.3 The Students

The student sample selected for this project consisted of those attempting the Three Unit Mathematics course in Year 12, with the following as intended antecedents :

(i) That such students had experienced significant success with the learning of Mathematics in the past, and were capable of achieving the Syllabus requirements for this course;

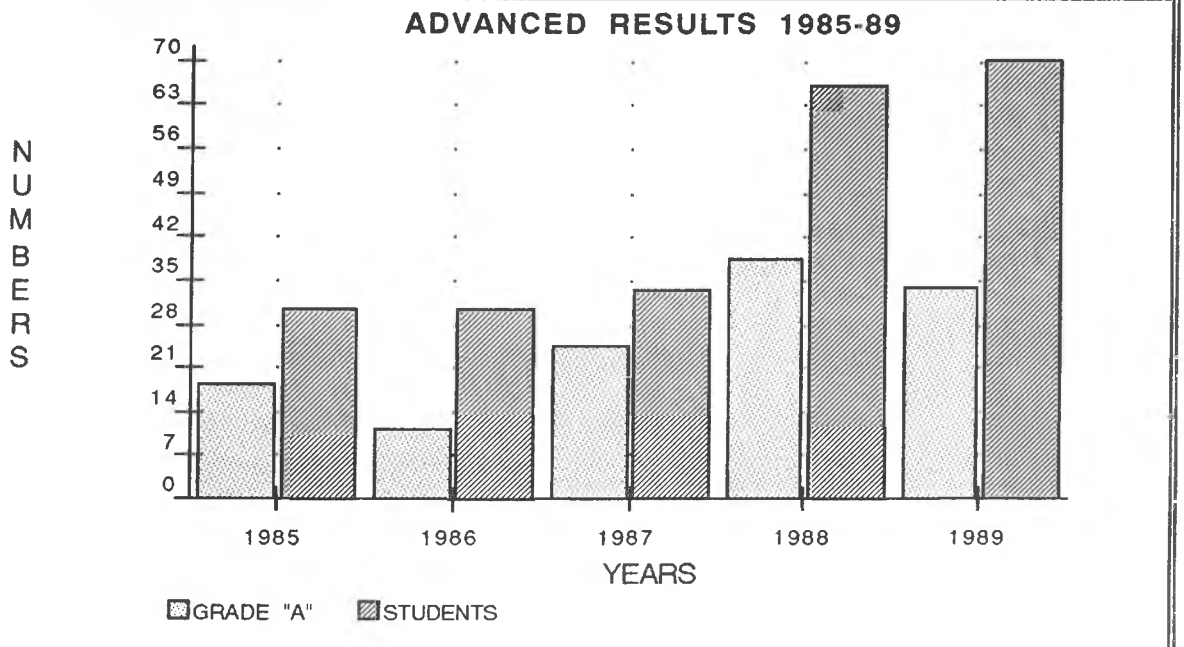
(ii) That while attitudes towards Mathematics might vary, overall these should be positive and the subject be perceived as important;

(iii) That the students be sufficiently familiar with the types of computer technology proposed for this project to be able to use these effectively.

### 4.3.1 Student Abilities

The relative ability of the sample group compared with similar groups may be initially described by considering the performance of the group at the Year 10 School Certificate examination, and their performance and participation at the optional Australian Mathematics Competition examination. Both examinations provide external criteria by which the mathematical ability of the sample group may be objectively gauged (see Appendix A).

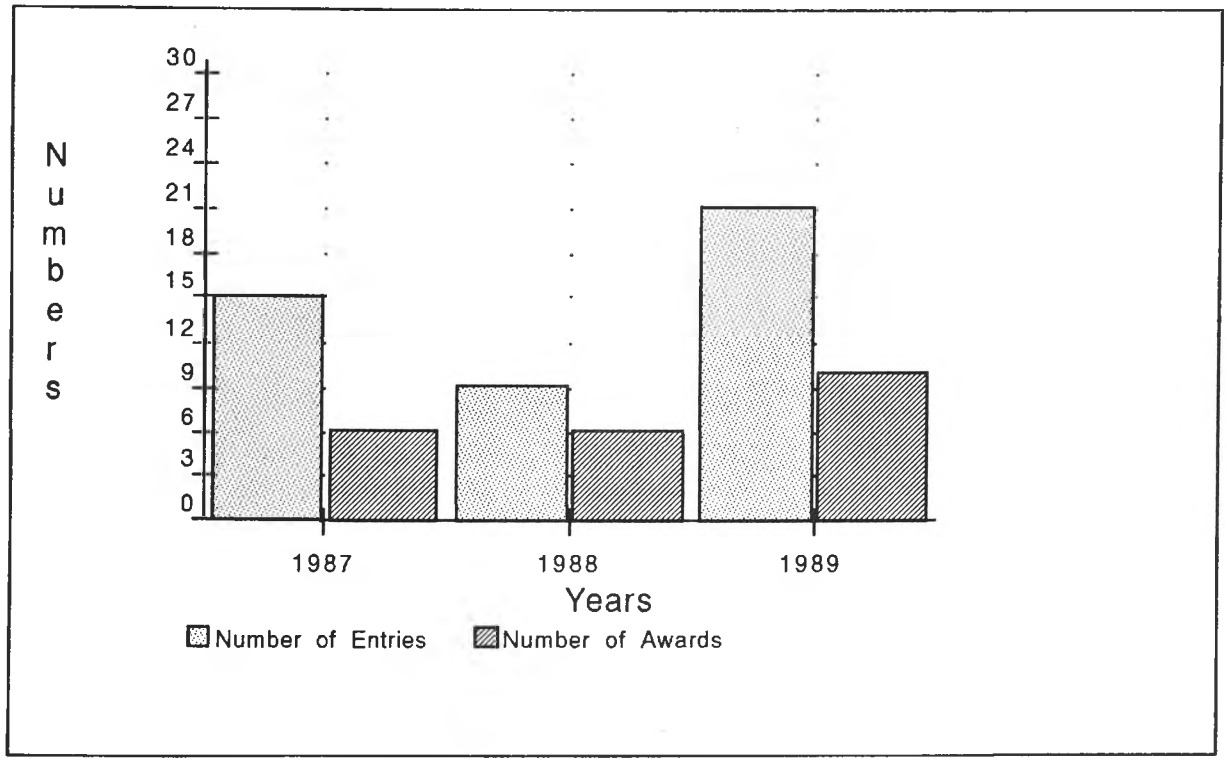
The sample group (here labelled "1987") was one of the first three groups presented for the School Certificate examination, with only one class of Advanced students. The past two years has seen this number doubled, with a corresponding increase in the award of "A" grades. Although difficult to draw conclusions as to the relative ability of the sample group from this, it is worthwhile noting that the proportion of "A" grades awarded across the State is 50% - an "average" school might expect half its Advanced candidates to score the highest grade.




Graph 4.1 : School Certificate Results 1985-89 (Advanced)

We may observe from the graph that the sample group demonstrates greater than average ability in this respect, and moreso than each of the other Year 10 groups from the past five years.

The Australian Mathematics Competition is held each year through the Canberra College of Advanced Education (now the University of Canberra) over the whole of Australia and the Pacific region. It is optional for students at Saint Joseph's, although those in the higher ability groupings are encouraged to participate. It is relevant, then, to consider the pattern of participation of the Sample group in their final year of schooling, in comparison with the previous Year 12 groups, as well as their relative achievements. (see Graph 4.2)




Graph 4.2: Numbers of Entries and Awards in A.M.C. Year 12 1987-89

We note again that the number of students participating has varied widely over the past three years, and that the current Year 12 group

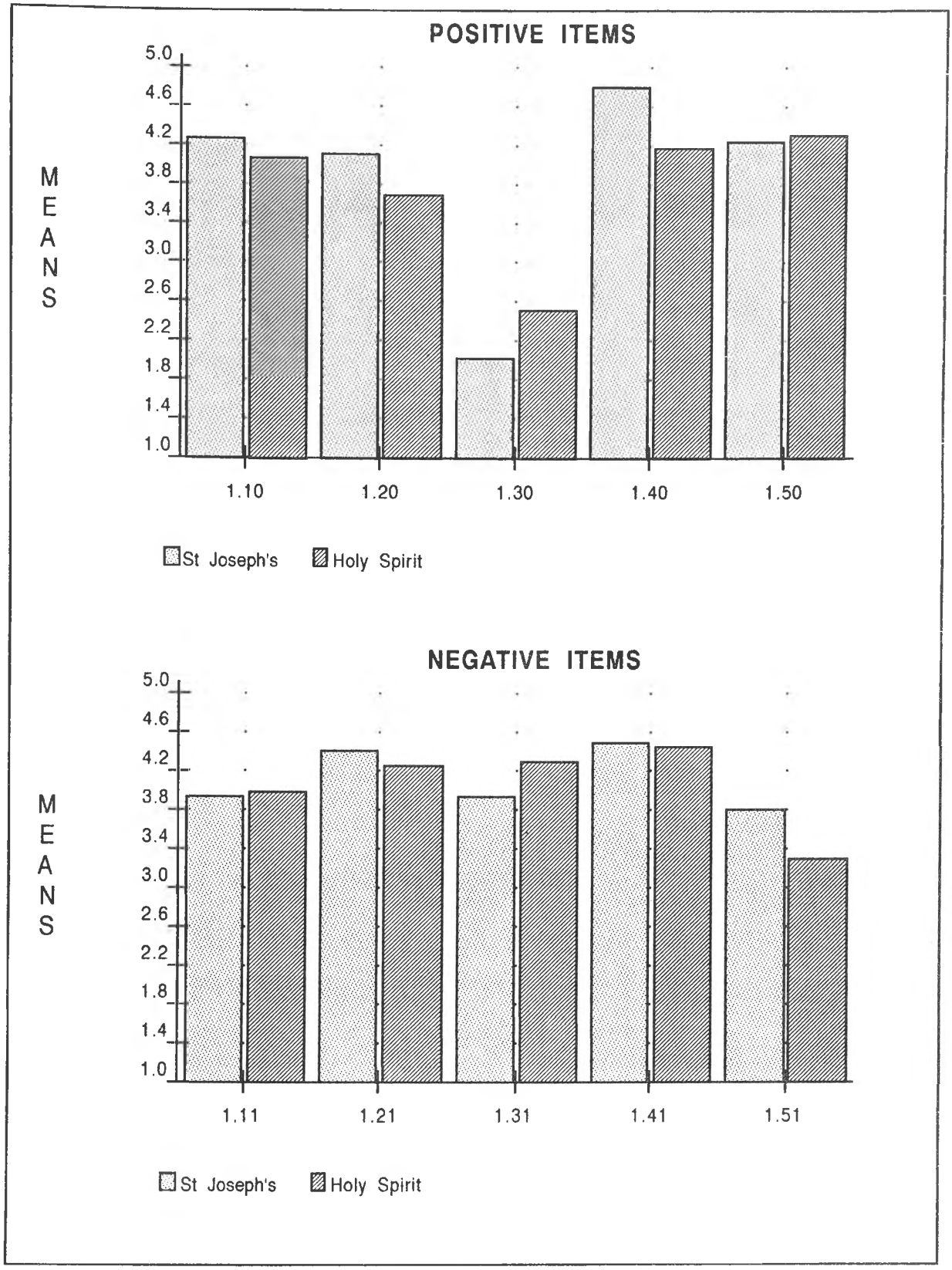
(labelled 1989) both participated more strongly and scored notably higher in the number of awards received than any of the previous groups. Again, the proportion of awards was higher for this group than the average for the Competition.

On the basis of these two external measures of mathematical achievement it is possible to conclude that the sample group (labelled 1987 in Graph 4.1 and 1989 in Graph 4.2) demonstrates above average ability in this subject, both in comparison with peer groups within the school, and in fact with larger groups at both State and National levels. Further, the strong participation in the optional Australian Mathematics Competition suggests a positive attitude towards the subject, a factor which is dealt with in greater detail below.

### 4.3.2 Student Attitudes

The attitudes of the students towards Mathematics were assessed using an attitude scale constructed for this purpose [see Appendix B]. In order to provide comparative data, this scale was also administered to the Year 12 Three Unit class at another Catholic co-educational high school in the region. This group consisted of twenty students (13 males, 7 females). It consisted of four sections, described below.

**(1) General Attitudes to Mathematics** : A number of features may be readily observed from the information in Graph 4.3.




Graphs 4.3 and 4.4: General Attitudes towards Mathematics (Positive and Adjusted Negative Items)

As might be expected, the differences between the two groups on the majority of items are not striking. If the groups had been widely varying in their attitudes at the commencement of the study, this would have made it difficult to draw conclusions about possible effects of the intervention.

On the first five items, we observe the Saint Joseph's group to present slightly more positive attitudes on items 1.10 (*"Even though it can be hard, I get a lot of satisfaction from doing Maths"*), 1.20 (*"I enjoy learning about new and different areas of Maths"*) and 1.40 (*"Maths is one subject I would really like to improve in"*). The responses of the two groups on item 1.50 (*"Maths is one of the most important subjects I study at school"*) are almost identical in both means and standard deviations - hardly surprising considering the select nature of both groups, and their obvious high ability in Mathematics.

Of greater interest, then, is the item in which the trend is reversed - item 1.30 (*"Maths is easy"*). Not only do both groups rate this item low (less than 3.0) but the Saint Joseph's group appears to disagree more strongly than their peers at the other school. Thus, although these are groups of students of high mathematical ability, they do not consider it (in general) as an easy subject.

In the adjusted negative items above, several further points may be noted. Once again we note general agreement between the two groups on most items. Both groups disagreed strongly with item 1.11 (*"People who like maths are often weird"* ), most likely because it tends to reflect badly upon themselves and their own self-images. When asked if *"School would be a better place without maths"* (item 1.21), Saint

Joseph's students disagreed slightly more strongly than those at Holy Spirit. This slight tendency was reversed on item 1.31 (*"It is impossible to understand maths"*) with Holy Spirit demonstrating perhaps a more confident approach than the sample group. Both groups strongly disagreed when asked if they wished that they *"did not have to study any maths at all"* - again, hardly surprising in groups who have chosen to study proportionately more Mathematics than their less able peers.

Only the last item saw any real divergence between the two groups : *"Being able to add, subtract, multiply and divide is all the maths the average person needs"* (item 1.51). We observe that, while Saint Joseph's students showed moderate disagreement with this statement, those at Holy Spirit tended to be neutral about it - neither agreeing nor disagreeing.

The overall picture derived from the items based on a "general attitude to mathematics" would appear to be one which shows a positive attitude towards the subject on the part of both groups. This positive initial attitude must be noted carefully and compared with that demonstrated later in the year. It will also be important to compare any changes in the attitudes of the groups over the same period which cannot be attributed to the computer enhancement under investigation, but which may result rather from maturity and increased mathematical experience.

**(2) Knowledge of Applications of Mathematics :**

In studying the items related to the knowledge of students regarding applications of Mathematics, a number of features stand out. Initially, we note again almost identical responses from the two groups on the

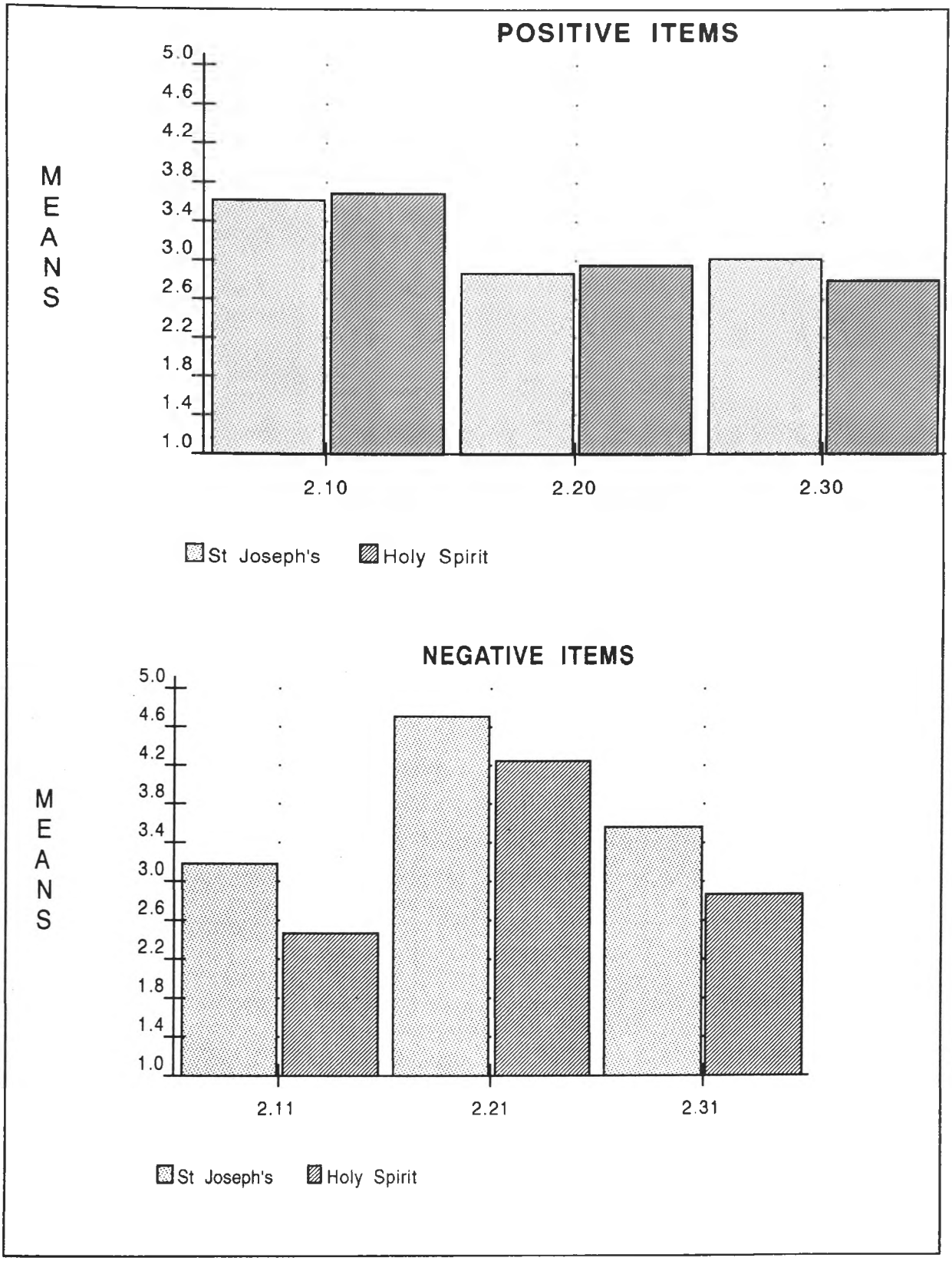
positive items. It is significant, however, that these appear somewhat depressed in comparison with previous results. Only item 2.10 (*"Maths is needed in most areas of life"*) rated more than a lukewarm response from both groups. The other two positive items (*"The maths questions I enjoy most are those involving real-life situations"* (2.20) and *"Using computer technology is a great way to learn maths"* (2.30)) both achieved indifferent responses. This last item is obviously an extremely significant one in the context of the current study, and the neutral response from both groups is advantageous at this point in time, before the intervention to be evaluated.

The negative items again tended to separate the groups more effectively then the positive ones. Item 2.11 (*"I will probably never use most of the maths I do at school"*) not only saw Saint Joseph's students disagree more strongly with the item than those from Holy Spirit, but they also disagreed with each other - the standard deviation for Saint Joseph's on this item was relatively high, indicating less consistency in the responses. This is consistent with the observation that a number of students in the Saint Joseph's sample group do not intend to pursue Mathematics-related careers upon leaving school. The reverse was true for item 2.21 (*"Only mathematicians need to study maths"* ) - Saint Joseph's students produced a highly consistent response to this item, disagreeing more strongly than their peers at Holy Spirit. The same tendency emerged from item 2.31 (*"Computers are not much use in learning maths at higher levels"* ) with the groups disagreeing on this. Saint Joseph's felt that this was not true, Holy Spirit felt that it was. Both groups, however, varied only slightly from a neutral result of 3.0.

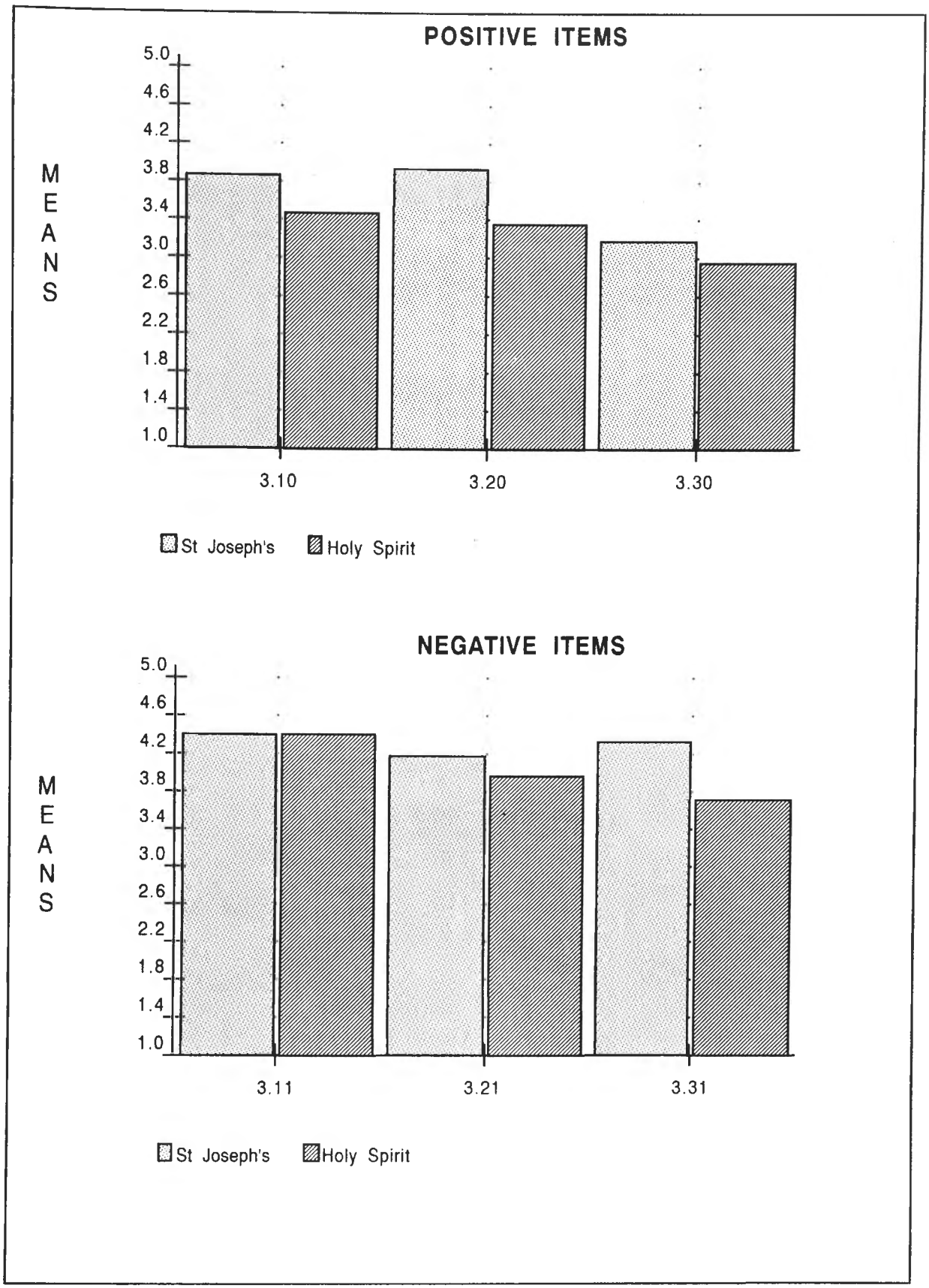
Overall, then, we may perceive of few strong differences between the two schools on these items related to the applications of Mathematics. It seems possible that a lack of knowledge on the part of the students in this regard may have led them to be more cautious in their responses. The main differences appear to relate to the attitudes towards the usefulness of the mathematics they study and the role of computers in mathematics. As mentioned above, the responses to these items at the later application of the Attitude Scale should prove to be interesting and significant for the evaluation.

**(3) Attitudes to Problem Solving**

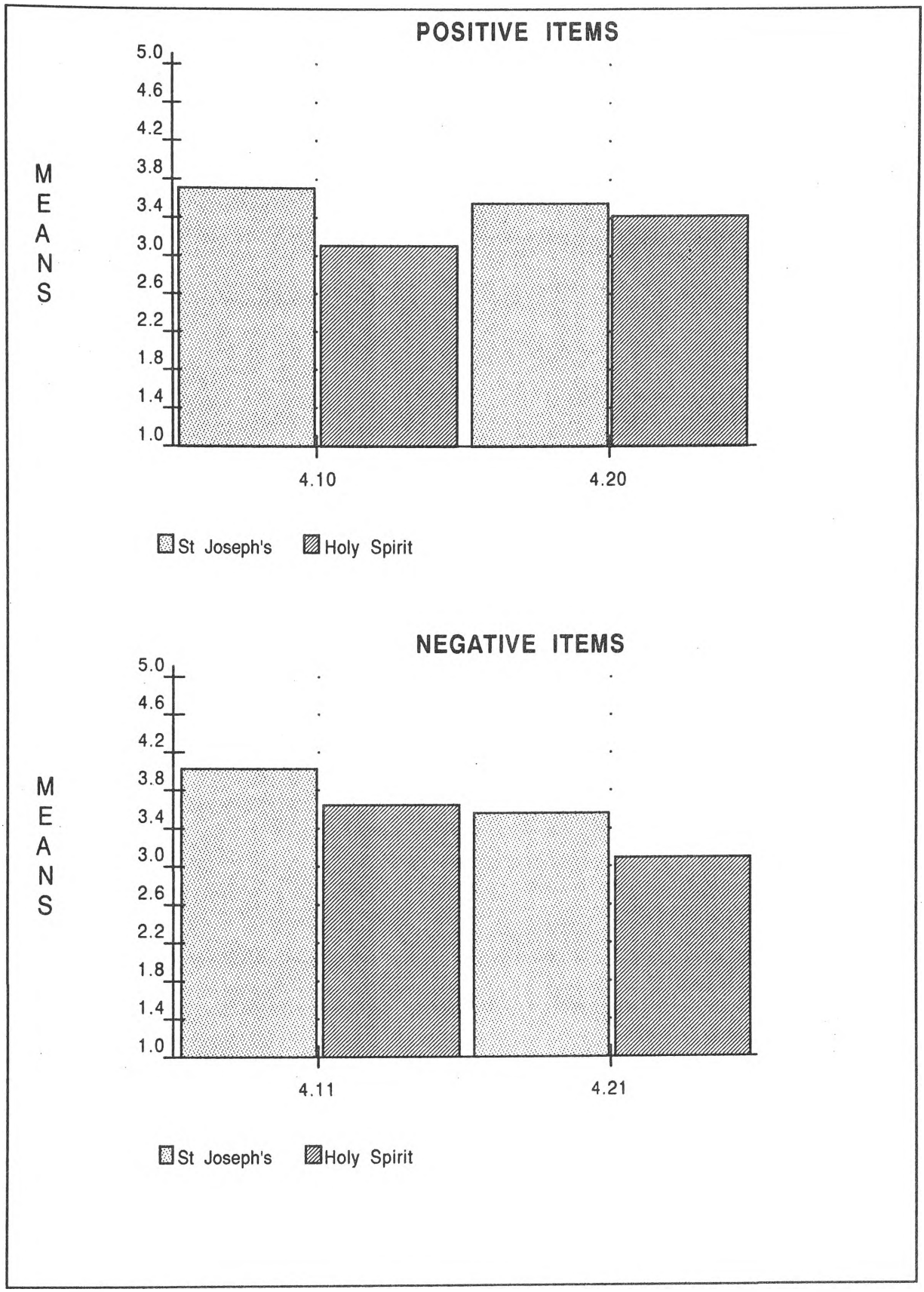
The items on attitudes to problem solving did little to distinguish between the two groups at the outset of the evaluation. All three positive items (3.10 : *"Working out maths problems is fun, like solving a puzzle"*; 3.20 : *"I like doing problems which show unexpected relationships"* and 3.30 : *"I never know what to expect in my maths class"* ) drew agreement from both groups, with a stronger positive response from Saint Joseph's. Relatively strong disagreement was the response to the three negative items (3.11 : *"Doing maths problems is a waste of time"*; 3.21 : *"If I can't work out a problem quickly, I prefer to give up"* and 3.31 : *"Maths is doing the same thing over and over again"* ). Neither group provided support for the old stereotype of mathematics lessons as being repetitive and monotonous. Again, we may perceive an overall positive attitude towards the subject reflected in the responses from both groups.




Graphs 4.5 and 4.6 : Knowledge of Applications of Mathematics (positive items and adjusted negative items)




Graphs 4.7 and 4.8 : Attitudes to Problem Solving




Graphs 4.9 and 4.10 : The Place of Mathematics in the Future

**(4) The Place of Mathematics in the Future** : (Graphs 4.9 and 4.10 )

Once again, we may observe the pattern which has become familiar - little difference between the two groups, but with a slightly more positive overall response from the students at Saint Joseph's than from their Holy Spirit counterparts. This tendency may be readily attributable to a Hawthorne-type effect , in which the group is influenced by the enthusiasm of their teacher. The four items in this section were 4.10 : *"I expect to have to do a lot more maths after I leave school"*, 4.20 : *"I would like to continue to learn more about maths in the future"*, 4.11 : *"Maths will play little part in my future career"* and 4.21 : *"I will be glad to be finished with maths after Year 12"*. Both pairs of positive and negative items in this section allow a consistency check on the responses as a means of internal validation of the instrument. If the results had been inconsistent serious doubts would have been cast upon the Attitude Scale being used. Again, most of the results in this section tend to be lukewarm rather than strongly positive or negative.

This section would suggest that there is a distinct possibility that many of the students are studying Mathematics for decidedly utilitarian reasons (most probably involving scaling advantages at the Higher School Certificate) rather than for any deep-seated love of the subject. This should come as no real surprise, nor is it surprising that an overall positive attitude emerges consistently from both groups. While this characteristic may not make the sample, perhaps, a "typical" group for our purposes, it does improve their standing in the role of "collaborators" in the research project at hand. Their neutral responses towards computer technology would further support their appropriateness in such a role, and their overall positive attitude

towards the subject should not preclude them from offering valuable criticism of the process and the outcomes which are the focus of the evaluation.

## 4.4 The Curriculum

The final antecedent factor which should be considered concerns the nature of the curriculum which was in place before the intervention which we seek to evaluate. Two aspects of Curriculum are relevant here - **process** and **content.** Essentially, the changes which are being evaluated here concern the former - exploring ways in which we may more effectively teach Mathematics with the aid of computer technology. This is, in a sense, a short-term goal for Mathematics Education. As emphasised by two American workers in the field,

> *"First we must see how the teaching and learning of traditional topics can be improved with the full use of technology. Can these topics be taught better, faster, and with greater student understanding? When this question has been answered, curricular issues can be addressed."*
>
> [Waits and Demana, 1988; 332-333]

Such is the goal of the present project. In the longer term, however, content issues will also be addressed in response to the need to adequately prepare high ability Mathematics students for life and careers in an increasingly technological world - one in which they will possess many of the empowering skills.

The current curriculum available for the New South Wales Higher School Certificate is most notable for its failure to incorporate any real preparation for life in a computer-dominated technology. While

increasingly recognised both overseas (Hirsch, 1985; Steen, 1986; Howson and Kahane, 1986) and in other states of Australia (Czerneskyj, 1988; Ryan, 1987; Clayton *et al*, 1988), the need for a computer-relevant mathematics syllabus has yet to be recognised in New South Wales.

In an overview of Senior Mathematics courses from all states of Australia prepared by Dr John Mack (Mack, 1987), New South Wales was shown to be the only state whose Senior Mathematics course had no advanced component in Statistics, and one of the few which did not include a computing component, a study of vectors and matrices or of difference equations. These are the components of what is termed **discrete mathematics** (which is the language and working structure of computers) as opposed to **continuous mathematics** (the language of the calculus), which is the traditional preparation for higher mathematics. While in no way discounting the importance of the latter, the vast majority of capable mathematics students entering the workforce now and in the future will be applying their skills in fields dominated by two looming and intertwined influences - statistics and computing. Neither is currently available for study in the higher Senior levels of Mathematics in New South Wales.

In 1989, after a decade of work in the area of Mathematics Curriculum in the United States, the National Council of Teachers of Mathematics (N.C.T.M.) published an ambitious document entitled **Curriculum and Evaluation Standards for School Mathematics** [Romberg, 1989]. Although some of the Standards are particularly applicable to the teaching of Mathematics in the United States, much is reflected in worldwide trends, such as those outlined in the Cockroft Report in

Great Britain (1980) and the International Commission of Mathematics Instruction Symposium in Kuwait (1986) on **School Mathematics in the 1990s**. In addition to recommending increased emphases upon problem solving, real life applications, computer technology and verbalisation and group work as the preferred means of mathematics instruction [Romberg, 1989; 125-128], the inclusion of discrete mathematics is also recommended. Further, the report goes so far as to list among its "Underlying Assumptions" that ". . *scientific calculators with graphing capabilities will be available to students at all times.*" Such standards are obviously reflected in the current study.

## 4.5 Conclusion

The antecedent situation, then, sees a curriculum dominated by the study of the calculus in a wide range of physical applications. The course is extensive but thorough, and students are given little time to wander far from the path prescribed by the external examination. Traditional teaching methods are favoured, although the target Year 12 group had been exposed to group work and discussion as a normal part of Mathematics lessons throughout their Year 11 studies. Computers, on the other hand, had not been used previously with this group, largely owing to an absence of suitable software which could be utilised on the school's computer system. With the arrival of eight hand-held calculators capable of algebraic manipulation and graphing, the stage was set for a change in the way in which the traditional curriculum might be presented, a change which promised both short term process and outcome improvements, and the possibility of anticipating more significant curriculum change which seems inevitable on a larger scale.

# Chapter Five

# Transactions

In seeking to gather data concerning the **processes** by which the current curriculum innovation was implemented, it was necessary to use a variety of modes. As one of the means of minimising the effects of bias in this internal evaluation, the attempt to observe the situation from a variety of perspectives was considered important. In order to better portray the scene:

- other teachers were involved as observers and co-presenters of the materials;
- classroom observations focussed on a variety of features (from the utilisation of time within the lesson to the group dynamics which accompanied a problem solving exercise);
- a second class was involved in using the series of worksheets and assignments developed for the project (in addition to the sample group);
- data was gathered by means of standard evaluation sheets which accompanied each lesson (Appendix C ), and also from student interviews which were conducted at a number of stages throughout the trial.

In all it was hoped to obtain a balanced view of the various features accompanying the implementation of the computer-enhanced programme which is the subject of this evaluation.

## 5.1 The Intended Transactions

As a result of the Review of the Literature above, it was decided that the programme should be based around a series of structured worksheets (called **Investigations**) which utilised the features of the HP-28C calculators in a way which was intended to aid the development, not only of important mathematical concepts, but also of mathematical skills related to problem solving (such as the systematic investigation of a situation). It was further hoped to encourage students in the use of group work and verbalisation, and the accurate and structured recording of observations and results. These were seen as potentially significant outcomes of this process in which computer technology is used to improve aspects of mathematics instruction.

The two features of the calculators which were perceived as most relevant to the current course were their **graphical** and **calculus** capabilities. If students could be assisted to more readily visualise the graphical shapes of various functions from their algebraic form, improving their ability to **estimate** the graph of a function from its equation, then this would help them to better appreciate important properties of such functions which form the basis for later work, particularly in calculus. Curve Sketching is a central component in each of the mathematics courses in New South Wales designed for those intending to pursue further study of the subject after leaving

school. By allowing immediate feedback on the shape of a curve from its equation, and by allowing students the opportunity to investigate the effects which changes to the equation may have upon the shape and position of its graph, it was felt that the graphical capabilities of the HP-28C calculator could aid in the development of such skills.

Studies over the past six years (particularly in the United States) increasingly point to alternative strategies in the teaching of calculus which may further utilise computer technology [Hansen and Zweng, 1984; Hirsch, 1985; Heid, 1988]. The traditional sequence by which calculus is introduced in the Senior school involves considerable early emphasis upon algebraic skills and co-ordinate geometry techniques. Such an approach emphasises the manipulative skills necessary for the calculation of derivatives of a variety of functions; applications, at least initially, are confined to the properties of curves on the two-dimensional cartesian plane. Although later applications are extended to selected "real-world" examples, these tend to be extremely limited in scope, giving little opportunity for the students to observe wider applications of the objects of their study. Further, while they may become *mechanically* proficient in the calculation of such functions, students frequently demonstrate a minimal understanding of the central concepts underlying these calculations.

In line with the findings of studies by Fey and Good [1985] and Heid [1988], the approach trialled in this study involved the delaying of the mechanical, algebraic aspects of the introduction to calculus. Instead, students began with the applications, aided by a tool such as the HP-28C which could calculate derivatives readily. Such an approach begins with the study of the ideas associated with rates of change in a

variety of situations. Relating these ideas to concepts of "function", students are taught to visualise various relationships, and thus led back to graphical representations. Relating the rate of change of the function to various features of the graph (especially the gradient) develops the notion of the "gradient function" which may then be formalised in the notion of "derivative". The process may be greatly assisted by tools such as the HP-28C, which have both graphical and calculus capabilities. It might be noted here that computer software capable of both these features effectively requires far greater memory and speed capabilities than those which may be supported by most micro-computers in schools.

The programme took the form of seven worksheets and a problem solving exercise (see Appendix D). The worksheets investigated the topics of:

(1) Absolute Value

(2) Curve Sketching and Reciprocal Functions

(3) Equation Solving

(4) Roots of Polynomials

(5) The Gradient Function

(6) Rates of Change

(7) The Derivative.

The last three represented an introduction to calculus for the Year 11 group. The sample (Year 12) group had already begun their study of calculus the previous year in the usual way. This treatment allowed a consolidation and extension of this work pursuing the central concepts in greater depth through both a visual (graphical) approach and at the same time relating these concepts to practical "real world" situations.

It should be noted at this point that the worksheets were not developed "in advance"; rather, they were deliberately developed as the programme unfolded, in response to the student and teacher reactions to preceding work. This was in line with the Action Research nature of the evaluation. As each unit was taught, it was observed, reflected upon, and thus provided the basis for the planning of the next unit. This cyclic relationship between the intended transactions and those observed was an important strategy in this evaluation.

## 5.2 The Observed Transactions

As mentioned above, data on the transactions were gathered from a variety of sources. Each unit was accompanied by a brief student evaluation; some were followed by interviews, some were observed by teaching colleagues. Such triangulation must be considered to be essential in situations where teachers seek to judge the effectiveness of their own work. These methods are summarised in the following table, which also provides a time-line and sequence for the units which were the subject of this evaluation.

| WEEK | UNIT/FOCUS | GROUP | SOURCE OF DATA |
|---|---|---|---|
| 1 | Attitudes | 12/3u | Attitude Scale administered |
| 2 | Attitudes | 11/3u | Attitude Scale administered |
| | *"Absolute Value"* | 11/3u | Student Evaluation Report |
| | | | Teacher Interview |
| 3 | *"Inversions"* | 11/3u | Teacher Observation |
| | | | Lesson Activity Breakdown |
| | | | Student/Teacher Interviews |
| | | | Participant Observation |
| 5 | *"Inversions"* | 12/4u | Student Evaluation Report |
| | | | Participant Observation |
| 6 | *"Equation Solving"* | 12/3u | Student Evaluation Report |
| | | | Participant Observation |
| 11 | *"The Gradient Fn"* | 11/3u | Teacher Interview |
| 12 | *"Approximation of* | 12/3u | Outside Observer |
| | *Roots"* | | Student Evaluation Report |
| | | | Student Interviews |
| | *"Rates of Change"* | 11/3u | Written Assignment |
| | *"The Derivative"* | 11/3u | Written Assignment |
| | | | Student Evaluation Report |
| 15 | *"Introduction to* | 12/3u | Class Discussion |
| | *Calculus"* | | Participant Observation |
| 16 | *"Inversions"* | 12/3u | Student Evaluation Report |
| | | | Participant Observation |
| | | | Student Interviews |
| 23 | Assessment Task | 12/3u | Non-Participant Observation |

Table 5.1 : Time Line for Project

### 5.2.1 Unit One : Absolute Value

The first investigation was intended largely as a pilot, in which students were introduced both to the style and format of the worksheets which were to follow, and also to the Student Evaluation sheets which were to accompany these. It was presented to the Year 11 Three Unit class by their teacher unobserved, since this was agreed to be a less threatening introduction to the programme for both students and teacher. While the students in this class had been introduced to the HP-28C calculators as part of their Year 10 Mathematics course, the teacher had not had this same experience, and was not entirely comfortable with their use at this stage.

The response from students and teacher to this lesson was positive, but with reservations. Altogether, thirteen rated the effectiveness of the calculators as "1" ("Very Good") and eight as "2" ("Of some help"); none rated it "3" ("Of no use") (one student could not decide between 1 and 2, and two chose not to rate them at all). Nonetheless, students and teacher were critical of the effectiveness of the calculator as a teaching tool in this topic - they felt the screen was too small and the graphing process too slow. They pointed out that the graphs were so simple that they could have been explained more easily and drawn more quickly without computer technology.

Students were required to submit the products of their investigation in written form, and they had been instructed to make notes of their observations as they went through the process of the exercise. Still, ten students handed up answers without any comments, and five with minimal comment. Nine students were considered to have presented "good" comments (detailing their observations), all from the same two

groups. In light of the stress which had been placed upon the importance of these comments immediately before the exercise, this result seemed to indicate a poor ability on the part of many of the students to express their observations clearly, and a perception by them that this was not really an important part of Mathematics. This sentiment was addressed later, when the class was given details of the Component "B" requirements for Assessment for the Higher School Certificate - it was pointed out strongly that their ability to demonstrate skills of communication, comprehension, analysis and reasoning were all essential components of the assessment of their Mathematics performance.

**Conclusions :** It was not desired that the calculators should be perceived by the students as a "gimmick", but rather as providing a service which could not readily be achieved without computer technology. In this regard, this unit was unsuccessful, since the functions portrayed were simple enough for students to visualise and graph by more traditional means. Realising this, the next unit was planned to utilise the calculators on the graphing of more difficult functions which could not be so readily visualised.

### 5.2.2 Unit Two : Inversions

This unit was devised as a means of increasing students' familiarity with a wide range of functions. It was developed as a problem solving exercise in which groups investigated the effects of certain transformations upon the graphs of simple functions - in particular, the effect of "inverting" the function, or finding its reciprocal function. Students were to observe any relationships between the graphs of a

function and its reciprocal, and attempt to generalise some "rules for inversion" from these observations. The calculator was to aid in the graphing of these functions, and the testing of their rules on other functions.

This investigation was presented to three groups by the evaluator - the Year 11 group (with their teacher as observer), the Year 12 Four Unit class (a subset of the sample group) and, later, to the Year 12 Three Unit class. At all presentations, students responded positively, indeed enthusiastically - a common sentiment from students and the teacher who observed the lesson was that "this was a most constructive treatment of a difficult topic". Of twenty-two Year 11 students, fourteen rated the lesson "Very Good", eight rated it "Of Some Help". These ratings were consistent across the groups in which the students worked, even to the extent that, of the four class members who indicated that they "would not be interested in using the HP-28's again in other areas", three of these formed one of the groups. These three (all males) found "the calculators interesting, but the work boring". They had rated the lesson "2" - "Of some help".

The individual from another group who had indicated disinterest in using the calculators again, had nonetheless labelled the lesson "1" - "Very Good". When interviewed later, he explained that, while he enjoyed the lesson, he found the graphing capabilities of the calculator "too slow and tedious in practice".

Another student who had rated the lesson highly was also interviewed. He described the "good points" of the lesson as helping his understanding of the concepts of "domain", "range" and

"asymptote". A "bad" feature of the lesson was that "some functions were hard to visualise", although he noted that the HP calculator helped in this regard. He described the lesson as being "different from a normal Maths lesson" in two ways : it was a "move away from textbook questions" and it introduced "new and different concepts".

These student interviews were conducted informally in the week following the lesson, usually in the playground. The students selected represented the full range of responses - those who had rated the lesson in either a strongly positive or negative way, and any who had given responses which seemed to need further explanation. In this way it was hoped to "flesh out" the simple responses given on the Student Evaluation Reports, which had been designed for quick and easy completion by the students, not for in-depth responses.

The teacher who had observed the lesson had agreed to make a record of student activities throughout the lesson - each minute of lesson time was to be classified as "active", "passive" (which included listening to the teacher) or "unspecified" (if they had received no specific instructions for their use of that time). Of the fifty-four minutes of the lesson, eighteen were classed as "passive" (33%), thirty-five as "active" (65%) and the first minute as "unspecified" (2%). It was further decided beforehand, that the lesson would be run with fairly strict teacher time-control - the teacher would introduce each question, allow groups a fixed time to work through that part of the investigation, and then move on to the next section. This was intended to keep the class moving at about the same pace, not allowing groups to move too far ahead or to fall too far behind. After about ten

minutes on a section, the class was brought back together for discussion of their observations and results.

The students interviewed approved of this strategy - they found it helpful in some of the more difficult parts, and it kept the lesson moving. The major purpose of this strategy was to prevent the class from moving too far apart : those who found the problems too difficult (especially in the early stages of the lesson) were given time to work and then helped towards the solution by those who had already achieved it. Used carefully and in moderation, this technique served to minimise the frustration often felt by both ends of the "ability spectrum". It was suggested that such "stop-start" control would probably not be as necessary in future lessons when students were more comfortable with this investigatory style of lesson.

The teacher/observer also considered the time control to be necessary for the effective running of this particular lesson. Problems identified by the observer included :

- how to motivate the better students (a number of the high achievers were among those most critical of the effectiveness of the calculators, since they found that they were able to visualise the graphs and answer many of the questions more quickly than the calculator);
- how to balance the time constraints of the course against the normal "time-expensiveness" of such an inquiry approach;
- the screen size of the calculator again made it difficult to visualise some functions.

It was noted that the students were, in general, very "user-competent" (probably moreso than the teacher in some cases). The main differences from a "normal" Mathematics lesson were seen to centre around:

- the use of group work (22 students and eight calculators required groups of two or three);
- more rigid teacher control of the time;
- the novelty of the calculators, the investigative approach and the change from their usual teacher all meant that students were perhaps more attentive than usual;
- the notion of "a different sort of active" on the part of the students - a wider range of activities than they would perhaps engage in during a "normal" lesson.

When the same approach was taken with the Year 12 students, the responses were very much the same - even though these students were much further along in their course and might be expected to have a very good understanding of curve sketching, they still responded enthusiastically, and expressed very positive comments as to the effectiveness of the approach in developing these skills. The response from the initial Four Unit group (of six students) was so strong that the lesson was repeated with the rest of the class.

**Conclusions :** There was no doubt that this approach was successful in both capitalising on the capabilities of the graphics calculators as an aid to concept development, and in the effective use of group work and investigation techniques as a means of approaching an important but difficult topic. It would seem likely that these should go together - that the effective use of computer technology implies an active, co-operative and inquiry-based approach to learning.

Although this had been a very effective use of the calculators by all criteria, the Year 11 group was showing definite signs of tiring of their use. They were beginning to see them as "gimmicks" and view themselves as "guinea pigs"; the Year 12 group, which had been more involved in planning and evaluating the programme, were showing no such feelings.

In consultation with the teacher of the Year 11 group (who had acted as collaborator thus far in the project) it was felt that a change of strategy was needed. In this way the flexibility afforded by the Action Research Model could be used to advantage : the cyclic process of Plan-Act-Reflect-Revise Plan proved most suitable here.

Two options were devised :

(1) It was decided to move away from the graph-plotting capabilities of the calculators to the use of one of the other functions. Possibilities included equation-solving (a powerful function which took the graph-plotting mode further), matrices and simultaneous equations, or the use of the symbolic manipulation capabilities of the calculator, in terms of algebraic manipulation. Other possibilities recognised at this stage (which did not involve the use of HP-28 calculators) included graph interpretation exercises (for the development of function concepts), the use of computer spreadsheets to investigate limits, or an introduction to the gradient function as a lead-in to calculus.

(ii) It was further recognised that individual use of the calculators by students would probably increase both their familiarity and confidence in this regard - it had been noted that some groups had allowed their use to be dominated by some of the more confident members, while others less inclined were avoiding their use. It was decided to organise "week-end borrowing rights" for the students in both the Year 11 and Year 12 groups. It was hoped that this would make students more comfortable and competent in their use, and perhaps allow the pursuit of some further investigation.

**5.2.3 Introduction to Calculus :**

In keeping with the suggestion of the Year 11 class teacher, it was decided also to allow a "cooling off" period for this class which might help to head off the negative feelings which were beginning to emerge towards the use of the calculators. Instead, a series of three worksheets were developed as an Introduction to Calculus, focussing upon rates of change and the gradient function. The calculators were not required until the third of these projects, in which their calculus capabilities could be utilised in order to explore the concept of "derivative". These were set as a major assignment, and students were given a number of weeks to complete it; regular class lessons were devoted to questions and discussion of the work over this period.

The Year 12 group, having already been introduced to calculus in Year 11, were given the opportunity to work through and discuss these three worksheets in class. They found the experience extremely beneficial, describing numerous "a-ha" experiences in which terms such as "derivative", "gradient function" and "point of inflexion" assumed

new meaning. These students were all competent in the mechanical calculation of these terms, but agreed that they did not fully understand them and their implications prior to this graphical approach.

The Year 11 group, on the other hand, did not fare so well. Most achieved a satisfactory result on the assignment, and demonstrated a fair understanding of the relationship between such features as velocity, tangent and gradient of given real-world functions. Only four, however, showed any appreciation of the nature and applications of the second derivative, or its relationship to points of inflexion.

**Conclusions :** It would appear that, in many respects, this method did not provide a fully successful introduction to calculus. While students did appear to gain a good understanding of the relationship between velocity and gradient, and the nature of the derivative, other more difficult concepts eluded them. The Year 12 students, on the other hand, found the process of enormous benefit, after having used these tools of calculus with apparently minimal understanding of their nature for some twelve months.

An approach such as this to calculus, which relates the concepts to real-world examples and encourages students to visualise the functions involved in graphical form, does hold promise for senior students. It is certainly effective if used to consolidate the more traditional approach, and would still prove an effective introduction if carefully planned and presented, and given sufficient time for the difficult concepts to be appreciated. Such a project, developed by Mary Barnes of the University of Sydney (*Introductory Calculus Project*,

1990) is currently being trialled and appears to fulfil much of this promise. Trials with the current Year 11 Three Unit class at Saint Joseph's indicate that this approach to the teaching of calculus can indeed be more effective than traditional methods.

**5.2.4 Other Investigations :**

Although the Year 11 group required a break from the HP calculators, the Year 12 sample group showed no such desire. Consequently, an investigation which utilised the Equation-Solving capabilities of the calculators was developed, and students were permitted to borrow them for a number of nights in order to complete the investigation. Once again, while remaining positive, this was not perceived by students as being a worthwhile application of computer technology - it did not sufficiently extend their current knowledge and skills enough to justify its use. Only 50% of students rated it "Very Good" and the other 50% as "Of some help".

Two features here are worth noting. The first is that no student in either class had yet rated the effectiveness of the worksheets or the calculator with a "3" : "Of no use". While some had not been keen to use them again for a variety of reasons, they were not perceived as useless. At the same time, the Year 12 class, while positive in attitude towards the project, is nonetheless a critical audience. They are not awed or overcome by the use of technology; rather they have been quick to point out deficiencies in the use of the calculators. In particular, they have recognised those times when it was appropriate and effective to use computer technology, and those times when it was not. In short, they have proved able collaborators in this internal evaluation.

One other investigation is worthy of mention, involving the use, not of HP-28 calculators, but of APPLE IIe computers using a spreadsheet program with graphics capabilities (APPLEWORKS with TIMEOUT GRAPH application). In order to provide students with an experience of some other application of computer technology, an investigation was prepared in which the Year 12 students used the computer spreadsheet to calculate the Roots of Polynomials by successively "zooming in" on the irrational point where the graph of the polynomial crosses the horizontal axis. While the students were able to use the equation solving capabilities of the HP-28C to calculate the value of such roots almost instantly, this method of successive approximation gave far more meaning to the nature of the process by which such approximations may be calculated. It also further developed the students' understanding of polynomials - as one pointed out, ". . I also gained a different perspective of what a polynomial is - a number -> number function rather than a graph or an algebraic equation only". The capacity of the spreadsheet to "expose the inner workings of the process" was invaluable. Another Mathematics Faculty colleague who offered to observe the lesson found it to be extremely positive; she was keen afterwards to look at ways of using computer spreadsheets with some of her junior mathematics classes.

The calculators were also used to explore the two approximation methods for roots of polynomials - Newton's method involving the calculation of derivatives, and the method of successive approximation usually described as "halving the interval". By using the algebraic substitution capabilities of the calculators, the usual tedious calculations involved in both these methods were minimised; students were free to explore the relative efficiency of each method for various

types of functions. This was again found to be a very worthwhile application of computer technology.

### 5.2.5. A Problem Solving Task :

As one of the outcome measures of the evaluation, a major assessment task was developed which sought to test the extent to which students had understood the central concepts of calculus and their ability to estimate the graphs of given functions. These were the two main features of the programme of worksheets described above, and it was necessary to gauge the effectiveness of the process. Most of the questions for this task were drawn from a similar task reported in Heid [1988], based on real-life situations and demanding a good understanding of the core concepts of calculus. Since these questions were quite unlike any that the students had encountered in their traditional studies, it was decided to provide a practice question which was to be attempted in small groups, and then discussed in class. The class teacher had the opportunity to act as "non-participant observer" while the students worked in their groups, and so to closely watch the dynamics of the problem solving process which unfolded.

The question was chosen so as to be as open-ended as possible, and to draw on their knowledge of the concept of "derivative" applied in an unfamiliar situation. The following question was presented to the students :

> *"Suppose transportation specialists have determined that* ***G(v)****, the number of miles per gallon that a vehicle gets, is a function of the vehicle's speed in miles per hour.*
>
> *(a) Interpret, in terms of mileage and speed the fact that*
>
> **G'(55) = 0.4.**

*(b) How might that fact be used in a debate about setting an appropriate national speed limit?"*

The students were free to choose their own groups, and the processes of these groups were observed over the period of some twenty minutes given. Initially, students grouped into two groups of three, another two pairs, and two individuals. After some five to eight minutes, the individuals had joined with larger groups, and after fifteen minutes, all groups had merged into more or less a whole class discussion. At this stage, each student reported their progress, and together with the teacher worked towards a solution.

The investigation of such problems, both in groups and individually, was perceived by students as contributing a great deal to their overall understanding of the concepts involved. The use of carefully chosen problems would appear to be as useful a tool as carefully structured worksheets in the process of concept development.

## 5.3 Conclusions

The potential for computer enhancement of the senior Mathematics syllabus is clearly great. A range of areas have been explored, many others await such a treatment. As the power and flexibility of such "classroom technology" increases, it would appear to be the skill and imagination of teachers which provide the greatest limitations to the widespread use of such aids as a means of improving many facets of the teaching of Mathematics at this level.

As suggested in the literature, structured worksheets would appear to be an effective tool in facilitating the use of computer technology in

the classroom. From the evidence of the processes observed above, these need to be inquiry-based and open-ended where possible; they should also be supplemented by open-ended assessment tasks as a a stimulus for both class discussion and individual investigation.

Finally, the message coming from the polarisation of attitudes towards the calculators between the two groups is clear : If computer technology is to effectively enhance the curriculum, it must be perceived as **genuine** - not some "add-on optional extra", but a natural part of the programme. This was the perception of the Year 12 group - as topics occurred, the calculators fitted into the topic. The Year 11 group perceived the reverse - they felt that the curriculum was being manipulated in order to accommodate the technology. The further fact that they did not feel the same "ownership" of the process that the Year 12 group experienced is also an important feature of which teachers should take note. As in all good Action Research projects, the process must be **democratic**; those involved, like the curriculum, must not be manipulated in order to make the technology "fit".

# Chapter Six

# Outcomes

A classroom is a complex and unpredictable place. Made up as it is of individuals, the effects of any interaction will vary significantly among those involved. Attempting to assess the full impact of a curriculum innovation, then, becomes a difficult if not impossible task. Since no two individuals will respond identically to any given situation, it becomes important to describe many of the outcomes of the present study, not as the results of a "group" or "class", but as the responses of individuals. While such tools as the Attitude Scale may prove useful in indicating general trends which have occurred over the period of the evaluation, judgement of the effectiveness of the implementation must derive from consideration of the effects upon individual participants.

As described in the outline of the research methodology, data on the outcomes of this curriculum innovation is derived from four major sources :

(i) the Attitude Scale administered previously at the start of the year, and then again at the end;

(ii) a major assessment task which provides a measure of the extent of understanding which the students have attained regarding the central concepts dealt with in this innovation;

(iii) formal interviews with students, and

(iv) observations and perceptions of the teacher regarding the attitudes and responses of the students over the course of the programme. As a participant-observer, the insights available to the teacher figure significantly in describing and understanding the situation.

Of interest also are two other factors - the performance of the students at the Higher School Certificate, and their subsequent choice of career. Taken together, these diverse sources provide a rich account of the effects of the intervention upon the participants, and allow judgements to be made regarding the effectiveness of the implementation.

## 6.1 Changes in Attitude

It was noted at the outset of the study that the differences between the sample group of Year 12 students at Saint Joseph's and their peers at Holy Spirit College were not extensive - in fact, there were few items for which the differences could be considered large enough to be thought of as significant, either statistically or otherwise. At the end of the period of the study, it is possible to consider two features from the responses to this scale : changes *within* the groups (which may be thought of as *internal* changes) and changes *across* the groups over the year (*external* or *comparative* changes). It is interesting to note that the majority of changes observed were of the former type - in fact, the most significant variations occurred within the sample group between the start and the finish of the year. The Holy Spirit group showed small variation over that period. The major observable trend was for the Saint Joseph's students to move to responses more similar

to their counterparts at the end of the year than was the case at the start.

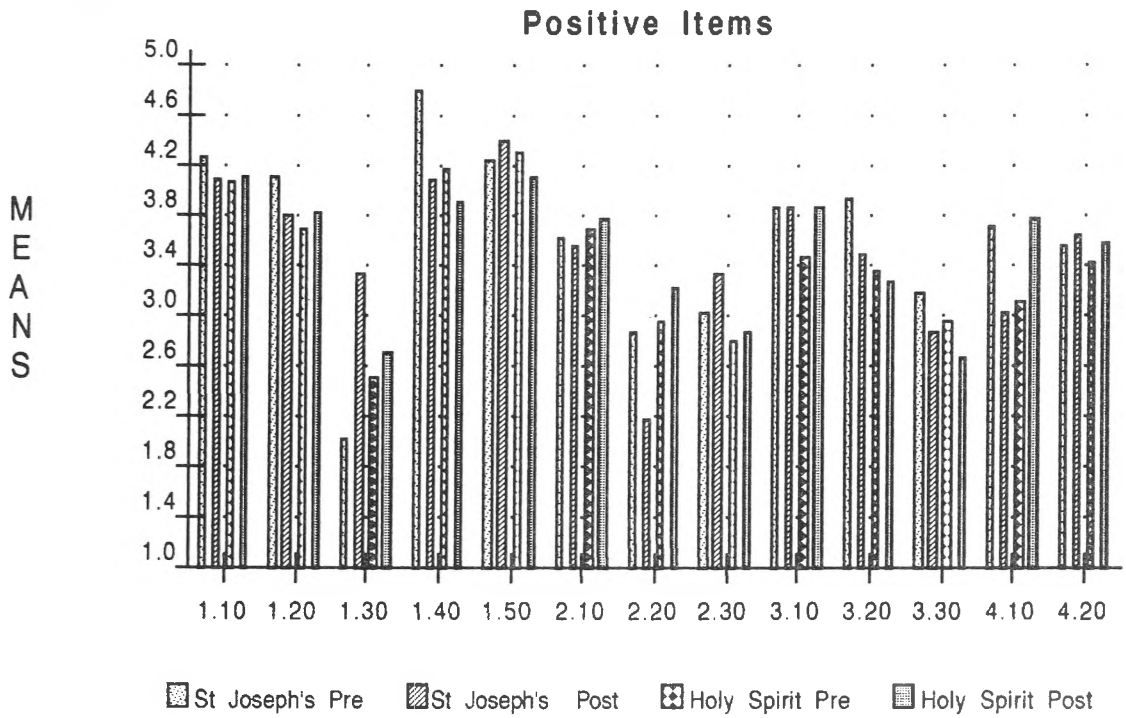
In order to observe trends across the items on the scale in addition to those across the two groups, the results of the pre- and post-applications of the Attitude Scale have been presented on two graphs only - one for the positive items, and the other for the negative items. Presented in this way it is clear that certain items drew more emphatic responses from the groups than others.

In addition, it was decided to apply a t-test (Fischer's T-Test for Unrelated Samples) to the results derived from each school in order to obtain a more objective measure of the significance of any differences observed between the initial and final applications of the Attitude Scale. The null hypothesis in this instance is that there will be no significant differences between the two results for each group - in other words, that each could be assumed to be drawn from the same respective statistical population at both applications of the Attitude Scale. Between the two schools, it was most notable that the two groups had effectively become more similar over the period of the evaluation. This is perhaps not surprising in light of the common Year 12 experience they shared, preparing for the same examination.

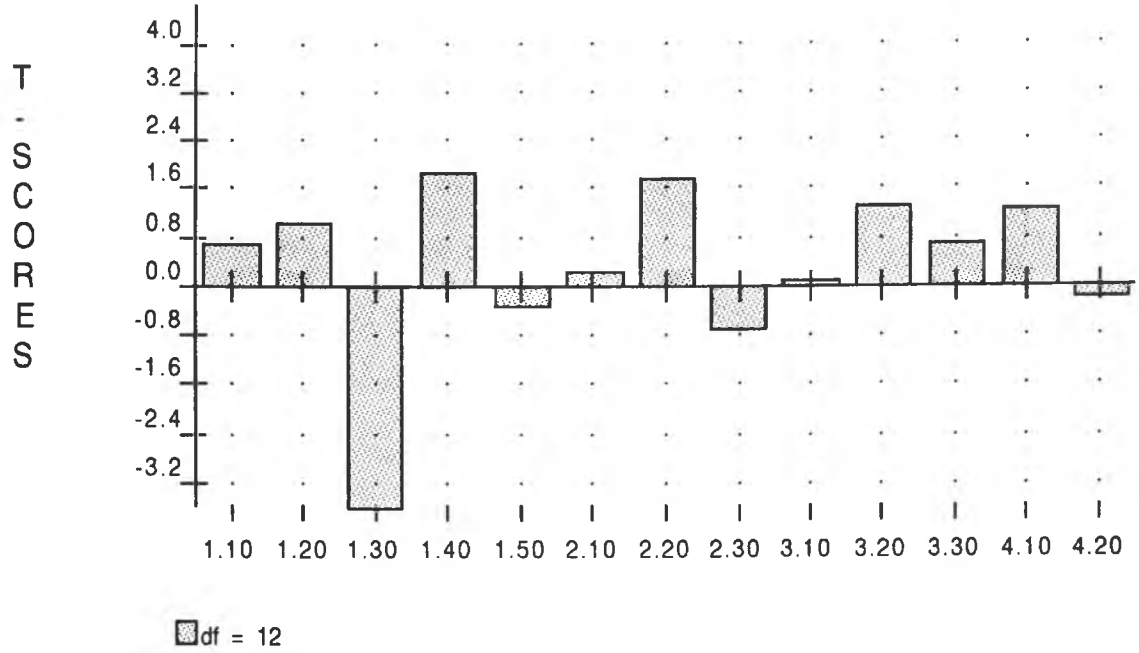
A statistically significant difference between the Pre- and Post applications for the Saint Joseph's group would add weight to the argument that the computer enhancement of their Senior Mathematics course was a worthwhile innovation. At the same time, the Holy Spirit group cannot be considered in any real sense a "control" group - while their experience may not have involved computer technology, changes

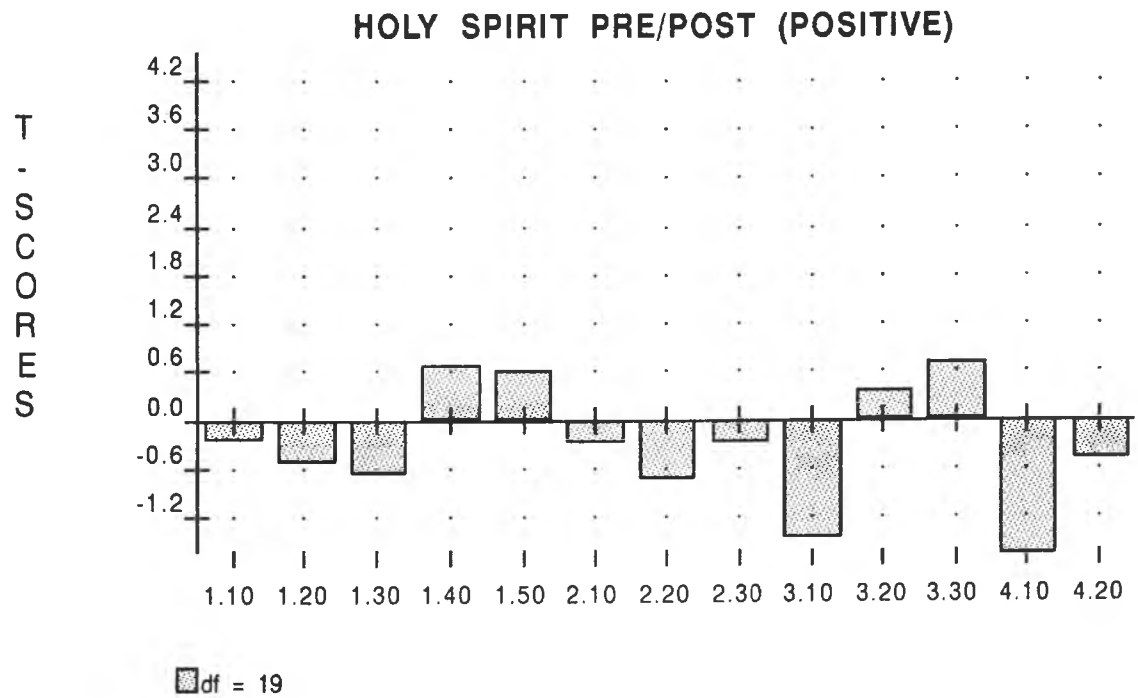
over the period of the evaluation may well have derived from other factors such as the effects of the teaching techniques, personality of the teacher, the sequencing of the instruction or even socio-economic factors for which there were no means of control. As noted previously, statistical techniques in such circumstances can in no way be considered as allowing clear and distinct interpretation - rather, they indicate tendencies which may add further weight to the arguments regarding the efficacy of the innovation.

Graph 6.1: Mathematics Attitude Scale (Pre- and Post-Results) Means of Positive Items.




The T-Scores for each school are presented in graphical form in order to better present the results for each item, allowing comparisons regarding the strength and direction of each score.




Graphs 6.2 and 6.3 : T-Scores for Positive Items (Pre- and Post)

For a class of thirteen students (with twelve degrees of freedom) the critical values of "t" at the 0.05 and the 0.01 levels of significance are given as **2.179** and **3.055** respectively. Thus for the Saint Joseph's

group we observe one positive item which may be regarded as having undergone significant change over the period of the evaluation - item 1.30 ( *"Maths is easy"* ). We observe no significant changes in the Holy Spirit group at either level of significance, the group of twenty students dictating nineteen degrees of freedom, with critical values of "t" at **2.093** (0.05 level) and **2.861** (0.01 level of significance). It was decided to use a "two-tailed" test since we were interested in a change in either direction on the items of the attitude scale. Such small group sizes make it unlikely that *any* significant difference will be detected, the probablility of a change being large enough to be considered significant is very slight. Thus, the fact that an item proved significant at the 0.01 level is important; further, others which came close to the critical values may also be worthy of note.

Item 1.30 proved to be a most revealing indicator concerning attitudes of the students towards mathematics and their abilities in this regard. While both groups showed a strong upward movement over the year (indicative of increasing confidence in their abilities), only the sample group moved from a "negative" response (less than the "neutral" mean of 3.00) to a "positive" one. The Holy Spirit group remained less than 3.00 at the end of the year - in other words, they were reluctant to agree that "maths was easy". It was interesting to note that the sample group began the year **less** confident in their abilities than their counterparts. The increased confidence indicated by this item, then, would appear to be a significant outcome of their Year 12 Mathematics experience.

The T-scores reveal another trend between the two schools in the **direction** of the change over the twelve months. Since the calculation of the group T-scores involved a subtraction of the final mean from the

initial mean, an item whose mean score increased over the period would register as a **negative** score. Of the thirteen positive items, Saint Joseph's produced only four such negative scores, while the Holy Spirit group produced nine. Although this indicates that the latter group became substantially more positive towards the aspects of Mathematics under consideration over the year than did the sample group, such a result must be read in context. At the start of the year, the means of the Holy Spirit group were in general far lower than those for Saint Joseph's - their attitudes were far less positive. Hence they moved upwards over the time in many cases only to become more similar to the sample group. In fact this was an overall trend - the two groups over the Year 12 period became in many respects more similar to each other than they had been previously. As mentioned above, this may well be attributable to the common Year 12 experience of the two groups - a relatively short period in their school careers, but nonetheless a most important one.

It is worth distinguishing at this stage between items which are **statistically significant** and those which we might describe as **educationally significant** in this context. Thus, items 2.20 (*"The maths questions I enjoy most are those that involve real-life situations"* ) and 2.30 (*"Using computer technology is a great way to learn maths"* ) are important in the context of this study, even though they did not display significant change over the period. Like item 1.30 above, these two items stood out in that their means (both initially and finally) were below those for most other positive items. Initial responses were largely neutral, although final responses had tended to polarise in most cases. The sample group actually disagreed quite strongly with the item concerning real-life situations by the end of the year. This is

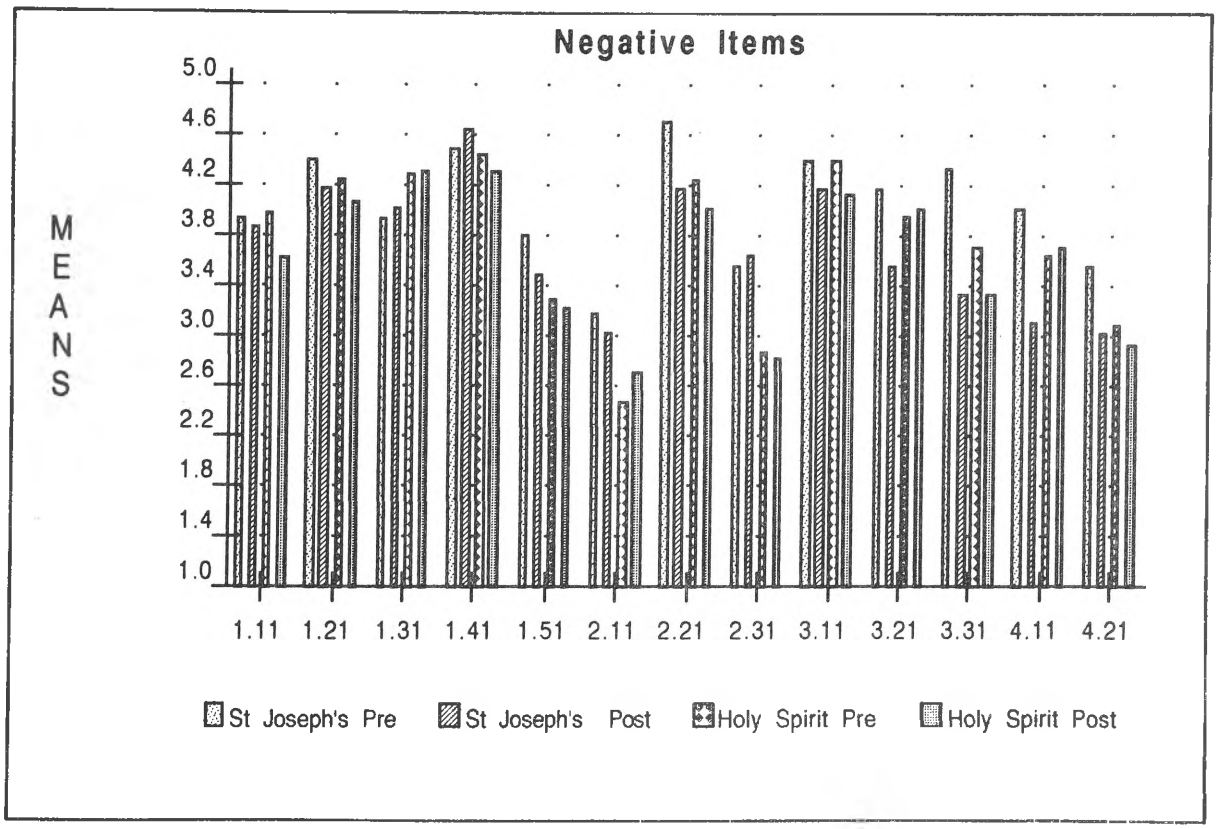
important, since much of the emphasis in the Calculus components of the innovation was upon such real-life applications. Discussion and interviews with the students later indicated that, in spite of this, they felt quite positively about such questions. The decline observed in the sample group on item 3.20 (*"I like doing problems which show unexpected relationships"* ) may help us to understand this apparent inconsistency. By the latter part of the year, the students were acutely aware of the pressures of external examinations, and showed impatience with tasks which were not perceived as contributing towards their preparation for this important hurdle. Questions which were unexpected and unrelated to the examination were found to be frustrating. There may perhaps have also been in some senses an "overdose" of "real-life" questions for the sample group - the Holy Spirit group showed a slight improvement in their attitudes towards this item. Such attitudes could well benefit from further investigation in learning how best to implement such an enhancement to the senior course.

While the improvement in attitude towards computer technology demonstrated by the Saint Joseph's group (item 2.30) was not large enough to be considered significant, it is worthwhile noting that it corresponded to a negligible change in the Holy Spirit group in this regard over the term of the evaluation. The approval shown by the sample group towards the use of computers appears to be very much tempered with caution - they are not agreeing unconditionally with the statement, perhaps because they perceive the inadequacies as well as the advantages associated with the use of such technology.

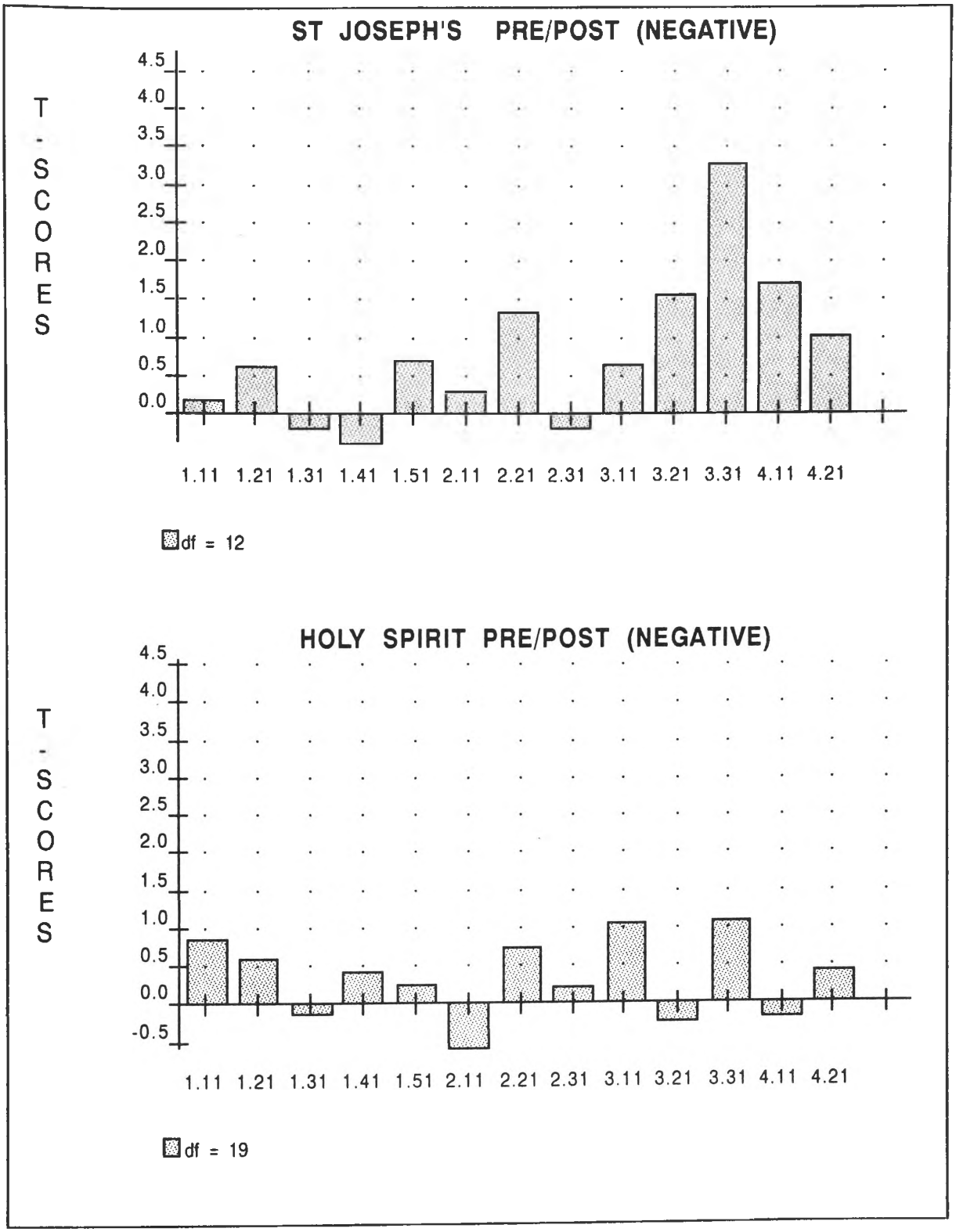
The only other substantial (if not statistically significant) changes which may be observed from graph 6.1 are those for items 1.40 (*"Maths is one subject I would really like to improve in"*) and 4.10 (*"I expect to have to do a lot of maths after I leave school"*). The former item saw the Saint Joseph's group move from a high initial mean to one which corresponded to that displayed by the Holy Spirit group at both samplings. It is possible to consider this high initial response in the light of the previous item, 1.30 (discussed above). At the start of the year, the sample group were considerably less confident about their mathematical ability than their counterparts, and hence saw considerable room for improvement. By the end of the year, their confidence had blossomed, with less urgency perceived in this regard.

Item 4.10 displayed a strong change in the means of both groups in opposite directions. The sample group moved from a position of fairly strong agreement to one which was decidedly neutral, while their peers moved the other way. Since it may be assumed that the students would have a very clear idea of their future career paths by the end of the year, this would indicate quite a strong change over the time - in other words, a number of these students have changed their minds in that time. More particularly, less of the sample group anticipate a strongly mathematical future. Light may be shed upon this result by consideration of the last positive item, 4.20 (*"I would like to continue to learn more about maths in the future"*). Here we note that the sample group have agreed even more strongly at the end of the year than at the beginning - even though some of them do not expect to do much maths in the future, they would still like to learn more about it. This is indicative of a positive attitude towards the subject quite distinct from any utilitarian purposes which students may perceive.

We note that on the adjusted negative items, the higher the mean, the greater the disagreement with that particular item and so in general the more positive the attitude towards mathematics. Again we may observe an overall tendency for the two groups to move towards more similar positions at the end than they held at the outset, with the majority of shifts for both groups being in a downward direction. When one considers that the final year of schooling, while obviously a significant and crucial year for the students, accounts for only one thirteenth of their school experience, it would be surprising if there occurred very much in the way of strong changes in attitude at all in this relatively short period. In this sense, too, changes too small to be considered statistically significant may yet be worthy of consideration in understanding the attitudes of the students towards the subject.




Graph 6.4 : Mathematics Attitude Scale (Pre- and Post-Results) Means of Adjusted Negative Items.




Graphs 6.5 and 6.6 : T-Scores for Negative Items (Pre- and Post)

Once again we observe no significant change at all for the Holy Spirit group. Statistically they may be considered as having come from the same population at both applications of the Attitude Scale. Although

changes have taken place as part of their Year 12 experience, these have not been large enough to substantially alter the attitudes of the group over the observed period.

The sample group, on the other hand again displays a change large enough to be significant at the 0.01 level of significance - item 3.31 (*"Maths is doing the same thing over and over again"* ). Both groups disagreed less strongly with this item by the end of the year than at the start, the sample group significantly so. Again, it is likely that the latter part of the year in Year 12 having been spent in preparation for the examination, allowed little opportunity for creative teaching. It is interesting that the two groups moved at the end of the year to almost identical positions on this item, adding further support to this argument. Such routine was however perceived by the sample group as quite different to that experienced previously. With regard to the computer enhancement factor, however, the item appears to have little to say.

The item related directly to computer technology, 2.31 (*"Computers are not much use in learning maths at higher levels"* ) drew similar responses to its paired counterpart, 2.30. The Holy Spirit group changed little from its neutral position - they appeared to have little opinion about this item one way or the other. The Saint Joseph's group disagreed with the statement, and changed little in this view over the period of the evaluation. It is possible then, in light of their new awareness of both the advantages and disadvantages of computer technology, that a "cancelling out" effect may have occurred in terms of positive and negative attitudes of students towards its use. Such questions remain problematic, requiring further information from the students in order to

be resolved, information which may well be forthcoming from the interviews at the conclusion of the study.

Other changes of interest concerned items 3.21 (*"If I can't work out a problem quickly I prefer to give up"* ), 4.11 (*"Maths will play little part in my future career"* ) and 4.21 (*"I will be glad to be finished with maths after Year 12"* ). Each adds insight into the attitudes, particularly of the sample group, and each may best be understood in the context of the approaching external examination.

While responses to the item on problem solving (3.21) may give an indication of greater honesty on the part of individuals who earlier may not have wanted to admit such a weakness, it may also represent the results of training in examination technique as the Higher School Certificate approaches. Students are advised not to waste time on the most difficult questions, but to leave these and come back to them if time permits. In this context, it is a reasonable response to the item.

The two items on the role of mathematics in the future provide further evidence that less of the sample group anticipate a mathematical career, while those at Holy Spirit are moreso inclined than previously. Further, the pressures of the examination would appear to be taking their toll - both groups moved to a position where they did not disagree that they would be glad to finish with maths after Year 12. This needs to be read in the context of "school maths", since the sample group at least had already indicated that they would like to learn more about the subject later on.

**Conclusions** : The changes which occurred in both groups over the period of the study are highly informative as to the nature and attitudes of these groups. It is clear that the sample group is still positive in attitude towards the subject - moreso than their peers at Holy Spirit, although there has been a decided improvement in the attitude displayed by this group. While this latter group has become more uniform in their determination to pursue careers which involve mathematics, there is evidence that at least some of the sample group have no such intentions. Nonetheless, this has not stopped them from enjoying the subject, nor from displaying improved confidence in their abilities in this area.

Finally, the attitudes of the sample group towards the use of computer technology as an aid to the teaching of mathematics are certainly more positive than those of their peers, who had enjoyed no such experience. These attitudes, however, are far from strong, and there would appear to be distinct hesitation on the part of the Saint Joseph's students in giving any type of wholesale endorsement of the effectiveness of such means. It will be a major purpose of the student interviews which follow to attempt to explain this hesitation: to give students the opportunity to express the strengths and the weaknesses which they have perceived in the use of such technology. The interviews would be expected to provide further detail to the picture so far painted from the results of the attitude scale above. In order that the interview situation would not be influenced even unconsciously, the responses to the scale were not sighted by the evaluator prior to speaking with the students. There is always a temptation on the part of the researcher to ask leading questions and to pre-empt responses which

suit the purposes of the study, rather than presenting the views of the participants.

## 6.2 Evidence of Concept Development

As mentioned previously in describing features of the Transactional level of the evaluation, the use of carefully chosen open-ended assessment tasks can be a very valuable learning experience. The ability to apply a concept in an unfamiliar situation would appear to distinguish between the "mimicking" of such a concept and its "internalisation", which may be thought of as "true" learning. In the observation of students in classroom situations, it is often the application of a concept which seals the student's understanding. As part of a research project entitled "*Resequencing skills and concepts in Applied Calculus using the Computer as a Tool*" [Heid, 1988] Heid developed a series of questions which tested the understanding of many of the concepts central to calculus. These questions formed the basis of the assessment task designed for this project [see Appendix D], testing the extent of understanding of the nature of the derivative and points of inflexion (expressed verbally and by application to both graphical questions and real-life situations) and the ability of students to estimate the properties of the graphs of functions given their algebraic form. In most cases, students were required to explain their reasoning; they were also required to apply their knowledge of functions and calculus in a number of unfamiliar situations. While the questions chosen are not of the type to be found on the Higher School Certificate examination (requiring less mechanical calculation using calculus techniques), they do assess the ability of students to communicate, to analyse information presented in a variety of forms, to

comprehend the nature of the question and to reason through to a solution - the four central assessment areas labelled "Component B" in the Syllabus which are required to be addressed as part of the overall assessment of the Senior Mathematics course.

The first question was chosen as a relatively simple application of fundamental skills - students needed to understand the nature of gradient, and to correctly interpret the question. Only one student failed to read the question carefully and scored poorly; ten others scored full marks, the other two lost only a single mark.

Question Two was more difficult, assessing students' understanding of the differences between the function itself increasing and decreasing, and increases and decreases in the rate of change of the function. By relating the question to a deer population, students had to distinguish between periods of time when the population was declining, and those when the rate of change of the population was increasing. Only one student answered all parts correctly; most had problems with the second part of the question, although nearly all were able to state when the population was increasing at the greatest rate.

The third question involved the application of familiar calculus notation to an unfamiliar real-life situation, namely unemployment rates. All students performed well on this question, indicating an ability, not only to apply their understanding of calculus to a given situation, but also to express this understanding verbally in an effective way. This would appear to be an important outcome of the course; such questions are not normally part of the Three Unit Mathematics course, and are not to be found in any of the texts. It

would be anticipated that students without such preparation would find such questions difficult, not because the concepts are too difficult, but simply because they are not usually expected to express such concepts verbally.

The ability of students to reason through an unfamiliar calculus problem (Question 4) and to verbally explain the concepts of "derivative" and "point of inflexion" (Question 5) was variable. These questions tended to discriminate well over the whole group, although seven of the thirteen still scored more than seventy-five per cent on these two questions. Overall, the concept of derivative was well understood, although point of inflexion still eluded some. These are of course difficult concepts to verbalise, and they tend to highlight the deficiencies of the traditional treatment of calculus (in which students become proficient at *calculating* such functions without any real understanding of their nature).

The final question was intended to test the ability of students to *estimate* the graph of a function from its algebraic form - a skill which we would hope to have been encouraged through the use of the HP-28C calculators. All but one of the students scored better than twelve out of a possible sixteen marks on this question (this one scored ten). This would indicate a high level of proficiency in this area - the graphs chosen were difficult ones and students had to not only use their reasoning abilities, but also to justify their answers.

**Conclusions :** The results on this Assessment task provide evidence that the approach taken in this computer enhancement of the senior mathematics course was effective in a number of areas. Most

importantly, students demonstrated, not only a good understanding of the important concepts required by the course, but also an ability both to express this understanding verbally and to apply these concepts in unfamiliar situations. Such Assessment Tasks appear to encapsulate the major emphases of the Component B requirements of the Syllabus, thus fulfilling formal requirements as well as providing students with a deeper understanding and appreciation of the central concepts of the Calculus.

## 6.3 Informant Interviews

Of all the information gathered throughout the course of this evaluation, from all the various sources and in many different forms, that which was found to be most valuable from the point of view of the classroom teacher was undoubtedly that derived from student interviews. In particular, the use of such ethnographic techniques as the "Grand Tour" question [Spradley, 1979] proved to be an invaluable source of interesting and useful insights into the workings of the class from the "other side of the desk". The interviews were conducted singly or in pairs, each of some twenty minutes duration. Although it was stressed that the questions be taken seriously and answered with candour, the atmosphere was friendly and open.

The opening question was "Describe a Typical Maths lesson"; invariably, after a moment of stunned silence, the description would begin - no two the same, and yet the same common themes easily recognisable throughout. The usual embarrassment and awkward silence often associated with teacher-student interviews was dispelled; instead the students waxed eloquent, encouraged to comment at last on something

in which they were the experts. The teacher had only to listen and be educated.

The "typical maths lesson" for this group is easily recognisable by its "lazy start". Certain students are "always asleep", others are eating, the teacher is "usually late". The "settling down period" at the start is seen as an important social time - "different from Year 10", "fun", people talking about their problems with the homework, "at least three jokes", "constantly learning" in a "relaxed atmosphere".

The lesson then branches into one of two types - a "questioning lesson" in which students pursue questions not fully understood from previous work, or a "new concept lesson", in which the teacher "presents an idea from the syllabus, its applications, hints and related concepts and (sometimes) a piece of mathematical history". Students describe "hitting the wall" halfway through the lesson - some people get on with their work and continue this way through the remainder of the lesson; others stop to talk. The teacher "wanders around, talking about work, answering questions, sometimes just talking".

Having described a "typical maths lesson", students were then asked to describe a "typical lesson with the HP calculators". These were commonly seen as "more organised", more "structured" than a "normal" lesson. Such lessons are characterised by more formal groupwork, by "discovery" and "research"; the use of structured worksheets made the lesson more "well-defined"; there was a more easily recognisable "objective" to the lesson.

The calculators were seen to "expand upon a small area of what we are doing"; "because it is usually a well-explored topic, (the calculator) aids concrete understanding". It was noted that the "concept development in groups does promote understanding, but takes much more time". Such calculator lessons seemed to be "looking at something from a different point of view", helping you to "understand things you took for granted". It was felt by one student that the purpose of the calculators was "not to learn more, but more to *appreciate* maths more". They were seen as a "really good way to illustrate different ways to approach problems", and to "learn different things about maths generally rather than just doing 'book learning' ".

One student mentioned that "some people sometimes left the lesson thinking we had not really covered anything new" and that some people "felt defeated by the calculators". An "awareness of how to use the calculators" became an issue in addition to the mathematics being learnt. Some felt it "counter-productive working in groups", preferring to work alone. There were also "dominant people" who would "emerge and overpower others in a group". Generally, however, most people worked effectively together - more people were "interested in getting the work done", both because of the novelty effect and also because the "aim of the lesson was clear". Sometimes two groups would join together to tackle a problem, and from this "pooling of ideas, everyone's knowledge was expanded".

"Normal" lessons were most characterised by features such as "concept development", "skill development" and "discussion". "Calculator" lessons were characterised by "problem solving", "verbalisation" and the study of "applications".

**Conclusions :** Much is made in traditional research modes of the need for data to be *objective* - free of the taint of the individual, precise and replicable, valid and reliable. It seems that the attempt is to understand human behaviour by removing as much that is human as is possible. The data above gives the lie to such a stand; while it may be difficult to replicate, it offers insights into the situation denied by any other means. It effectively presents the impact of the innovation from the most important perspective of all - from that of the participants. The responses of the students may be validated through triangulation with two other sources - the classroom teacher as "participant-observer", and other teachers who had observed the class in action both formally and informally. Both sources recognise the descriptions above as capturing in some sense much that was characteristic of the class situation. As researcher and classroom teacher, the interviews which gave rise to the data above became sharing experiences with the students in which common perceptions were recognised. Two other teachers who had been associated with the group provided further validation of these observations, recognising in them an experience consistent with their own.

In seeking to identify and to understand the successes and failures of any implementation, it is the participants who hold the answers. It is these responses which lend support and validity to the other types of data collected; the statistics and graphs of attitude scales and the like must face the judgement of the people in the classroom, and it is there that their accuracy and appropriateness must be decided.

## 6.4 The Final Outcomes

So where did they go from there?

It is hardly surprising given the evidence of their obvious mathematical ability that this group performed well at the Higher School Certificate. While it was never the claim of this project that the use of computer technology would lead to improvements in examination performance, it was always of critical concern that it should in no way detract from such performance. In particular, the danger of taking too much time to achieve improvements in concept development was an ever-present concern. The students noted correctly that approaches which utilise discussion, group work and discovery learning will often take longer than more traditional methods, and supposed greater understanding of concepts does not always guarantee improved performance in a skill-based examination.

Concern for time, then, was always of high priority - it always is in any preparation for the Higher School Certificate, but especially in an innovative one. Nonetheless, it was found to be quite possible to complete the content of the course in the allotted time, even allowing for such diversions as have been described here. The performance of the sample group at the Three Unit examination paper was the best of any of the three groups which the school has so far presented. Such a result would of course have been likely without any innovation, and it is not claimed that the use of HP-28 calculators will help students to score better marks in an external examination - merely that it should in no way hinder such capable students.

A final point of interest, however, cannot be fully appreciated without some reference to the nature of the innovation under discussion. Of the previous Year 12 Three Unit classes, it has been usual for the greater number of such students to pursue careers in the fields of Commerce and Engineering. An obvious factor in this has been the availability of traineeships for students of high ability from firms involved in the local steel industry.

Of the thirteen students in the sample group, however, only three opted for Commerce and one for Engineering; two others have gone into the arts. The other seven have begun studies in Mathematics and Computing. This proportion is significant - in previous years, the proportion of the Three Unit classes which pursued Mathematics-based careers had always been less than thirty percent; this group saw over fifty percent choose such paths. In the light of the current project it is not inconceivable that the positive attitude towards the subject, the greater understanding of the major concepts and relationships and the increased appreciation of the applications of computer technology and mathematics afforded by the computer enhancement of their Mathematics course may have in some measure contributed to these choices. While such an outcome may have been unlooked for, and certainly not deliberately planned, it may nonetheless be an interesting result of such an approach, if as yet one difficult to verify. Certainly it is a further element of this study which may prove worthy of further investigation.

# Chapter Seven

# Conclusions

It is probably worthwhile reminding oneself at this point that the current study, while laying claim to elements of research, is essentially a case study of an evaluation. As a case study, its primary purpose is to provide *". . an opportunity for teachers to observe their colleagues, their successes and failures, and apply this experience to their own situation"* (to quote from the Introduction, page 1). The value of such an endeavour then is to be judged, not so much by the significance of its outcomes, but by the extent to which :

(i) the classroom teacher is provided with information from which decisions can be made regarding the effectiveness of the implementation, and

(ii) the reader is able to understand the particular situation, to interpret and make meaningful the processes described, and so to draw inferences which may be applicable to other situations.

This is not the "generalisability" aimed for in traditional research; it is rather a recognition of the uniqueness of each situation while allowing that a knowledge of other situations may inform one's own.

As an evaluation, also, the value of the work may be judged in the extent to which it provides meaningful information from which decisions may be made. The extent to which the evaluation is *responsive* to the needs of its audiences is a critical factor in providing such information; as far as possible, the data is collected and presented in such a way as to reflect the needs and concerns of the various audiences. While the evaluation may attempt to respond to the various interested parties, it is not possible to cater to the priorities of all. Thus the conclusions which follow, while based upon the data, present only one possible analysis of the process and the results. As noted in Stake's definition of "Responsive Evaluation", it is the "program activities" rather than the "program intents" which are the focus for such an evaluation. The greater part of the value of such an evaluation for classroom teachers lies in the description of the classroom situation - the Transactional level - and the extent to which this provides a meaningful portrayal of the processes involved.

The conclusions which may be drawn from such an endeavour, then, are in some ways not the most important part of the account. The casual reader who hopes to gain insight into the project by referring only to this chapter may well miss the vital insights and understandings which are to be found earlier, particularly in descriptions of the processes which accompanied the project. Nonetheless that which follows is an attempt to recount the most significant features of the process in terms of the focus questions posed at the outset :

- How may I implement a particular curriculum innovation?
- How may I judge the effectiveness of this implementation?

- What tentative generalisations may be induced from such an evaluation concerning both the use of computer technology in the teaching of mathematics, and the roles of the classroom teacher as evaluator and researcher?

In fact, the study explores three major processes - the **changes in teaching** which accompany the use of computer technology in this context, especially with respect to the classroom processes (group work, verbalisation and problem solving) by which such teaching may be effected; the **changes in the students** which occur as a result of such an innovation (in particular, changes in attitude towards the subject and its applications, and changes in understanding of concepts associated with this study); and **the research process** by which the classroom teacher may study and evaluate the whole.

## 7.1 Implementation and Formative Outcomes

Studies described in the literature reviewed in Chapter Two suggest that computer technology can be used to enhance the traditional teaching of mathematics at the senior levels, particularly in those aspects of the syllabus which relate to Curve Sketching. Further, by carefully sequencing the presentation of key concepts, such as the rates of change of functions and curves, the traditional introduction to calculus can be made more meaningful and relevant, with respect both to the understanding by students of the central concepts and to the applications of calculus to real-life situations. Studies in the United States involving the teaching of algebra in a similar type of sequence [Heid, 1988] strongly suggest that other aspects of the syllabus may

also benefit from such enhancement, although such areas are beyond the capabilities of most current computer hardware which is readily available for classroom use.

The particular computer application chosen for this study - the Hewlett-Packard HP-28C calculator - may be seen as having several intrinsic strengths and weaknesses. Capable of performing algebraic manipulations, representing the graphical forms of functions and calculating the derivatives of such functions easily, this hand-held device surpasses the capabilities of most of the computer software available for classroom use. Portable and relatively inexpensive, these devices appear to have decided advantages over computers with perhaps far more power, but lacking in the same capabilities and flexibility. The evidence of this study supports the view that they may provide significant advantages to classroom teachers of mathematics, particularly at the Senior secondary level. [see Section 1.3]

At the same time, their limitations are also significant. Without the memory capabilities of their more powerful cousins, hand-held tools such as these do not have the same speed of calculation nor the same size viewing "window", factors which are quickly found to be frustrating for those attempting to use them in a classroom situation. The graphing process is slow, and it is not easy to view different parts of curves, nor to "zoom" in or out as is possible on many of the computer graphics programs. These capabilities are there, but they are somewhat time-consuming to effect. [see especially 5.2.1] A fine line exists, then, between those features which may make such tools invaluable aids to the teaching of Senior Mathematics, and those

which count against them, possibly causing them to be viewed as more a hindrance than a help. These were the features recognised within the present study as the various strengths and weaknesses of the calculators.

The effective use of such technology in the current project was found to be greatly enhanced by the use of structured but open-ended worksheets, which stated the Aim of the exercise, provided the necessary knowledge (both with respect to Mathematics and the use of the calculators or computers) and then led students through a series of investigatory activities. [see Section 2.1] These provided the structure necessary both for the introduction of new concepts in Mathematics, and for the use of new functions on the computer hardware; students who were uncertain in either area were assisted by this format. The investigations were also designed to be open-ended so that students who were confident could work ahead and not be held back by others. This was observed to occur, although students were reluctant to work too far ahead of the class for fear of missing important points; teacher control of the "pace"of the lesson was found to be effective in this regard. [see Section 5.2.2] Bringing the class back together for discussion and questions was found to be a valuable means of aiding student understanding of concepts. In this way the worksheets catered for both ends of the ability spectrum, rather than attempting to keep the group moving at the same pace, which is often the case in such situations.

The use of group work was another key feature in the implementation of the innovation. Groups which encompassed a range of ability levels and varying degrees of confidence and competence in the use of the calculators provided support for the less able, while at the same time

deepening the understanding of the more capable. The need for verbalisation in group situations cannot be stressed strongly enough as a means of reinforcing and clarifying essential concepts and understandings [see Section 6.3].

Finally, the use of the Action Research model proved to be of great benefit in two ways :

- By involving the students as participants and collaborators their support and ownership of the project was greatly enhanced. They proved to be critical informants, willingly giving of their time and opinions in discussing the positive and negative aspects of the innovation [see 6.3].
- The cyclic nature of the Action Research approach allowed the programme to be developed in a way which was responsive to the needs and perceptions of the participants. By observing and reflecting upon features of the implementation as it was occurring, it was possible to produce a series of worksheets and tasks which were far more appropriate than those which might have been developed prior to the implementation [refer 5.1].

## 7.2 A Summative Evaluation

As described above, it is possible to recognise a number of useful and important features arising from the implementation of the computer-enhanced Mathematics course in question. It is necessary at this stage to examine each of these features critically and in more detail if a useful summative evaluation is to be produced. The worth of each is to be judged against the standards made explicit earlier, and each is

to be addressed in terms of the aims for the project which were stated on page 11 of the Introduction.

**7.2.1 Understanding of Concepts and Relationships :** It is to be remembered that the current project initially arose from a dissatisfaction with the traditional treatment accorded to the study of Mathematics at the Senior level, in which the development of skills of algebraic manipulation appears to have pre-eminence over the understanding of concepts and relationships. Nowhere is this more obvious than in the study of the Calculus, and this consequently became a central feature of this innovation.

The performance of the sample group at the Higher School Certificate examination clearly demonstrated no lack of ability in the required skills of manipulation as a result of following an alternative course; their results on the assessment tasks set to test their understanding of concepts, their ability to apply these in unfamiliar situations and their ability to express and explain these results verbally gave clear indication that these elements, too, had been well catered for [see Section 6.2].

As noted previously, an important indicator of understanding is to be found in one's ability to *estimate*, and this factor also was tested with regard to curve sketching. A significant result of the use of computer technology in this context was an increasing familiarity with a variety of functions in both their graphical and algebraic forms. As mooted at the outset of this project, the immediate feedback, coupled with the opportunity to quickly and easily explore the effects which variations

to the equations have upon the shape and location of a curve, provided an effective means of encouraging such familiarity [see 5.2.2].

The ability to apply the use of the calculators to more difficult functions (including some from real world examples) further proved a useful asset. The selection of questions from a range of real-world sources (including economics, population growth and even pollution control) for discussion and assessment purposes proved to be a motivating and satisfying experience for the students. They invariably found such questions challenging and interesting, and were keen to continue discussion after the end of the lesson in many cases. The question cited in Chapter Five on the rate of petrol consumption led a number of the students to continue working over the weekend and to bring their efforts back to class for further discussion.

**7.2.2 Encouraging a Positive Attitude towards Mathematics :** It was clear from the initial application of the Attitude Scale and from previous experience with the sample group that they began the project with a positive attitude towards this subject. At the same time, it appeared that they held fairly neutral feelings concerning the use of computers in Mathematics. By the end of the year, three significant factors had emerged :

(i) Their positive attitudes had in no way declined - although an increase in the positive attitudes of the Holy Spirit group had also occurred over that period, implying the possibility that this may be an effect of Year 12 study at this level rather than factors specific to this sample. While both groups indicated some relief at nearing the end of their studies, the Saint Joseph's group clearly indicated that they would like

to continue to learn more about Mathematics after they left school - even though some had no intention of pursuing mathematics-related careers. Certainly a significant part of this group did, in fact, choose to study in the areas of mathematics or computing, but even those who went into such unrelated fields as the performing arts appeared to exhibit positive attitudes towards the subject.

(ii) The confidence of the sample group had certainly improved over the last year of their senior studies. From a group with a relatively low level of confidence in their own abilities (as indicated by the Attitude Scale) the responses to this question had increased significantly. Such an increase in confidence is a most important factor in assessing the results of the programme - certainly the peer group at Holy Spirit showed no such dramatic improvement. Whether it is to be attributed to the use of computer technology, to the interactive nature of the lessons, or to any of a number of uncontrolled factors cannot be decided with any degree of certainty. However, it seems likely that students who feel that they *understand* the concepts behind the work they are doing are more likely to feel confident about it than those who may have satisfied themselves with a certain amount of rote learning.

(iii) The majority of students in the sample group demonstrated an ease and competence with computer technology which they had not shown initially. Although some complained of groupwork being dominated by others, all students showed a willingness to work with the calculators when they were offered, and an enthusiasm to continue such use at home

when such opportunities arose. Although aware of the limitations of such technology, the students were also comfortable with it, and capable in its use. Nearly all the students indicated that they expected to use computers to a greater or lesser extent in their chosen careers, and the disproportionate number who chose mathematics and computer science careers has already been noted [see 6.4]. Clearly, the programme fulfilled its goal of "encouraging a positive attitude towards Mathematics, and the non-trivial role it is likely to play in the future."

**7.2.3 Understanding of the Subject and its Applications :** The factors cited above also provide support for the achievement of this goal. The unusual exposure of students at this level to the relationship between Mathematics and Computer Technology provided them with numerous insights into the support which each field provides for the other [see 6.3]. Since the vast majority of careers which require a high level of mathematical ability are now inseparable from computer technology, such understandings "should stand the students in good stead in the future."

The use of a wide range of mathematical applications was also a significant feature of the programme, already commented upon. The applications offered in the traditional coverage of the course are severely restricted in type, almost entirely drawn from particle motion and exponential growth. Although important, they may leave students with little understanding of the ways in which mathematics may contribute to such diverse fields as ecology, economics, politics and the environment. As well as increasing their understanding of

mathematics, students in this sample demonstrated an interest and awareness of such issues; it seems likely that their inclusion can only prove beneficial. As mentioned earlier, it is a cause of some regret that there is no formal mention of Statistics in the study of mathematics at these levels - an application which is relevant and significant in almost every aspect of our modern technological society.

**7.2.4 Improvements in Communication, Reasoning and Problem Solving :** Undoubtedly, the single most important factor in contributing to the abilities of students in these areas lies in the use of group work and class discussion. While in some few instances it may prove beneficial for students to have access to computer technology on a one-to-one basis, in the majority of cases this is not so. The students in this situation showed evidence that they benefitted from the forced verbalisation and the analysis of their own reasoning which is required when working with others; they appeared to gain from the greater pool of ideas and alternatives available from their peers; they sorted out problems far more easily within a small group, problems which they may otherwise have been reluctant to ask in front of the whole class.

At the same time, a group will not work well if it does not have a task which is appropriate. One may observe the comparison with computer technology - the hardware is only as good as the software which runs it. Such tasks must be carefully chosen; in some cases, they must be carefully sequenced, so that sufficient information is available without giving too much away. They must be perceived as being worthwhile - ideally they will be both relevant and interesting. Tasks such as that described in section 5.2.5 are suitably open-ended and challenging, while relating the mathematical concepts to the types of real-world

situations which students are likely to perceive as topical and important.

**7.2.5 The Negative Features of the Implementation :** Thus far the emphasis has clearly been upon the positive features which have emerged from the evaluation. Of equal significance, however, are those features of the implementation which were not perceived of as having been entirely successful. Invariably we learn more from our failures than our successes.

The element of group work is probably a good place to start. While an effective group can prove to be an invaluable aid to any lesson, groups are not always effective nor were they in this evaluation. On the whole, the students worked together extremely well - this was a common feature mentioned during interviews with the students at the end of the year. At the same time, another common feature centred around complaints about group domination and feelings of frustration by some members. It was the recognition of these factors which prompted the loan of calculators to students overnight and at weekends. If they had only been available in class, some students would have preferred to sit back and let others "take over". In this way they were given as much time as they needed to come to terms with the technology, and all reported positively after such experiences. It would appear that, as with most teaching strategies, a "blanket" application is less than effective; rather the application must allow for alternative modes for different students. The instruction should be individualised as much as possible.

Effective groups do not "just happen". Although they had experienced group work previously in mathematics classes and elsewhere, a certain structure and organisation is necessary if a group is to "work". Sometimes this occurs by chance; often it does not. The difficulties encountered in some of the group situations suggest that it may be more effective for students to be given some training in group dynamics, even if only the allocation of roles to members of the group - roles such as "scribe", "time-keeper", "spokes-person" and "encourager". Such an approach is taken in Mary Barnes' *"Investigating Change"* programme [Barnes, 1990], an investigative approach to calculus based upon very similar principles to that described here, although far more extensive than the current programme. Instead of a series of worksheets, Barnes' programme involves a series of booklets which examine the theory and applications of calculus using computer technology as an aid and the type of groupwork and investigative approach which has characterised the present implementation. This new programme has appeared very effective in trials this year, subsequent to the study under consideration.

As mentioned in a number of previous contexts, it was recognised early in the programme that the innovation should not be perceived as a "gimmick", as appeared to be the case with the Year 11 Three Unit class initially. It is possible that this early experience may have contributed to the gradual polarisation which occurred between the two groups - the Year 12 group becoming more and more interested and supportive, their Year 11 counterparts more and more negative. A lack of confidence on their teacher's part in the use of the calculators coupled perhaps with a lack of ownership of the process may also have contributed. At the end of the year, the Year 11 group indicated that

they recognised the calculators as an "optional extra", "tacked on" to their programme rather than as an integral part of it. The Year 12 group, on the other hand, felt that the work with the calculators complemented their classwork effectively - they perceived it as arising naturally within their topics. This would appear to be a critical factor in the effective use of such technology.

Finally, it is necessary to note the criticism of the calculators themselves. Initially, students found them difficult to use; they frequently noted that the graphing screen was too small to present a satisfactory image of the graph they were studying, and the graphing process was frustratingly slow. As time passed, they became proficient with the main functions of the calculators, and this allowed them more flexibility in adjusting such features as the size of the "window" on the viewing screen, allowing them to "zoom in" and "zoom out" on parts of the graphs. Undoubtedly, computer software which could perform the same functions would be preferable; however, such software with both graphics and calculus capabilities are simply not yet available in a usable form - although programs such as *Mathematica* and *Milo* for the Macintosh appear to come closest to fulfilling these requirements.

Of course, the physical limitations of the HP-28C calculators also allow advantages of cost and portability which are significant features in this context. Most schools cannot afford powerful microcomputers and the software needed for such applications; nor are such computers readily available for mathematics classes, or even for students to take home. In noting that a school may purchase five of these hand-held calculators for the cost of a single computer, such practical

considerations may well outweigh their shortcomings in the decision as to whether or not they are a worthwhile educational investment in a climate of increasing economic accountability.

**7.2.6 Conclusion :** Overall, then, the implementation may be considered to have been successful in achieving its major goals. The processes chosen for the implementation were appropriate to the situation, and discernible benefits flowed on to the students who participated in the programme. In keeping with the Action Research tradition, the shortcomings of the implementation provide the basis for further reflection and the means by which improvements may be made in subsequent years. As Stenhouse observed :

> *"A curriculum without shortcomings has no prospect of improvement and has therefore been insufficiently ambitious."*
>
> [Stenhouse, 1975; 125]

Decisions which follow from this evaluation include :

- The continued use of the HP-28 calculators in senior classes as an aid to their learning of concepts in curve sketching and calculus; these tools are to be complemented with the use of other computer software more appropriate to particular aspects of the course;
- The support of such technology with appropriate worksheets, including programmes such as that mentioned previously by Barnes. The development of such programmes and their implementation into this particular school situation should continue to be based upon Action Research procedures, and should remain an on-going priority both as a viable means of

improving mathematics teaching in this situation and possibly on a wider scale;

- The training of students in the fundamentals of group dynamics so that they may more readily take advantage of such factors as verbalisation and group problem-solving as means of more effectively learning mathematical concepts - this should occur in the Junior school, since such methods are applicable at all levels of mathematics instruction [Cockroft, 1980];
- A continued emphasis within the Mathematics Faculty and the school as a whole upon teachers assuming the role of evaluators of their own work as a means both of better understanding and improving their own practice, and of adding to knowledge about teaching in general. Such an approach would be consistent with Stenhouse's notion of *extended professionalism*, and implies teachers developing what Stenhouse describes as a *research mentality* towards their work. This is to be encouraged through an emphasis upon inservice training, professional reading and learning from observations of colleagues. The involvement of students as critical informants is to be encouraged.

## 7.3 Generalisations

In relation to the research component of the present study it is possible to derive certain generalisations with regard to the two key areas of this thesis :

- the effective use of computer technology in the teaching of mathematics, and
- The role of the classroom teacher as evaluator and researcher.

Such generalisations are more in the form of hypotheses than results; as is the way with qualitative research, they are not proffered as having statistical validity. Rather they are features which have arisen in the course of this evaluation which may provide some guidance for others who might wish to implement a similar innovation, or perhaps for classroom teachers who wish to evaluate their own work. At best they may provide questions worthy of further research in these important areas.

### 7.3.1 Using Computer Technology in Mathematics :

While changes to the curriculum will undoubtedly come about in response to the influence of computer technology, in implementing such tools the technology must fit the curriculum, not the other way around. If the students perceive that the curriculum is being manipulated in order to better use particular capabilities of the computer, then they may well come to see it as a "gimmick". It is necessary to derive from the Syllabus those content areas which are suitable for computer enhancement, and to distinguish these from those that are not. Essentially, if a concept cannot be taught faster or more effectively by use of the computer than it could by traditional methods, then the computer should not be used. As noted previously, if computer technology is to effectively enhance the curriculum it must be perceived as being consistent with the existing programme, and of significant advantage over traditional methods. These were the factors hypothesised initially on the basis of readings and classroom observations; they have been supported by the findings of this study.

Considerably more preparation is necessary for the effective use of computer technology than for more traditional methods. The development of structured, open-ended worksheets was found to be effective in catering for all ability levels within a class; these allowed instruction to be relatively individualised, while still allowing the option for the teacher to control the pace of the lesson, and to draw students together at times for class discussion. It is advisable for such worksheets to at least end with an open-ended investigation which will challenge the more capable students - these will be the more critical audience and will need to be occupied with more than simply assisting their less able peers.

The value of *verbalisation* in the teaching of mathematical concepts cannot be overemphasised. A student who is forced to put a concept into words will quickly reveal gaps in understanding if they exist. The use of class discussion and group work both encourage such verbalisation. It is to be expected of students that they not only know "how", but also "why". The interactive classroom environment which is encouraged by such processes implies far-reaching effects in terms of concept development, improved attitudes towards the subject, analysis of their own reasoning processes, better problem solving skills, as well as improvements in social skills.

Finally, some of the most effective and significant lessons which occurred as part of this computer enhancement of the senior mathematics course did not actually involve using any form of computer technology. These were lessons which explored the *concepts* which arose from such an enhancement. The types of questions which formed the Assessment Tasks for this programme investigated the

familiar concepts of senior mathematics from altogether different perspectives. Questions about the derivative and points of inflexion which had nothing to do with curve sketching forced students to explore their own understanding of these concepts, and usually to find such understandings wanting. It is this potential for the use of computer technology to give rise to a new and different approach to old problems which is perhaps the more exciting and innovative - the ability to view and investigate concepts from new and different points of view, and to observe their use in unusual and interesting applications. The computer provides one tool for mathematics teaching and learning which offers insights into more effective concept development.

### 7.3.2 The Role of the Teacher as Evaluator and Researcher:

In a climate of increasing accountability at all levels of education, teachers more than ever need to become evaluators of their own work. On the most fundamental level, however, such skills are necessary if teachers as professionals are to improve the quality of their own practice as individuals in addition to the quality of the profession. This study utilised three major tools in carrying out an internal evaluation with the intention both *". . to understand better* [one's] *own teaching and to contribute more to the science of teaching."* [Stake, 1967; 523]

First, at the very heart of this project, was the Action Research model. Its fundamental characteristics, of being *systematic, collaborative* and *self-evaluative* are essential requirements for such an internal evaluation. Involving the participants in the planning and implementation of the project proved to be extremely rewarding and

satisfying for both researcher and participants. If teachers wish to implement an innovation in their own classroom, it would seem to be highly advisable to attempt such a collaboration; if for no other reason, the criticisms offered by one's students tend to be both informative and humbling.

The cyclic nature of the Action Research model also proved of great value in effectively implementing the innovation. When working with unknowns, it is wise not to attempt to plan too far ahead into the dark - rather to act, to observe, to reflect and then to revise one's original plan in the light of this experience. If this is done systematically it is possible to draw much valuable information from each cycle.

The importance of a "research mentality" is also not to be overlooked. Experience with the use of Action Research methods in the past has shown that they tend to be addictive, and that teachers will continue to look beyond the day-to-day classroom situations and attempt to answer other questions regarding their own teaching, and teaching in general. This is an attitude prefaced in Stenhouse's notion of *extended professionalism*, described previously [Section 2.4].

Secondly teachers with such an attitude towards their work are inevitably also *evaluators* of their own work - they are reflective and systematic in their goal to improve their practice. At the same time, the lot of the internal evaluator is a difficult one, and one best served by working within certain guidelines and structure. In the case of the present study, this structure was provided by Stake's *Countenance Model* , providing as it does a comprehensive overview of the evaluation process, clearly describing the relationships which exist

between Description and Judgement components - between Intents and Observations in the former, and between Standards and Judgements in the latter. Viewing these elements in terms of the three stages of the process, Antecedents, Transactions and Outcomes, proves to be a valuable evaluation model for the teacher as evaluator.

Incorporating a cyclic approach at the transactional level, in keeping with the Action Research model, allowed the evaluation to be more flexible in the gathering of process data, and so more useful in providing information for a formative evaluation. Taken as a whole, of course, it was possible to derive much which supported a thorough summative evaluation as well.

Third, the final component of this study, the means by which some of the most valuable data was gathered, lies in the use of ethnographic techniques and a qualitative approach. In this way it was attempted to meaningfully portray the situation in which the innovation occurred, to sympathetically describe the effects of the innovation upon the participants, and so to provide rich and fine-grained data from which decisions could be made. The internal evaluator may learn much from those who are most familiar and most affected by the innovation - and that is usually the students. A final consideration here is the need to gather sufficient *baseline data* - a factor which may well be overlooked in school-based evaluations, and for which ethnographic approaches may again prove valuable. It is too late to target a programme or innovation for evaluation close to its conclusion, at which time it is impossible to decide what, if any, changes have come about because of it.

The teacher as evaluator and researcher is not in an easy position - that to be studied is a complex mixture of individuals, operated upon by diverse influences which are largely beyond the control of any individual. At the same time, the teacher is restricted in both time and resources. An effective evaluation makes great demands upon the evaluator - the data must be valid, accurate, carefully assembled, and yet extensive at the same time. In the face of a range of growing pressures, it is undoubtedly difficult for such a task to be carried out effectively by classroom teachers, and yet such are the demands of professionalism. It is only by assuming the roles of evaluator and researcher that classroom teachers may not only improve their own practice, but may provide example and guidance to their peers, and so ensure the continued advancement of educational practice.

# Appendix

## Appendix A : External Examination Results

### School Certificate 1985-89

| ADV | 1985 | 1986 | 1987 | 1988 | 1989 |
|---|---|---|---|---|---|
| GRADE "A" | 18 | 11 | 24 | 38 | 34 |
| GRADE "B" | 12 | 18 | 9 | 20 | 27 |
| GRADE "C" | 0 | 1 | 0 | 8 | 9 |
| STUDENTS | 30 | 30 | 33 | 66 | 70 |

### Australian Mathematics Competition (Year 12) 1987-89

| | 1987 | 1988 | 1989 |
|---|---|---|---|
| Number of Entries | 15 | 9 | 21 |
| DISTINCTIONS | 0 | 3 | 3 |
| CREDITS | 6 | 3 | 7 |
| TOTAL AWARDS | 6 | 6 | 10 |
| % AWARDS | 40 | 67 | 48 |

## Appendix B : Mathematics Attitude Scale Results

### (1) GENERAL ATTITUDE TOWARDS MATHEMATICS (Positive Items)

| QUESTION | 1.10 | 1.20 | 1.30 | 1.40 | 1.50 |
|---|---|---|---|---|---|
| Pre-Test Results : | | | | | |
| SJ MEAN | 4.25 | 4.08 | 2.00 | 4.77 | 4.23 |
| STAN.DEV | 0.45 | 0.49 | 0.71 | 0.44 | 0.93 |
| HS MEAN | 4.04 | 3.67 | 2.48 | 4.15 | 4.30 |
| STAN.DEV | 0.51 | 0.86 | 1.10 | 1.20 | 0.92 |
| Post-Test Results : | | | | | |
| SJ MEAN | 4.08 | 3.77 | 3.31 | 4.08 | 4.38 |
| STAN.DEV | 0.86 | 1.01 | 1.03 | 1.26 | 1.33 |
| HS MEAN | 4.10 | 3.80 | 2.70 | 3.90 | 4.10 |
| STAN.DEV | 1.25 | 0.83 | 1.08 | 1.29 | 1.21 |
| **SJ T-SCORES** | **0.62** | **0.96** | **-3.62** | **1.80** | **-0.33** |
| **HS T-SCORES** | **-0.19** | **-0.47** | **-0.62** | **0.62** | **0.57** |

### GENERAL ATTITUDE TOWARDS MATHS (Adjusted Negative Items)

| QUESTION | 1.11 | 1.21 | 1.31 | 1.41 | 1.51 |
|---|---|---|---|---|---|
| Pre-Test Results : | | | | | |
| SJ MEAN | 3.92 | 4.38 | 3.92 | 4.46 | 3.77 |
| STAN.DEV. | 0.99 | 0.86 | 1.26 | 1.13 | 1.09 |
| HS MEAN | 3.96 | 4.22 | 4.26 | 4.42 | 3.27 |
| STAN.DEV. | 1.14 | 0.83 | 1.05 | 0.83 | 0.94 |

| QUESTION | 1.11 | 1.21 | 1.31 | 1.41 | 1.51 |
|---|---|---|---|---|---|
| Post-Test Results : | | | | | |
| SJ MEAN | 3.85 | 4.15 | 4.00 | 4.62 | 3.46 |
| STAN.DEV. | 1.34 | 1.07 | 1.15 | 0.87 | 1.20 |
| HS MEAN | 3.60 | 4.05 | 4.30 | 4.30 | 3.20 |
| STAN.DEV. | 1.57 | 1.10 | 1.03 | 0.92 | 1.01 |
| **SJ T-SCORES** | **0.15** | **0.58** | **-0.16** | **-0.39** | **0.66** |
| **HS T-SCORES** | **0.81** | **0.54** | **-0.12** | **0.42** | **0.22** |

**(2) KNOWLEDGE OF APPLICATIONS OF MATHS (Positive Items)**

| QUESTIONS | 2.10 | 2.20 | 2.30 |
|---|---|---|---|
| Pre-Test Results : | | | |
| SJ MEAN | 3.60 | 2.85 | 3.00 |
| STAN.DEV. | 0.87 | 1.21 | 1.08 |
| HS MEAN | 3.67 | 2.93 | 2.78 |
| STAN.DEV. | 1.15 | 0.98 | 0.83 |
| Post-Test Results : | | | |
| SJ MEAN | 3.54 | 2.15 | 3.31 |
| STAN.DEV. | 1.13 | 0.80 | 1.11 |
| HS MEAN | 3.75 | 3.20 | 2.85 |
| STAN.DEV. | 1.02 | 1.28 | 1.04 |
| **SJ T-SCORES** | **0.15** | **1.66** | **-0.69** |
| **HS T-SCORES** | **-0.23** | **-0.73** | **-0.23** |

**KNOWLEDGE OF APPLICATIONS OF MATHS (Adj. Negative Items)**

| QUESTIONS | 2.11 | 2.21 | 2.31 |
|---|---|---|---|
| Pre-Test Results : | | | |
| SJ MEAN | 3.15 | 4.69 | 3.54 |
| STAN.DEV. | 1.57 | 0.48 | 1.20 |
| HS MEAN | 2.45 | 4.22 | 2.85 |
| STAN.DEV. | 1.07 | 0.99 | 0.80 |
| Post-Test Results : | | | |
| SJ MEAN | 3.00 | 4.15 | 3.62 |
| STAN.DEV. | 1.15 | 1.34 | 1.04 |
| HS MEAN | 2.70 | 4.00 | 2.80 |
| STAN.DEV. | 1.49 | 1.03 | 0.89 |
| **SJ T-SCORES** | **0.27** | **1.31** | **-0.17** |
| **HS T-SCORES** | **-0.59** | **0.67** | **0.18** |

**(3) ATTITUDES TOWARDS PROBLEM SOLVING (Positive Items)**

| QUESTIONS | 3.10 | 3.20 | 3.30 |
|---|---|---|---|
| Pre-Test Results : | | | |
| SJ MEAN | 3.85 | 3.92 | 3.15 |
| STAN.DEV. | 0.80 | 0.64 | 1.14 |
| HS MEAN | 3.44 | 3.34 | 2.93 |
| STAN.DEV. | 0.92 | 0.99 | 0.80 |

| QUESTIONS | 3.10 | 3.20 | 3.30 |
|---|---|---|---|
| Post-Test Results : | | | |
| SJ MEAN | 3.85 | 3.46 | 2.85 |
| STAN.DEV. | 0.80 | 1.13 | 1.14 |
| HS MEAN | 3.85 | 3.25 | 2.65 |
| STAN.DEV. | 0.88 | 0.79 | 1.46 |
| **SJ T-SCORES** | **0.01** | **1.23** | **0.65** |
| **HS T-SCORES** | **-1.41** | **0.31** | **0.73** |

**ATTITUDES TOWARDS PROBLEM SOLVING (Adj. Negative Items)**

| QUESTIONS | 3.11 | 3.21 | 3.31 |
|---|---|---|---|
| Pre-Test Results : | | | |
| SJ MEAN | 4.38 | 4.15 | 4.31 |
| STAN.DEV. | 0.65 | 0.55 | 0.48 |
| HS MEAN | 4.37 | 3.93 | 3.70 |
| STAN.DEV. | 0.92 | 0.99 | 0.62 |
| Post-Test Results : | | | |
| SJ MEAN | 4.15 | 3.54 | 3.31 |
| STAN.DEV. | 1.14 | 1.27 | 0.95 |
| HS MEAN | 4.10 | 4.00 | 3.30 |
| STAN.DEV. | 0.97 | 0.97 | 1.38 |
| **SJ T-SCORES** | **0.61** | **1.53** | **3.26** |
| **HS T-SCORES** | **0.88** | **-0.22** | **1.15** |

**(4) ROLE OF MATHEMATICS IN THE FUTURE (Positive Items)**

| QUESTIONS | 4.10 | 4.20 |
|---|---|---|
| Pre-Test Results : | | |
| SJ MEAN | 3.69 | 3.54 |
| STAN.DEV. | 1.32 | 1.20 |
| HS MEAN | 3.09 | 3.39 |
| STAN.DEV. | 1.09 | 1.23 |
| Post-Test Results : | | |
| SJ MEAN | 3.00 | 3.62 |
| STAN.DEV. | 1.53 | 1.26 |
| HS MEAN | 3.75 | 3.55 |
| STAN.DEV. | 1.37 | 0.94 |
| **SJ T-SCORES** | **1.18** | **-0.15** |
| **HS T-SCORES** | **-1.64** | **-0.45** |

**ROLE OF MATHEMATICS IN THE FUTURE (Adj. Negative Items)**

| QUESTIONS | 4.11 | 4.21 |
|---|---|---|
| Pre-Test Results : | | |
| SJ MEAN | 4.00 | 3.54 |
| STAN.DEV. | 1.15 | 1.27 |
| HS MEAN | 3.63 | 3.07 |
| STAN.DEV. | 1.06 | 1.33 |

| QUESTIONS | 4.11 | 4.21 |
|---|---|---|
| Post-Test Results : | | |
| SJ MEAN | 3.08 | 3.00 |
| STAN.DEV. | 1.55 | 1.35 |
| HS MEAN | 3.70 | 2.90 |
| STAN.DEV. | 1.42 | 1.41 |
| **SJ T-SCORES** | **1.65** | **1.01** |
| **HS T-SCORES** | **-0.17** | **0.38** |

(**NOTE :** Degrees of Freedom for Saint Joseph's = 12 [n = 13] Criterion score for rejection of $H_0$ at 0.01 level of significance = 3.055

Degrees of Freedom for Holy Spirit = 19 [n = 20], criterion score for rejection = 2.861 at 0.01 level, or 2.093 at 0.05 level of significance.)

**Test used was Fischer's t-test.**

$$t = \frac{M_1 - M_2}{\sqrt{\frac{N_1 s_1^2 + N_2 s_2^2}{N_1 N_2}}}$$

with M = mean, s = standard deviation and N = number in sample.

## Appendix C : Student Evaluation Sheet

**Record Sheet - HP-28C** **Date :**

Your Name : _____ Class : _____

TOPIC : _____

Which FUNCTIONS did you use on the HP-28C (Please tick)

PLOT _____SOLVR_____ ALGEBRA_____REAL_____

TRIG _____LOG_____ COMPLEX_____d/dx_____

Integrate _____Other (please specify) _____

Did you use the HP-28C - INDIVIDUALLY _____

- IN A GROUP _____ GROUP SIZE_____

Describe briefly the way in which you used the HP-28C :

* To verify your answers ____* To draw graphs _____

* To learn a new concept_____

* As a tool for investigation _____

_____

* Other (please specify) _____

_____

How **effective/useful** was the HP-28C for this purpose

**(1) VERY GOOD**

**(2) OF SOME HELP**

**(3) A WASTE OF TIME**

Please give more detail in this regard if possible :

_____

_____

Are you interested in using the HP-28C again in other areas ?

**YES** _____ **NO** _____

IF YOU HAVE SUGGESTIONS FOR WAYS IN WHICH WE COULD BETTER USE THESE CALCULATORS, PLEASE WRITE THEM ON THE BACK.

# Appendix D : Investigations & Assessment Tasks

| UNIT | TITLE | PURPOSE |
|---|---|---|
| 1 | Absolute Value | INVESTIGATION |
| 2 | Inversions | INVESTIGATION |
| 3 | Equation Solving | INVESTIGATION |
| 4 | Roots of Polynomials | INVESTIGATION |
| 5 | The Gradient Function | INTRO TO CALCULUS |
| 6 | Rates of Change | INTRO TO CALCULUS |
| 7 | The Derivative | INTRO TO CALCULUS |
| 8 | Sample Question | ASSESSMENT TASK |
| 9 | Major Assessment Task | |

# Mathematics Investigation

**TOPIC** : ABSOLUTE VALUE

**AIM :** To familiarise students with the graphical representation of the absolute value of common functions.

**REQUIRED KNOWLEDGE** :

*MATHEMATICAL : The definition of absolute value is :

$$|a| = a \quad \text{for } a > 0$$

$$|a| = -a \quad \text{for } a < 0.$$

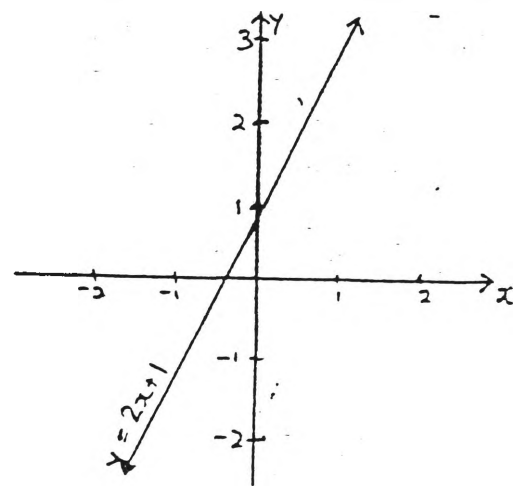
* CALCULATOR FUNCTIONS : - **ABS** (use either REAL or COMPLEX menu)
- **PLOT** (first STORE the equation (STEQ) then DRAW)

**INVESTIGATION**

(1) Using the HP-28C, enter and draw the graphs of $y = x$ and $y = |x|$. **Sketch these and record your observations.**

(2) Now try **$y = |2x - 3|$** [i.e. 'ABS(2*x - 3 ENTER ]
Sketch it on the same graph as $y = 2x - 3$ and **record what you notice.**

(3) Shown is the graph of $y = 2x + 1$.
Sketch what you think $y = |2x + 1|$ should look like.
**Verify** your answer with the HP-28C.



(Absolute Value)

(4) Now sketch graphs for :

(i) $y = 2 - x$ and $y = |2 - x|$

(ii) $y = 3x - 1$ and $y = |3x - 1|$

(iii) $y = x^2$ and $y = |x^2|$

(iv) $y = (x - 2)(x + 2)$ and $y = |(x - 2)(x + 2)|$

(v) $y = x^3 - 3x - 1$ and $y = |x^3 - 3x - 1|$

**Clearly describe your observations. Now make up FIVE of your own functions and do the same.**

**EXTENSION 1** : TWO graphs can be plotted at the same time by simply putting "=" between them. e.g. entering '$x + 4 = |x - 2|$ will graph **both** functions.

**HENCE,** solve (i) $|x - 2| = 3$

(ii) $|2x - 1| = |x + 2|$

(iii) $|x - 3| = x$ (What do you notice? **WHY ?**)

(iv) $|2x - 1| = -x$ (What do you notice? **WHY?**)

**Record your observations and make up TEN more (or get them from your textbook!)**

**EXTENSION 2** : GO BACK AND REPLACE THE " = " IN ALL THE ABOVE FUNCTIONS WITH INEQUALITIES ( > AND < , ≥ AND ≤ ) **NOW TRY THEM !**

# MATHEMATICS INVESTIGATION : Inversions

*AIMS* : To increase students' understanding of the DOMAIN and RANGE of function.

- To extend the ability of students to recognise features of the GRAPH of a function from its EQUATION.

## *REQUIRED KNOWLEDGE*

* *Mathematical* : The **DOMAIN** is the set of all possible x-values, the **RANGE** the set of all possible y-values which a given function may assume.

- We define the **INVERSION** of a function f(x) as the **reciprocal** of that function i.e. $^{1}/_{f(x)}$ (as in the verb "to invert")

- An **ASYMPTOTE** is a line of **discontinuity** at which the function is **undefined**. The function will **approach such a line but never touch it.**

* *CALCULATOR* : - The **PLOT** menu will allow you to STore a given function (**STEQ**) and then **DRAW** it.

- In order to graph functions with discontinuities, we must "disconnect" the part of the calculator which stops at these, called a "FLAG". This particular one is **FLAG 59**, and to do this go to the **PROGRAM TEST menu** (above the letter "0"), enter 59 and select **CF** ("clear flag").

(Inversions - 2)

*INVESTIGATION 1 :*

1. Draw a neat sketch of the graph of $y = x$ and on the same size number plane directly **below**, sketch the **INVERSION** of this, $y = 1/x$.

What do you notice in the second graph as the first one :

- approaches the origin from above ? Below ?
- approaches positive infinity ? Negative infinity ?
- What are the DOMAINS and RANGES of the two graphs ?

Record your **OBSERVATIONS** carefully, and any other points of interest you might notice.

2. Now do the same for the graphs of :

(i) $y = x - 3$ (ii) $y = 2 - x$.

(iii) $y = 2x + 1$ (iv) $y = 1 - 3x$

Try sketching the straight line first, and then work out what its inversion will look like from the graph. **VERIFY** your answers with the HP-28C.

3. Write down the **GENERAL RULES** for transforming a function into its inversion.

4. Now test these rules using these functions :

(i) $y = x^2 \Rightarrow y = 1/x2$ (What do you notice ?)

What are the similarities with the function in Question 1 ? What are the differences ? Can you understand why these similarities and differences occur ?

(Inversions - 3)

(ii) $y = (x - 2)^2 \Rightarrow y = {}^1/_{(x-2)^2}$

Again **record** your observations and try to account for them.

(iii) $y = x^2 - 4 \Rightarrow y = {}^1/_{x^2-4}$

Is this what you expected ? Record your observations and account for these.

5. Go back and revise your "**RULES FOR INVERSION**".
TEST your new rules on some functions of your own choice - consider th effects on CUBICS (like $x^3$ and $(x + 1)(x - 1)(x - 3)$ ) and QUARTRICS !

***EXTENSION 1***. Have you ever turned half a tennis ball inside out ? In many ways that is like what we are doing here - turning functions "inside out"!

What would a CIRCLE or ELLIPSE look like turned "inside out"?

Try starting with a semi-circle like $y = \sqrt{(4 - x^2)}$; see what happens and then conjecture about the effect of inversion upon the full circle. What about in **THREE DIMENSIONS** ? Could you imagine turning a **sphere** "inside out" ?

***EXTENSION 2*** : Most functions have a point which is **unchanged** by the function - we will call it a **STILLPOINT.** e.g. $F(x) = 2x - 3$ has a stillpoint at $x = 3$ since $F(3) = 3$. In general a stillpoint occurs at any point where $f(x) = x$. How are stillpoints affected by INVERSION ?

# Mathematical Investigation : EQUATION SOLVING

**AIM :** To equip students with the knowledge and skills required to find multiple solutions of equations using the HP-28C.

## REQUIRED KNOWLEDGE :

* Mathematical : Many equations have more than a single solution - quadratics, cubics, quartics; trigonometric equations may have an infinite number of solutions.

* Calculator : PLOT menu : STEQ, DRAW, *W, *H, PPAR
SOLVe menu : SOLVR

## INVESTIGATION :

Consider an equation such as $y = x^4 - 5x^2 + 4$

The HP-28C offers two options. We may **PLOT** the graph which shows the intersection of the above function and the x-axis. Each intersection is a solution of the given equation.

Alternatively, we may **SOLVe** the equation directly, using the **SOLV** menu, selecting the the **SOLVR** option, and pressing **INV "X"** to produce the SMALLEST solution (NOTE : the "X" is on the SOLVR option list, next to "EQ" and selected by pressing the grey key directly below.)

In either case, the equation must first be **STORED** by using the **STEQ** option from either the **PLOT** or **SOLVe** menus.

(Equation Solving -2)

If we wish to find more than a single solution, we may use **both** the above options as follows :

(1) Enter the given equation (first press ')

(2) Select the **PLOT** menu and **ST**ore the **EQ**uation using **STEQ.**

(3) **DRAW** the graph of the equation.

(4) We now use the four **cursor keys** below the screen to move the "cross-hairs" positioned initially at the origin to each of the required solutions. At each solution, press the **INS** key (the first grey key) to provide approximations for each solution.

(5) Press **ON** to return to the normal screen, where each estimate is presented as an ordered pair.

(6) Select the **SOLVe** menu, and the **SOLVR** option.

(7) Press the grey key below **"X"** to enter the first estimate (the one or LEVEL ONE of the memory stack).

(8) Now press **INV "X"** to solve for X. The calculator will produce the solution closest to your given estimate.

(9) Either **record** this solution and then **DROP** it from the stack or use the **ROLL option** to move this solution to the "top" of the stack and replace it with the next estimate. (e.g. to roll an item from LEVEL 3 to LEVEL 1 of the stack, simply enter "3" and then ROLL (i.e. INV DROP)

(Equation Solving - 3)

(10) Repeat steps (7) to (9) until all solutions have been found.

(NOTE : If the given equation produces a graph which does not fit onto the screen, the "PLOTTING PARAMETERS" may be changed using the **±W** and **±H** commands on the second page of the **PLOT** menu. When finished, restore the screen to its original size by entering **'PPAR PURGE** (INV 4).

**Investigation** :

Try the above techniques to find all solutions of equations like :

(i) $x^2 - x - 1$

(ii) $x^3 - x^2 + x - 1$

(iii) $\cos(x) = \sqrt{3}/2$ for $0 < \phi < 360$ ,

(Note that to plot trig functions the calculator must be in RADIANS mode using the **MODE** menu. Solutions may be converted back to degrees using the third page of the **TRIG** menu.)

## Mathematical Investigation : ROOTS OF POLYNOMIALS

**AIM** : To introduce to students the **computer spreadsheet** as an efficient tool for locating and approximating roots of polynomials to any desired degree of accuracy.

**REQUIRED KNOWLEDGE** :

Mathematical : The roots (or zeros) of a polynomial equation correspond to those Real points where the graph of the polynomial cuts the x-axis.

While the quadratic polynomial can be solved easily by use of either factorising or the formula, and formulae exist for cubic and quartic polynomials, it has been proven that no such formula is possible for polynomials of degree 5 or higher. The solution to polynomials of degree higher than two will often be most effectively found by **approximation methods**.

Computer : A suitable spreadsheet format for the approximation of roots may be constructed along the following lines :

| | A | B | C | D |
|---|---|---|---|---|
| 1 | INITIAL | X | F(X) | |
| 2 | 0 | +A2 | +A2^2 - 5 | |
| 3 | INCREMENT | +B2 + A4 | (Copy above) | |
| 4 | 1 | (Copy above) | | |
| 5 | | - | - | |

This template will produce the values for the function $F(x) = x^2 - 5$ from the initial x-value of 0 up to as far as you wish to **copy** the two given formulae - at least 10 values and no more than 20 is most effective

(Approximation of Roots - 2)

**NOTE** : When copying these formulae, references to cells A2 and A4 will always be "absolute", or **no change**, while all other references will be **relative**.

Pressing Open Apple - ESC activates the GRAPH option. For TYPE, select XY, for DATA, select the X-column as your X-range and the F(x) as the A-range, and then simply VIEW the graph.

## APPROXIMATING ROOTS

(1) When the table above has been produced, identify the range in which a **change of sign** occurs in the F(x) column (in the example given this should be between x = 2 and x = 3) This indicates the presence of a root in this interval

(2) Make the last value before the sign change the new **initial value** (here, place 2 in cell A2)

(3) Divide the increment by 10 - here, enter 0 1 in A4

(4) Repeat steps (1) - (3) to produce successive approximations to the desired root.

Appleworks Spreadsheet will give an accuracy up to 8 decimal places normally If greater accuracy is desired, the values above could be multiplied by a constant factor of 10 to "move the decimal point" - e g multiply by 10,000 to give 12-figure accuracy in your answers

(Approximation of Roots - 3)

**NOTE** : The Appleworks Spreadsheet tends to evaluate expressions in order from **left to right** rather than the usual algebraic **order of operations**. If you are using a complicated function, the order you desire may be specified by using **parentheses** or by breaking the function up into "bite-sized pieces", with a new column for each part, and then combine these columns to produce the result. e.g. a function such as $2x^2 - 3x + 5$ may be expressed as a formula such as

+ (A4^2)*2-(3*A4)+5

or by having a column for $x^2$, a column for $2 * (x^2)$, a column for $3*x$, and then a column to add these and 5 Experiment with these.

**EXERCISES**

(1) If $P(x) = 2x^3 + 5x - 4$, show that there is a real root of the equation $P(x) = 0$ in the interval $0 < x < 1$, and find a value of this root correct to 6 significant figures.

(2) Given that the equation $x^3 - 3x^2 - 9x + 4 = 0$ has 3 real roots, find in what interval these roots lie, and evaluate to 5 decimal places

(3) Use the method described above to estimate the **cube root of two** by solving the equation $x^3 - 2 = 0$

## Mathematical Investigation : THE GRADIENT FUNCTION

**AIM** : To deepen students' understanding of the properties and nature of continuous functions, particularly the **rate of change.**

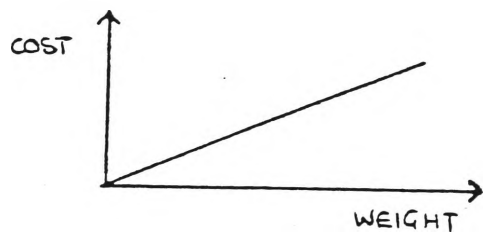
**DISCUSSION :**

* A **FUNCTION** describes a relationship between different things, in which a change in one (the **independent** variable) induces a change in the other (the **dependent** variable).

* Different functions will have different **rates of change.**

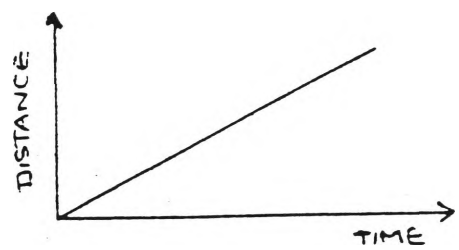
Examples

(1) The relationship between the **weight** of a grocery item (say, butter) and the **cost** of that item may be a **linear** relationship as shown As weight increases, cost increases at a steady rate



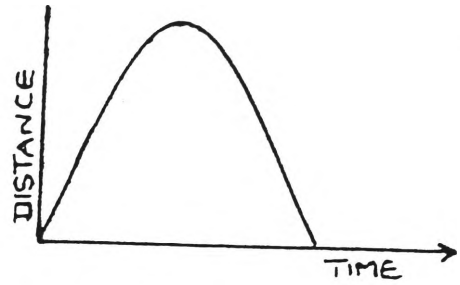

In fact, it is unlikely that the relationship would be this simple Draw a graph of what you think is more likely to describe the situation, and compare your results with others in your group

(2) **Velocity** is the function which relates **distance** and **time** for moving objects Thus a linear function describes an object moving at **constant** speed (where distance is interpreted as distance **away from** some location)



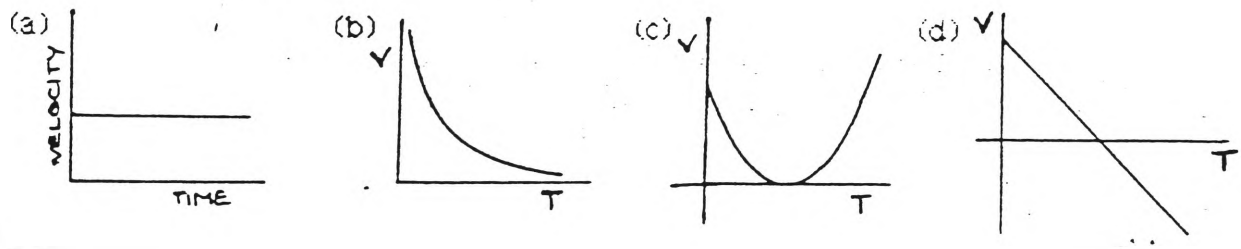
(The Gradient Function - 2)

(3) A moving object under the influence of some force (such as **gravity**) will appear as some other function. e.g. a ball thrown vertically upwards would be represented by an inverted parabola as shown.

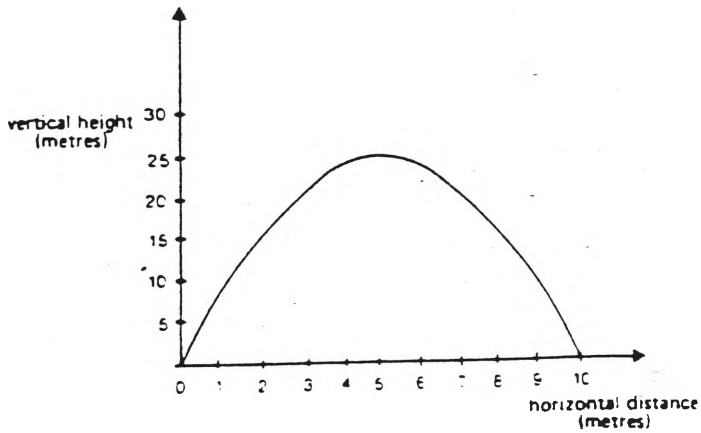



The **velocity** or **rate of change** of such a function may be considered as the **GRADIENT** function.

Which of the following best represents the **gradient** function (velocity) of the graph in (3) ? [First describe the changes in velocity which a ball thrown upwards would experience, then select the graph which you think best shows these changes **Remember** : velocity includes **direction** as well as speed - positive in one direction, negative in the other ]




(4) The path of a missile is represented by a quadratic graph shown below.



(The Gradient Function - 3)

(i) Which one of the following statements is true?

A The missile reaches a height of 30 metres.

B The missile bounces 5 metres from its point of projection.

C When the missile is 25 metres vertically above the point of projection, it has travelled 5 metres horizontally.

D The missile ascends more quickly than it descends.

(ii) From the graph in (i), describe in detail the **velocity** of the missile as it describes its path.

Draw the graph of the VELOCITY of the missile (vertical axis - maximum : 100 m/s) against the HORIZONTAL DISTANCE ( as shown on the graph above.)

(iii) On the first diagram, draw the **tangent** to the curve at the points (1, 5), (3, 20), (5, 25), (7, 20) and (9, 5)

Discuss the relationship between these tangent and the graph you drew in (ii).

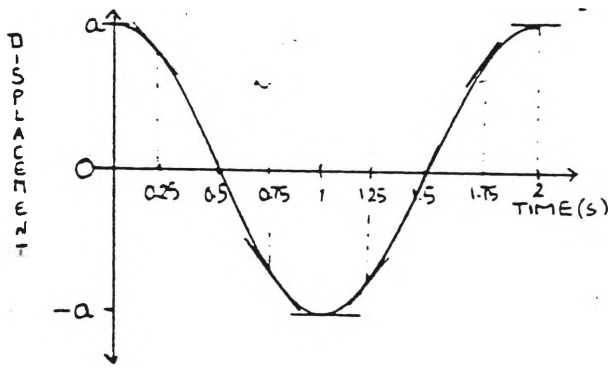
(5) Describe in words the motion of a child on a swing, in terms of both the **displacement** from the centre point, and its **velocity** On two graphs, with **time** as the horizontal axis, graph these situations

## Mathematical Investigation : RATES OF CHANGE

**AIM :** To extend students' understanding of the relationship between the **gradient** and the **rate of change** of a continuous function at any point in the domain.

**DISCUSSION :**

(1) Consider, from the previous investigation (**The Gradient Function**) the motion of a child on a swing. If we take the origin (O) as the rest position of the swing, and the maximum displacement from this position to be "a" units in a horizontal direction, then the graph of **displacement** against **time** would appear as :




(i) **Describe** in words the situation described in this graph

(ii) Where is the **velocity** greatest and least ? Positive and negative ?

(iii) Each of the **tangents** shown gives an indication of the **gradient** of the curve at that point. **Describe** in words the changes to these gradients at each $^1/_4$ sec interval

(iv) How does this relate to the **velocity** of the child at each point ?

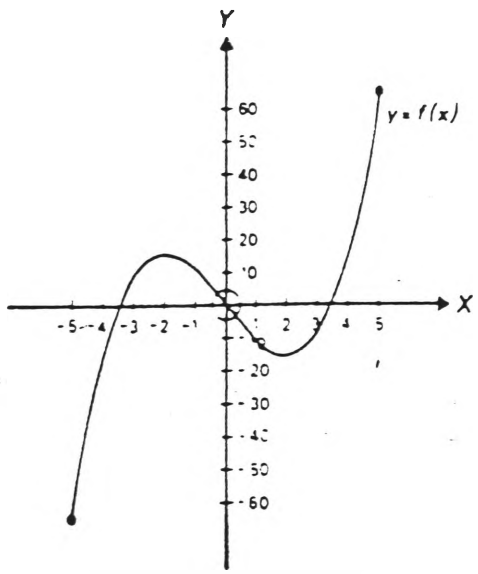
(v) Imagine the curve pictured above as a "roller-coaster" and describe the changes in velocity which you as the passenger would experience

(Rates of Change - 2)

(vi) On the same time-scale as shown, draw the **graph** of the velocity as you have described it What type of graph is produced ?

(vii) Describe the **force** (or **acceleration**) acting upon the child on the swing (assuming that the swing is pulled back to the starting position "a" and then released - **not** pushed.) The acceleration is the **rate of change of the velocity**, just as the velocity is the rate of change of the displacement over time

(2)




The graph represents the function with the rule

$$f(x) = x^3 - 12x, \quad -5 \leq x \leq 5.$$

(i) Which one of the following statements is true?

A The maximum value of the function is 16.

B The equation $f(x) = 0$ has exactly two roots, one negative and one positive.

C $f(x) = K$ has three real roots for all real K.

D The minimum value of the function is -65.

Briefly give reasons why you reject or accept each alternative

(Rates of Change - 3)

(ii) Identify on the graph those places where the gradient is **zero**, and where it is **greatest** and **least**; use this information to sketch a graph of the **gradient function** of this cubic.

(iii) What type of graph is produced ? Did you expect this ?

(iv) Do the same for the quadratic function $F(x) = x^2 - 4$ to produce its gradient function, $F'(x)$. What do you observe about the relationship between the two graphs ?

(3) The GRADIENT FUNCTION of a given function $f(x)$ is often called the **DERIVATIVE** of that function, and is denoted by symbols such as $^{dy}/_{dx}$ ("d" representing a "small change" - thus $^{dy}/_{dx}$ implies a "change in y with a change in x") or just $f'(x)$. Thus the derivative of a function $y = G(x)$ could be called the $^{dy}/_{dx}$ or $G'(x)$. It describes the **instantaneous rate of change** of that function at each point in the domain.

## Mathematical Investigation : THE DERIVATIVE

**AIM** : To introduce to students the basic concepts of the **differential calculus** in which we study the rate of change of a function through its "derived function", the **derivative**.

**REQUIRED KNOWLEDGE** :

Calculator : To calculate the derivative of a function

(1) Enter the function e.g. 'X^3 - 12*X [ENTER]

(2) Enter the variable name i.e. 'X [ENTER] (We say we "**differentiate** the function **with respect to** X")

(3) Press $^{d}/_{dX}$ (i.e. INV 6)

In this case, $^{d}/_{dX}$ ( $X^3$ - 12X ) = $3X^2$ - 12

This is the **derivative** or **gradient function**.

If we wish to graph both function and derivative on the same graph

(1) Enter the function as above and **STOre** it under some name, say F, by pressing 'F and then STO. Recall the function by simply entering F.

(2) Find the derivative as above, and store this as, say, 'D

(3) Both functions may be graphed by entering

'F = D [ENTER] EVAL

and then using the PLOT menu (Press STEQ and then DRAW)

Note : It may be necessary before DRAWing to increase the scale of the vertical axis using *H on the second page of the PLOT menu. In this case, 10 *H gives a good picture

(The Derivative - 2)

(1) **DISCUSS** how the parabola $3X^2 - 12$ describes the rate of change of the cubic $X^3 - 12X$.

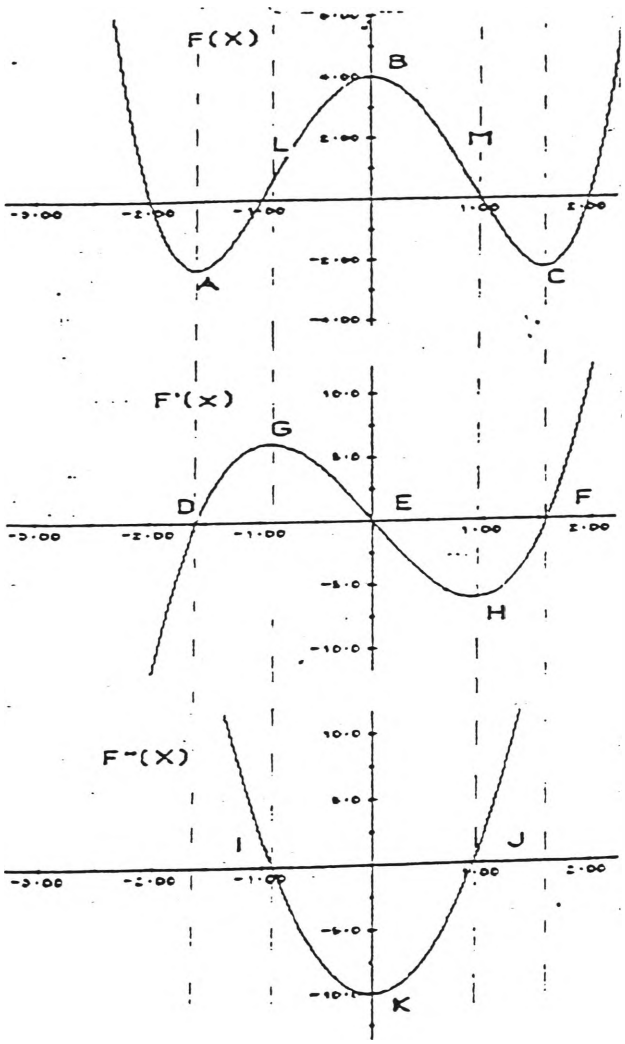
(i) State the x-value where the gradient of $f(x) = x^3 - 12x$ is least. (Look at the derivative $f'(x)$ to see this clearly.) What does this indicate about the **rate of change** of $f(x)$?

(ii) Between what intervals on the x-axis is the gradient (or velocity) of the original curve **positive** ? **Negative** ? (How can this be seen from the graph of $f'(x)$ ?

(iii) Where is the gradient (velocity) **increasing** ? **Decreasing** ?

(iv) What information would the **second derivative**, $f''(x)$, give about the derivative, $f'(x)$, and the original function? Explain.

**EXERCISES**




Consider a quartic function, F(x), its derivative, F'(x), and second derivative, F''(x) as shown.

(i) Points A, B and C are called **turning points** or **stationary points**. Which are the turning points on F'(x) and F''(x) ? What do these points tell you about the function ? How can they be found from the derivative ?

(ii) L and M are called **points of inflexion**, and they indicate a change in the **concavity** of the curve. What do you think this means ? How can points of inflexion be deduced from the first and second derivatives ? Name a point of inflexion on F'(x).

(The Derivative - 3)

(iii) The part of $F''(x)$ lying between I and J is negative, lying below the x-axis. What does this tell you about this region on $F'(x)$ and $F(x)$?

(2) As shown above, sketch on three graphs the following functions and their first and second derivatives, and use these to identify all turning points and points of inflexion on the original functions :

(i) $G(x) = x^3 - 12x + 6$ (ii) $Q(x) = x^4 - 5x^2 + 4$

(3) (i) If you are told that a function $\phi(x)$ has $\phi'(2) = 0$ and $\phi''(2) > 0$, what can you deduce about $\phi(2)$ ?

(ii) Given that $H'(-2) = 0$ and $H''(-2) = -8$, what does this tell you about $H(-2)$ ?

(4) A body moves so that its speed V m/s, t seconds after starting, is given by

$$V = t^2(10 - t)$$

After what time did its speed change from a state of increasing to a state of decreasing ? What is its maximum speed ?

## COMPONENT B SAMPLE QUESTION

Suppose transportation specialists have determined that **G(v)**, the number of miles per gallon that a vehicle gets, is a function of the vehicle's speed in miles per hour.

**(a) Interpret**, in terms of mileage and speed, the fact that

$$G'(55) = 0.4$$

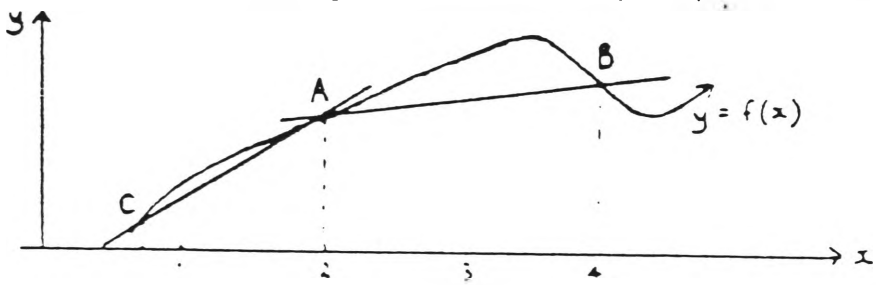
(b) How might that fact be used in a debate about setting an appropriate national speed limit ?

**Mathematics Assessment Task : Component B**

Select the correct answer to each of the following questions and, in the space provided, give reasons for your answer. Of the FOUR marks available for each response, a correct answer alone is worth only ONE.

1. You are given the graph below and the following two facts :

- The line containing A and B has slope equal to $\frac{1}{2}$.
- The line containing A and C has slope equal to 2.



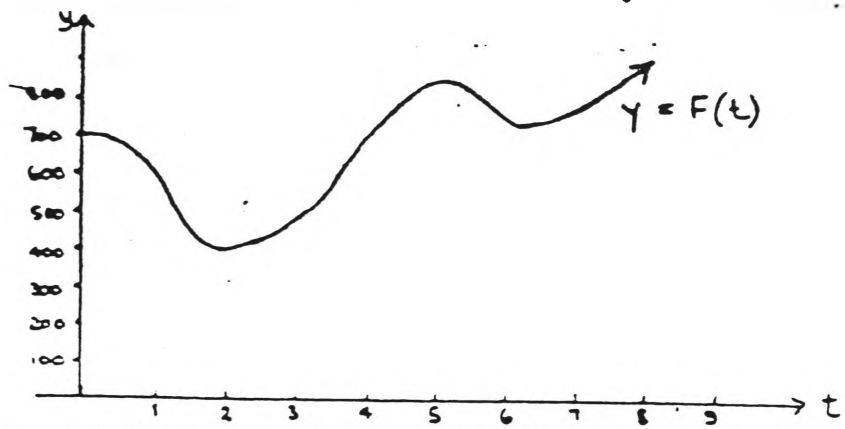

which, if any, of the following statements **must** be true? For each of the statements which is true, explain why. (You may wish to name more than one statement.)

a. The slope of the curve $y = f(x)$ at $x = 4$ is greater than $\frac{1}{2}$.

b. The slope of the curve $y = f(x)$ at $x = 2$ is less than 2 and greater than $\frac{1}{2}$.

c. The slope of the curve $y = f(x)$ at $x = .74$ is greater than 2.

2. The number of deer in a forest at time t years after the start of a conservation study is F(t), as shown on the graph below.




a. Between what two years is the population of deer declining at a rate of approximately 100 deer per year? Explain.

b. During what time periods is the rate of change in the deer population increasing? Explain in terms of the graph.

c. When is the population of deer increasing at the greatest rate? Explain in terms of the graph.

3. G(t) is the number of people unemployed in a country t weeks after the election of a fiscally conservative Prime Minister. Translate each of the following facts about the graph y = G(t) into statements about the unemployment situation.

a. The y-intercept of y = G(t) is 2,000,000.

b. G(20) = 3,000,000.

c. The slope of y = G(t) at t = 20 is 10,000.

d. G"(36) = 800 and G'(36) = 0.

4. Thus far in the course, you have learned no rule for finding the derivative of a function like $F(x) = 3^x$. Explain how you could find $F'(4)$.

5. Explain, as clearly and simply as possible, the meaning of the terms :

DERIVATIVE :

POINT OF INFLEXION :

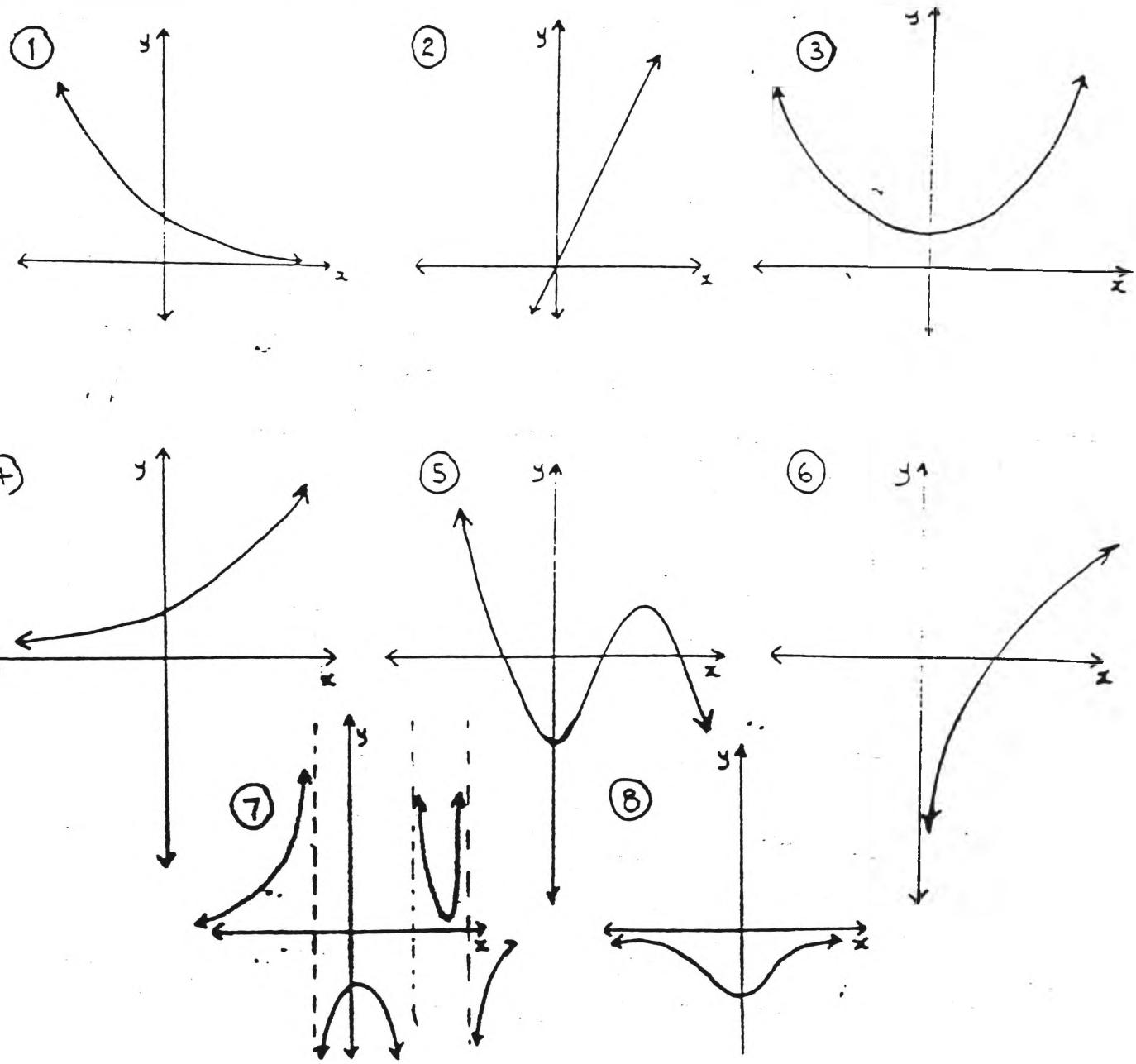
6 Match each equation below with the graph it determines. Justify each answer briefly

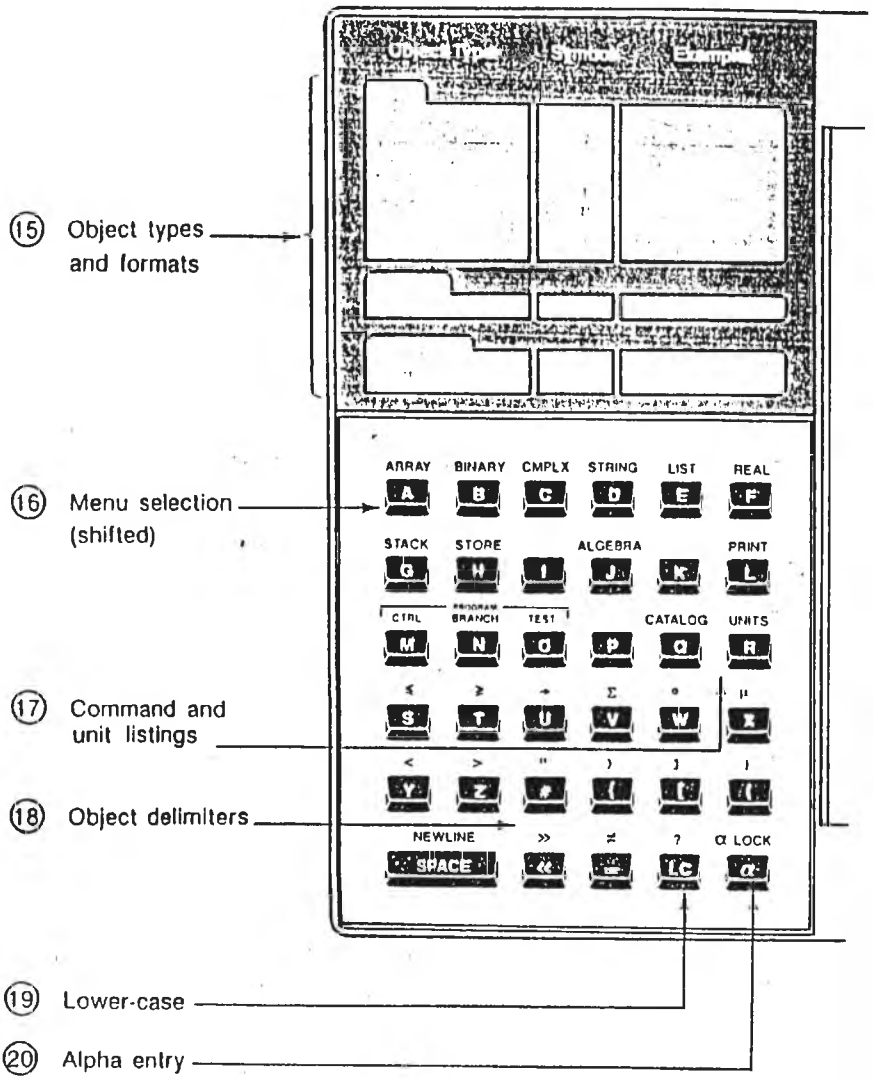
a. $y = 5e^{-3x}$

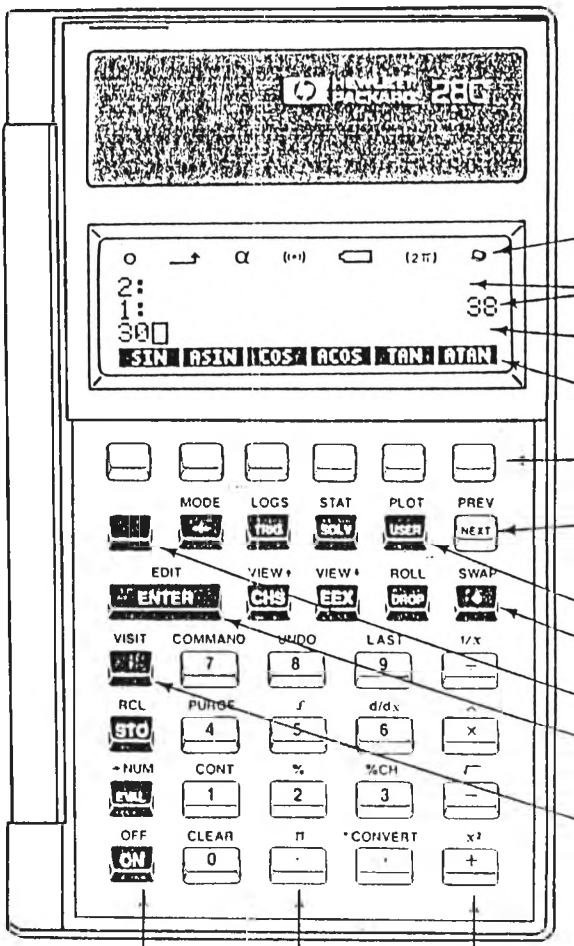
b. $y' = 2x$

c. $y = \ln x$

a. $y = 1/(x - 2)(x + 1)(4 - x)$




(15) Object types and formats
(16) Menu selection (shifted)
(17) Command and unit listings
(18) Object delimiters
(19) Lower-case
(20) Alpha entry
ARRAY
BINARY
CMPLX
STRING
LIST
REAL
STACK
STORE
ALGEBRA
PRINT
CTRL
BRANCH
TEST
CATALOG
UNITS
NEWLINE
SPACE
α LOCK




(14) Annunciators

(13) Stack levels

(12) Command line

(11) Menu labels

(10) Menu keys

(9) Next menu row

(8) Menu selection

(7) Backspace

(6) Shift

(5) Enter command line

(4) Delimiter for symbolic objects

(3) Arithmetic

(2) Number entry

(1) Power on/off; clear command line; stop program

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