Exploring the Sine Rule

ID: XXXX

Time required
45 minutes

Activity Overview

In this activity, students investigate the relationships between sides, angles and the ratios of these for right-angled and non-right-angled triangles, consolidating skills and understandings related to the Sine Rule.

Concepts

- *Sine as ratio of sides, sine rule as ratio of side and sine of the corresponding angle.*

Teacher Preparation

This investigation offers opportunities for review and consolidation of key concepts related to the sine rule. As such, care should be taken to provide ample time for ALL students to engage actively with the requirements of the task, allowing some who may have missed aspects of earlier work the opportunity to build new and deeper understanding.

- *This activity can serve to consolidate earlier work on trigonometry. It offers a suitable introduction to non-right-angled trigonometry, extending if desired to the cosine rule.*
- *Begin by discussing how some of the properties of right-angled triangles can be utilized to solve problem situations. This lesson extends this capability.*
- *The screenshots on pages X–X (top) demonstrate expected student results. Refer to the screenshots on page X (bottom) for a preview of the student .tns file.*
- ***To download the .tns file, go to http://education.ti.com/exchange and enter "XXXX" in the search box.***

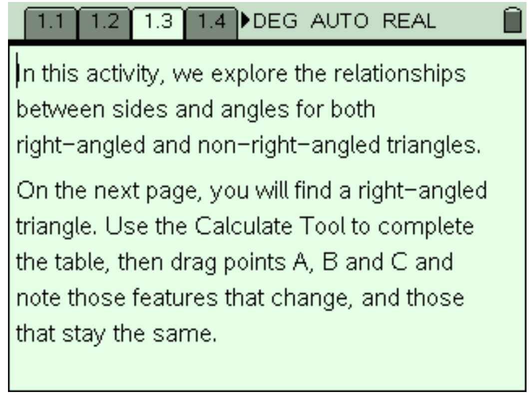
Classroom Management

- *This activity is intended to be* ***teacher led.*** *You should seat your students in pairs so they can work cooperatively on their handhelds. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds, although the majority of the ideas and concepts are only presented in* ***this*** *document; be sure to cover all the material necessary for students' total comprehension.*
- *Students can either record their answers on the handheld or you may wish to have the class record their answers on a separate sheet of paper.*

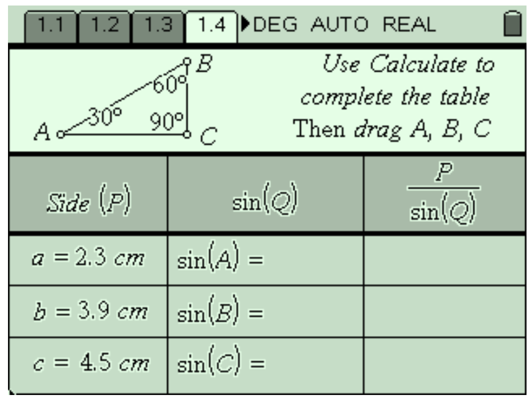
TI-Nspire™ Applications

Graphs & Geometry, Notes, Calculator

Step 1: Students should be encouraged to share what they already know and understand regarding trigonometry. In particular, they should discuss the relationship between sides and angles: general observations should include noting that the largest angle is found opposite the longest side, and the same for the smallest angle and side.

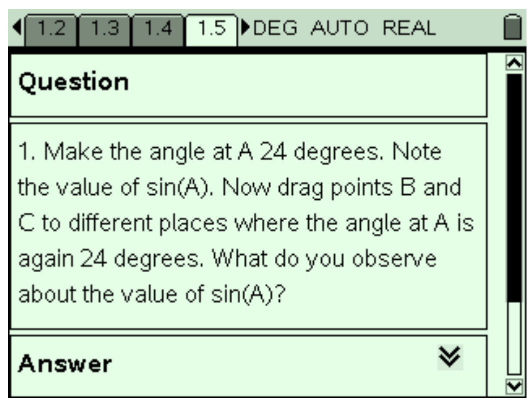


Step 2: Page 1.4 offers a geometric model by which students may explore the relationships between sides and angles more closely. Using the Calculate Tool, students should complete the table and then observe the results as they grab and drag the different vertices of the triangle. If desired, there may be value in having them calculate the sine, cosine and tangent angles fro the ratios of the sides.



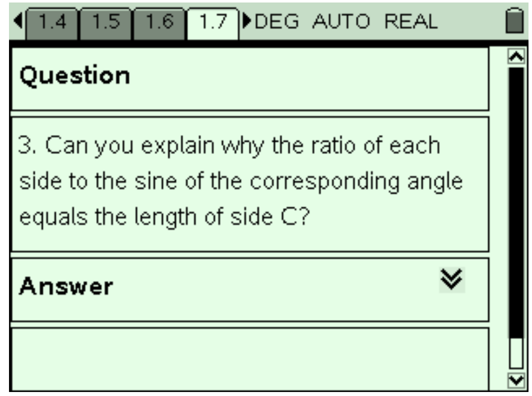
Step 3: It is important that students realize that the same angle can be produced from many different triangles. They should observe that, although the side lengths which make it up may vary, the value of a ratio such as sin(24 degrees) does not vary, and nor does the ratio of side to sine that we are now introducing.

Student understanding of ratios may need some careful monitoring, looking for misconceptions that may be disproved using the dynamic model.

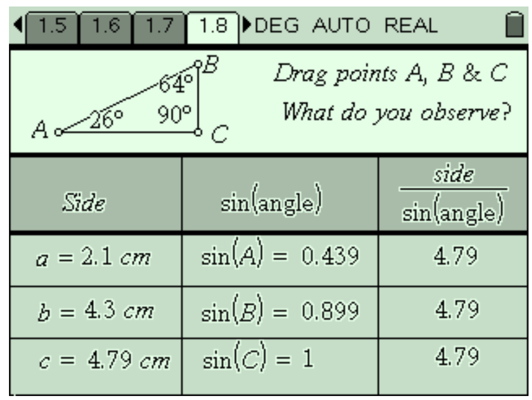


Step 4: Although students should be comfortable with Sine as a ratio of sides, this activity introduces a new type of ratio: side to the sine of the corresponding angle.

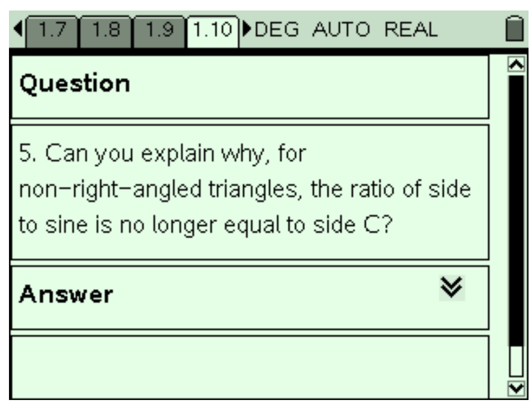
Using their prior knowledge of the sine ratio, students should be able to demonstrate algebraically why a ratio such as a : sin(A) will equal the hypotenuse c in a right-angled triangle, after observing it geometrically.



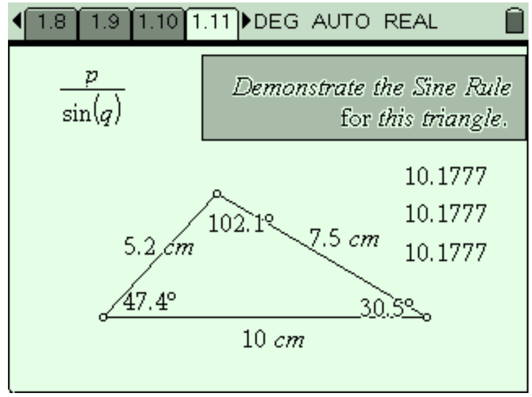
Step 5: We are now ready to extend this understanding to non-right-angled triangles. While the previous triangle ABC was constrained to being right-angled, this one is not and students should observe that, even for non-right-angled triangles, the ratio of side to sine remains equal for all three sides.



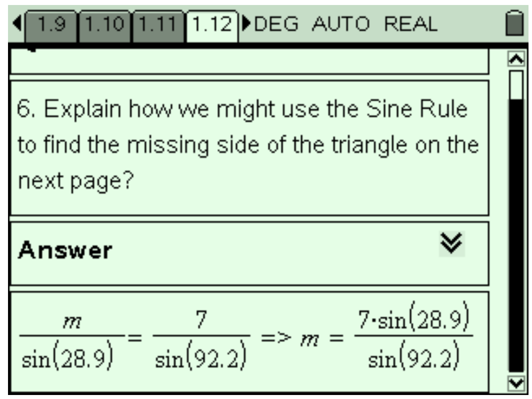
Step 6: Students may simply observe that, for a right-angled triangle, sin(C) = sin(90) = 1, and hence the ratio of c to sin(C) will naturally be equal to the length of the hypotenuse. When angle C is no longer equal to 90 degrees, then the ratio will no longer reduce to just the value c.



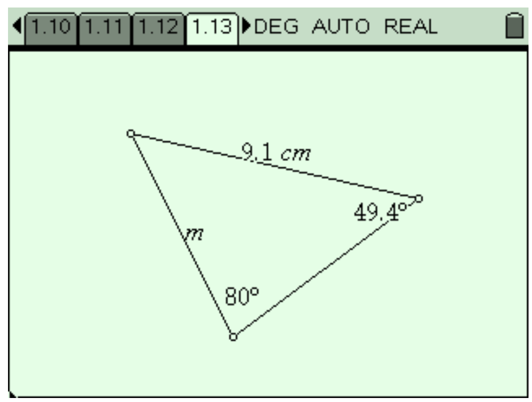
Step 7: Page 1.11 offers an opportunity for students to verify the relationship between sides and sines of corresponding angles, leading up to their statement of the sine rule in both words and algebraic notation.



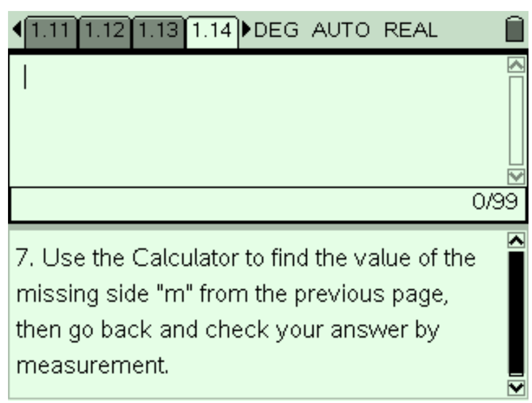
Using the Calculate Tool, students should evaluate the sine rule ratio for the three sides of the given triangle, and then observe that these three results remain equal to each other no matter how the triangle changes.



Using their own words, they should then be encouraged to explain carefully how we might use this result to find a missing side, and then verify this result algebraically and geometrically using the tools provided.

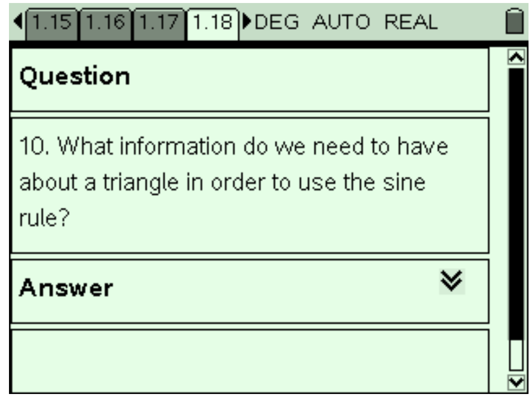


First, they demonstrate their result by verbal explanation, referring to the triangle provided on page 1.13. They use the Calculator on page 1.14 to calculate their result for the length of the side labeled "m". finally, they may return to the diagram and use the Length Measure Tool to verify their calculated result.



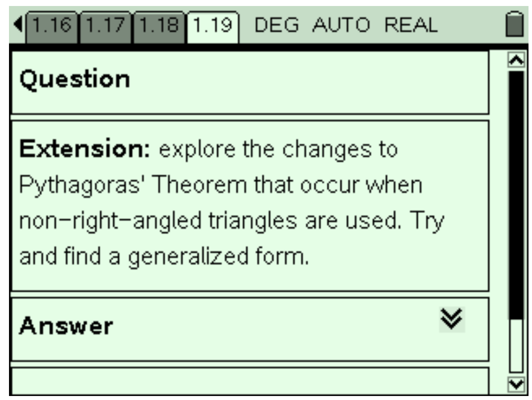
In the same way, students should establish, compute and verify a method for finding a missing angle, given two sides and another angle (pages 1.15 – 1.17).

Step 8: Finally, it is important that students reflect upon what they have learned, and put their knowledge into words in a practical way. A useful generalization of this knowledge would be a recognition of the circumstances under which the Sine Rule would be appropriate.



Extension: A suitably challenging extension for this activity would be to encourage students to attempt to generalize Pythagoras' Theorem in the same way that we have here generalized the Sine Ratio.

If scaffolding is required, then the idea of dropping an altitude and turning any non-right-angled triangle into two adjacent right-angled triangles may get students started.



Exploring the Sine Rule – ID: XXXX

(Student)TI-Nspire File: *Alg1ActXX_Sine_Rule_EN.tns*

1.1 DEG AUTO REAL

Exploring the Sine Rule

Algebra 1: Trigonometry

1.2 DEG AUTO REAL

Trigonometry is cool.

It uses triangles to show connections between two things that seem quite different: angles and lengths.

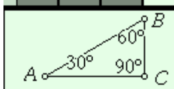
To start with, we use right-angled triangles to establish these relationships, but what about triangles that are not right-angled?

1.3 DEG AUTO REAL

In this activity, we explore the relationships between sides and angles for both right-angled and non-right-angled triangles.

On the next page, you will find a right-angled triangle with sides and angles measured. Use the Calculate Tool to complete the table, then drag points A, B and C and note those features that change, and those that stay the same.

1.4 DEG AUTO REAL



Use Calculate to complete the table
Then *drag A, B, C*

Side (P)	sin(Q)	$\frac{P}{\sin(Q)}$
$a = 2.3\ cm$	$\sin(A) = 0.5$	4.5
$b = 3.9\ cm$	$\sin(B) = 0.87$	4.5
$c = 4.5\ cm$	$\sin(C) = 1$	4.5

1.5 DEG AUTO REAL

Question

1. Make the angle at A 24 degrees. Note the value of sin(A). Now drag points B and C to different places where the angle at A is again 24 degrees. What do you observe about the value of sin(A)?

Answer

1.6 DEG AUTO REAL

2. What did you notice about the ratio of each side to the sine of the corresponding angle? Try different angles.

Answer

The ratio of side to sine is the same for all three sides and angles for any triangle.

1.7 DEG AUTO REAL

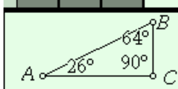
3. Can you explain why the ratio of each side to the sine of the corresponding angle equals the length of side C?

Answer

sin(A) = a/c (opposite/hypotenuse)

Then a/sin(A) = a/(a/c) = c.

1.8 DEG AUTO REAL



Drag points A, B & C
What do you observe?

Side	sin(angle)	$\frac{side}{\sin(angle)}$
$a = 2.1\ cm$	$\sin(A) = 0.439$	4.79
$b = 4.3\ cm$	$\sin(B) = 0.899$	4.79
$c = 4.79\ cm$	$\sin(C) = 1$	4.79

1.9 DEG AUTO REAL

4. What do you observe about the ratio of side to sine when the triangle is no longer right-angled?

Answer

The ratios of side to sine remain equal to each other, for all possible triangles.

1.10 DEG AUTO REAL

5. Can you explain why, for non-right-angled triangles, the ratio of side to sine is no longer equal to side C?

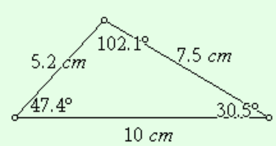
Answer

In a right-angled triangle, sin(C) = 1 since C = 90 degrees. So c/sin(C) = c. But for non-right-angled triangles, sin(C) is no longer 1, and so the ratios will no longer

1.11 DEG AUTO REAL

$\frac{p}{\sin(q)}$

Demonstrate the Sine Rule for this triangle.

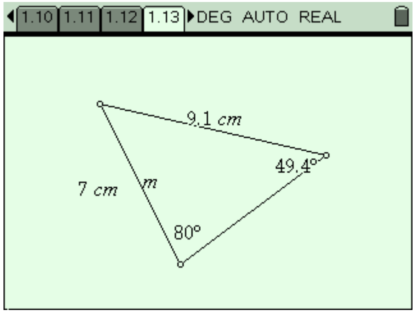


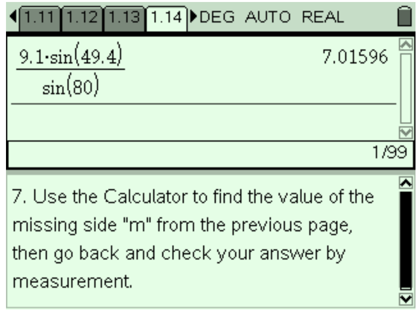
1.12 DEG AUTO REAL

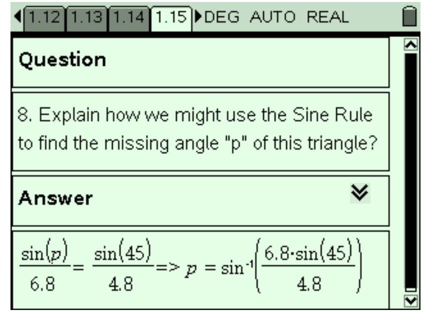
6. Explain how we might use the Sine Rule to find the missing side of the triangle on the next page?

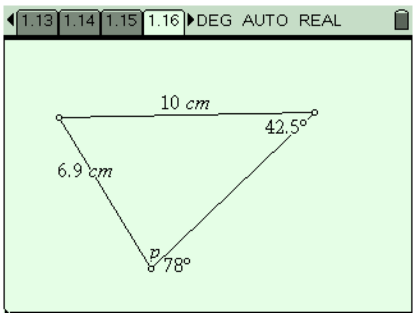
Answer

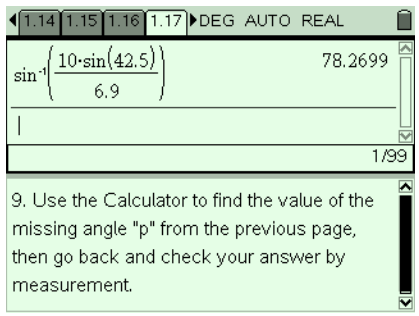
$\frac{m}{\sin(28.9)} = \frac{7}{\sin(92.2)} \Rightarrow m = \frac{7 \cdot \sin(28.9)}{\sin(92.2)}$

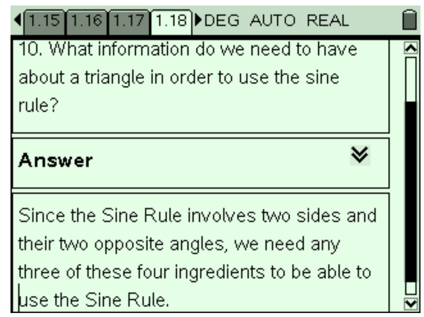

1.10 1.11 1.12 1.13 DEG AUTO REAL
9.1 cm
49.4°
7 cm
m
80°

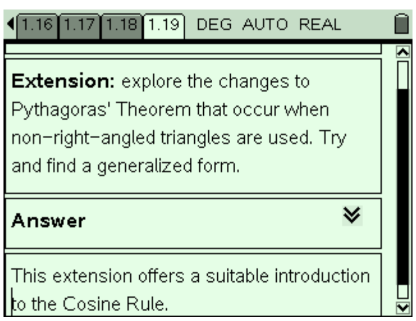

1.11 1.12 1.13 1.14 DEG AUTO REAL
$\frac{9.1\cdot\sin(49.4)}{\sin(80)}$ 7.01596
1/99
7. Use the Calculator to find the value of the missing side "m" from the previous page, then go back and check your answer by measurement.


1.12 1.13 1.14 1.15 DEG AUTO REAL
Question
8. Explain how we might use the Sine Rule to find the missing angle "p" of this triangle?
Answer
$\frac{\sin(p)}{6.8}=\frac{\sin(45)}{4.8} => p=\sin^{-1}\left(\frac{6.8\cdot\sin(45)}{4.8}\right)$


1.13 1.14 1.15 1.16 DEG AUTO REAL
10 cm
42.5°
6.9 cm
p
78°


1.14 1.15 1.16 1.17 DEG AUTO REAL
$\sin^{-1}\left(\frac{10\cdot\sin(42.5)}{6.9}\right)$ 78.2699
1/99
9. Use the Calculator to find the value of the missing angle "p" from the previous page, then go back and check your answer by measurement.


1.15 1.16 1.17 1.18 DEG AUTO REAL
10. What information do we need to have about a triangle in order to use the sine rule?
Answer
Since the Sine Rule involves two sides and their two opposite angles, we need any three of these four ingredients to be able to use the Sine Rule.


1.16 1.17 1.18 1.19 DEG AUTO REAL
Extension: explore the changes to Pythagoras' Theorem that occur when non-right-angled triangles are used. Try and find a generalized form.
Answer
This extension offers a suitable introduction to the Cosine Rule.