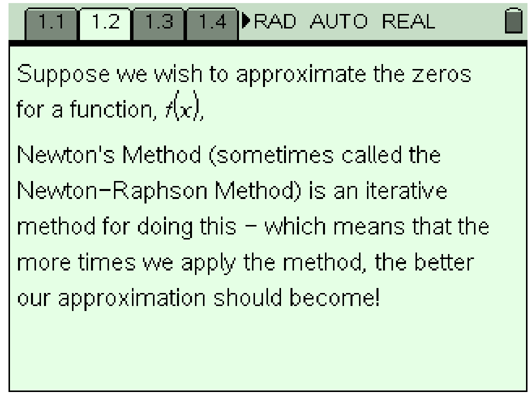
Exploring Newton's Method

ID: XXXX

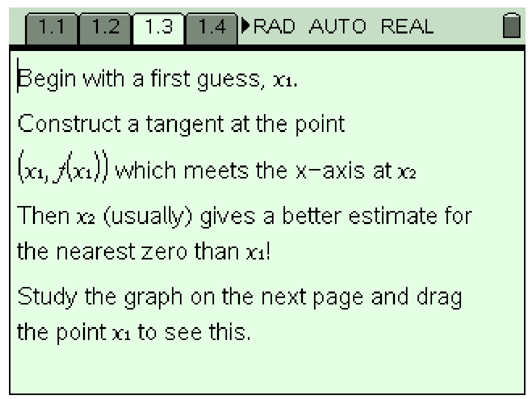
Name ______________________

Class ______________________

In this activity, you will build an understanding of Newton's Method for finding approximations for zeros of a given function. You will use a variety of tools, graphical, numerical, algebraic and programming, to observe the process and limitations of this important method.



Open the file *Alg1ActXX_Newtons_Method_EN.tns* on your handheld and follow along with your teacher to work through the activity. Use this document as a reference and to record your answers.



The Problem

Suppose we wish to approximate the zeros for a function, f(x),

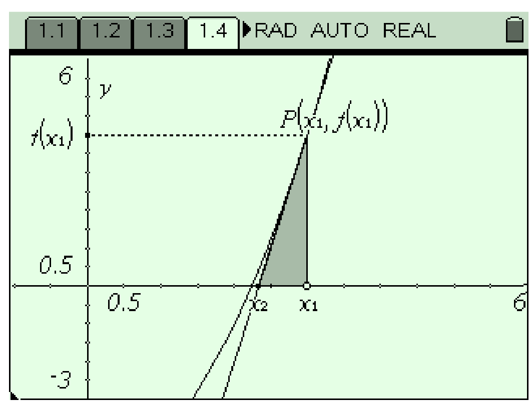
Newton's Method (sometimes called the Newton-Raphson Method) is an iterative method for doing this - which means that the more times we apply the method, the better our approximation should become!

Begin with a first guess, x_1.

Construct a tangent at the point $(x_1, f(x_1))$ which meets the x-axis at x_2

Then x_2 (usually) gives a better estimate for the nearest zero than x_1!

Study the graph on page 1.4 and drag the point x_1 to see this.



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EXERCISES

1. Can you see how this method works? Should it always work?
2. Find the gradient between the points P(x_1, f(x_1)) and (x_2, 0).
3. Since this is the gradient of the tangent to the curve at P, then it may be expressed in terms of f'(x). Show this relationship.
4. Using the equation you have just created, solve for x_2. This is Newton's formula.

Use the graph and spreadsheet tools on pages 2.2 and 2.3 to further explore this process. Try different functions and different starting values.

5. Another approach often used with iterative processes is to write a program which will perform the repetitive task quickly and efficiently. Outline the steps you would use here.

Use CAS tools if you have them to take advantage of more powerful spreadsheet, function definition and programming options available (Pages 4.1 – 4.5).

6. Using any or all of these available tools, explore Newton's Method and clearly explain when and why it fails.

SUGGESTED SOLUTIONS

1. The method uses the fact that the tangent at a point appears to cross the x-axis between that point and the nearest zero. It should work except when turning points get in the way.
2. Gradient = $((f(x_1)-0)/(x_1 - x_2)) = ((f(x_1))/(x_1-x_2))$
3. $f'(x_1) = ((f(x_1))/(x_1-x_2))$
4. $x_2 = x_1 - ((f(x_1))/(f'(x_1)))$
5. Step 1: Define the function, f(x).
    a. Step 2: Choose an initial guess, a.
    b. Step 3: Evaluate f(a)
    c. Step 4: Evaluate f'(a).
    d. Step 5: Use the formula a-((f(a))/(f'(a))) to find a better approximation.
6. The method cannot work when the guess lies on a turning point (since the tangent will never cut the x-axis), nor will it work when a turning point lies between the guess and the zero - it actually goes further away instead of closer.