Making Witches Hats from Cones

ID: XXXX

Time required
45 minutes

Activity Overview

In this activity, students make cones from circles, exploring the relationship between the size of the sector cut from the original circle, the radius of the base and the height of the cone produced.

Concepts

- *Sectors, arcs, circumference, areas of circles and properties of cones.*

Teacher Preparation

This investigation offers opportunities for review and consolidation of key concepts related to the properties of circles and cones. As such, care should be taken to provide ample time for ALL students to engage actively with the requirements of the task, allowing some who may have missed aspects of earlier work the opportunity to build new and deeper understanding.

- *This activity can serve to consolidate earlier work on circle properties. It offers a suitable introduction to three-dimensional figures, extending to volumes and surface areas.*
- *Begin by discussing how some of the properties of geometry are going to be utilized to solve problem situations.*
- *The screenshots on pages X–X (top) demonstrate expected student results.*
  Refer to the screenshots on page X (bottom) for a preview of the student .tns file.
- ***To download the .tns file, go to http://education.ti.com/exchange and enter "XXXX" in the search box.***

Classroom Management

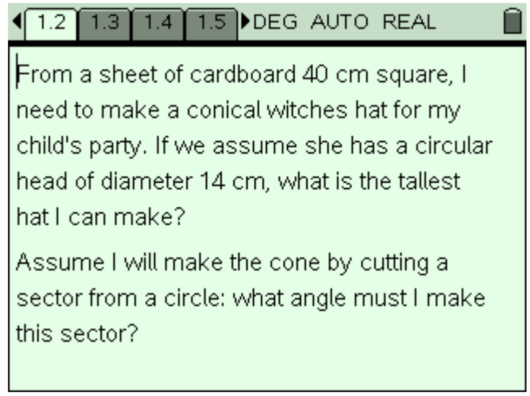
- *This activity is intended to be* ***teacher led.*** *You should seat your students in pairs so they can work cooperatively on their handhelds. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds, although the majority of the ideas and concepts are only presented in* ***this*** *document; be sure to cover all the material necessary for students' total comprehension.*
- *Students can either record their answers on the handheld or you may wish to have the class record their answers on a separate sheet of paper.*

TI-Nspire™ Applications

Graphs & Geometry, Notes, Lists & Spreadsheet, Calculator

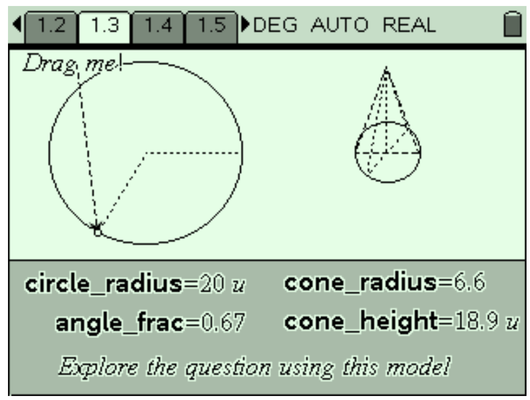
Step 1: Students are to go to page 1.2, where they are to read about the problem situation.

It is recommended that students physically model this problem by cutting out different sized sectors and making cones from the remainders. If desired, the radius and height of the resulting cones may be collected into a Lists & Spreadsheet page and the relationship studied both numerically and graphically.



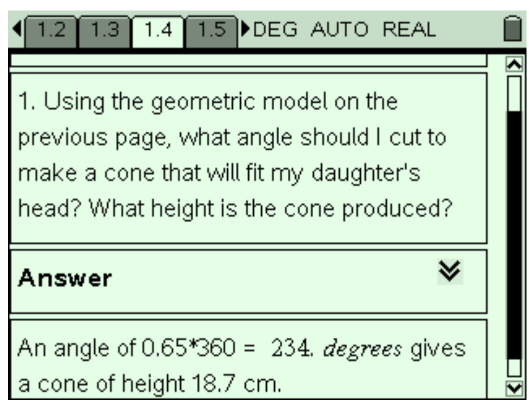
Step 2: Page 1.3 offers a geometric model by which students may explore the problem more generally than would be possible by actually cutting out sectors from paper circles.

Class discussion may focus upon the possible relationship between the size of the sector angle and the height and base radius of the resulting cone.



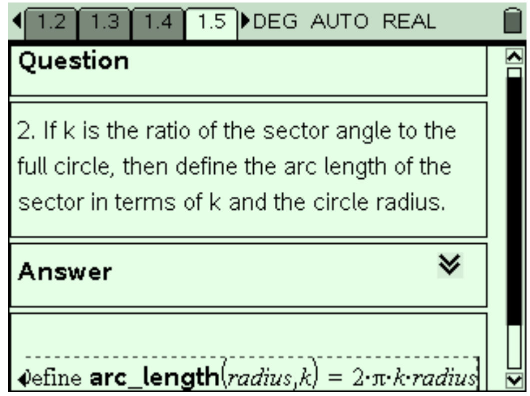
Step 3: Students should use the geometric model provided to find the answer to the original question: an angle of around 234 degrees will produce a cone which fits the child's head and produces a witches hat of 18.7 cm.

The challenge comes in seeking to generalise this result, and for this an algebraic approach (supported by the TI-Nspire) is needed,

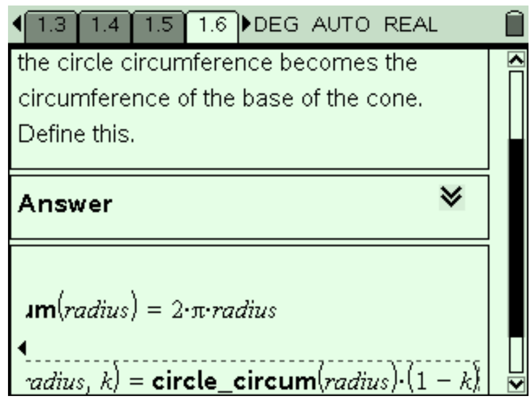


Step 4: It is helpful at this stage to introduce the idea of an angle ratio – what fraction is the sector angle of the entire circle? This variable (we will call it "k") will make it easier for students to develop the required functional relationships for this task.

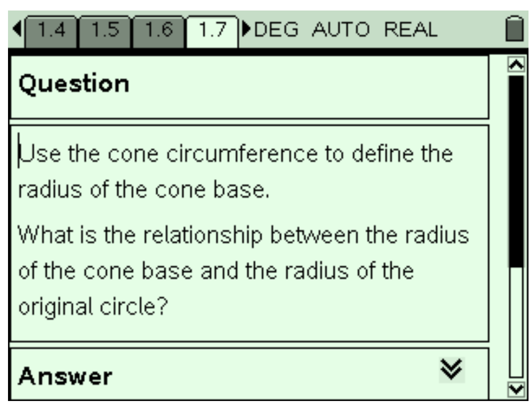
The concept of **circumference** may need to be reviewed, and from this developed the understanding that the arc length of the sector will be the same fraction of the total circumference as the central sector angle is of the full circle.



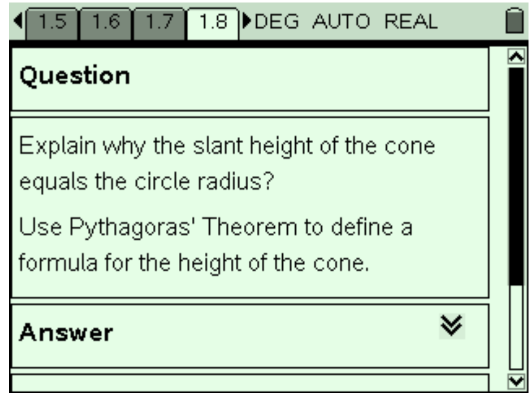
Step 5: By working with a concrete model, students should be led to appreciate that the remainder of the circle circumference once the sector is removed actually becomes the circumference of the base of the cone, leading to a function defined in terms of the circumference and radius of the circle, along with the ratio k. i.e. the circumference of the cone equals the circumference of the circle minus the arc length of the sector. *It is important that students are able to express this relationship in words before attempting an algebraic form.* Note that TI-Nspire allows use of such real language in the definition of functions.



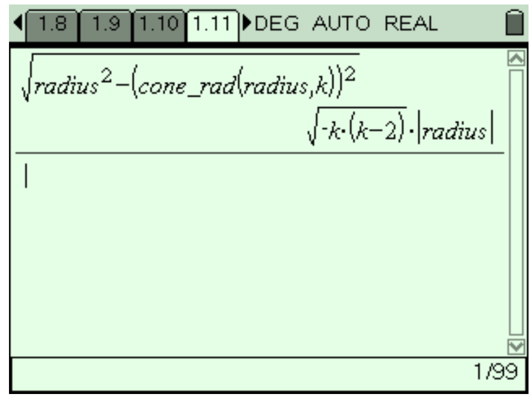
Step 6: The next step is firmly algebraic and students should be given the opportunity to try to develop the relationship between the cone circumference and the radius of the base themselves, working in pairs, before too much scaffolding is provided by the teacher. It involves solving the equation linking the standard circumference formula (2 * Pi * x where "x" is the radius of the cone base) with the new formula for the circumference of the cone base given above. The radius of the cone base turns out to be (1 – k) times the original circle radius.



Step 7: We now have one expression for the radius of the cone base, but we also know the slant height of the cone, since it is formed directly from the original radius of the circle – once the sector is removed, the two radii are joined together, giving the cone itself and equating to the slant height. We may now use Pythagoras' Theorem to develop an expression for the height of the cone in terms of the circle radius and the sector angle ratio, k.

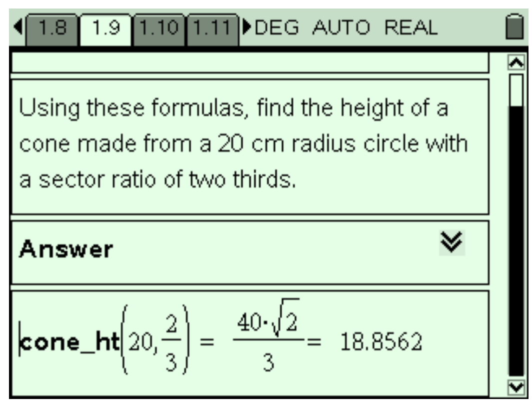


The use of CAS facilties at this point confirms a nice simplification of this height formula. Since the ratio of the slant height (equal to the radius of the original circle) of the cone to the base radius of the cone is 1 : 1 – k, then the simplified formula shown follows from Pythagoras, after removing "radius" as a common factor.

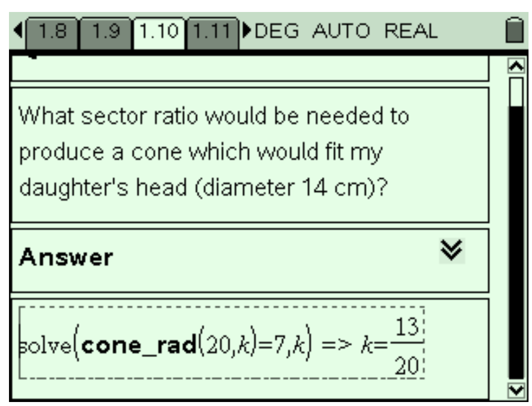


We are now in a position to answer some questions about our witches hat.

We can predict the height of a cone from a given circle, produced by cutting sectors of different angles.

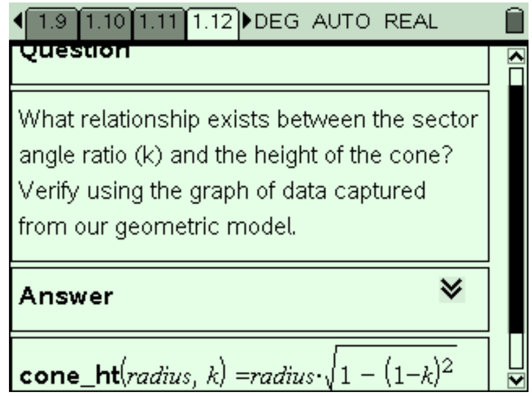


We may also answer the original question posed: from a 20 cm radius circle, what angle do I need to cut in order to produce a cone which will fit my daughter's head size? In this case, with a head radius of 7 cm, the ratio of the sector to the full circle needs to be 13/20, or 0.65, as predicted by the geometric model.

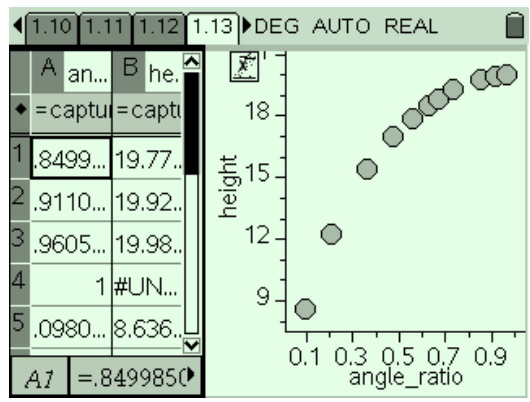


We are also now in a position to ask more general questions, concerning the nature of the relationship between the angle size and the height of the cone produced.

Step 8: It might first be established that the relationship between the radius of the original circle and that of the cone base is a linear one. Again, this may be done using the geometric model provided and capturing data from these two measurements or, more powerfully, by collecting student data from different sized cones they have made from different sized circles, and then plotting the graph of this data using **Quick Graph**.

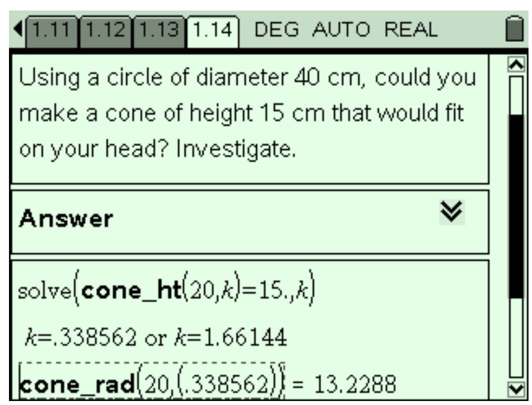


In the same way, it is readily established that the relationship between angle of sector and height of the resulting cone is non-linear, as shown. Algebraically, the relationship is a radical function, and students should be encouraged to try graphing their functions against the data shown. This is powerful and convincing for them in establishing whether their algebraic functions are correct.



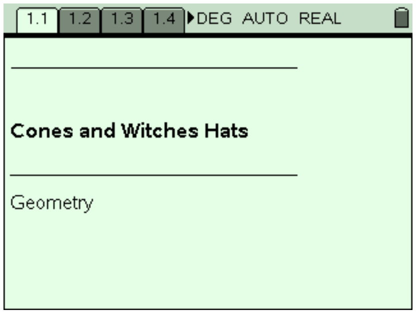
Extension: Finally, challenge questions and extensions may follow, including those which may be suggested by the students. In the example shown, it is possible to "work backwards" to predict the possibility of cones of different heights against a given head size.

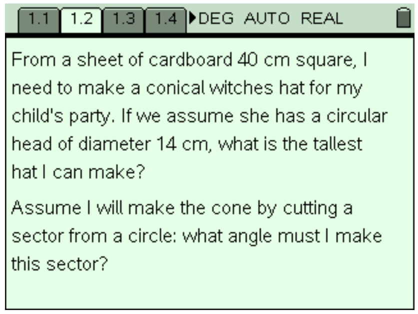
It should become quickly apparent that not all heights are possible for given head sizes, and further investigation may be worthwhile in establishing what these limitations are.

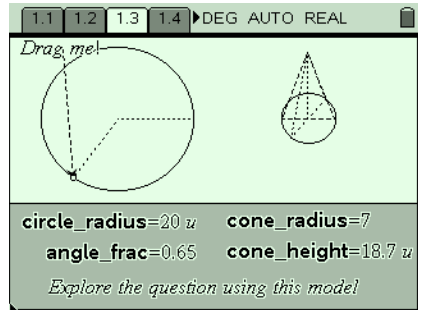


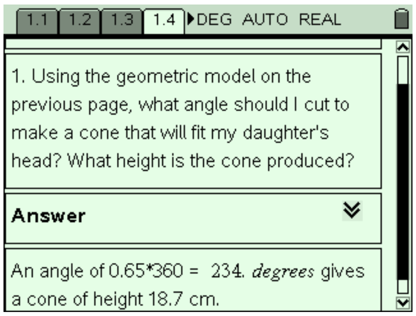
Cones and Witches Hats – ID: XXXX

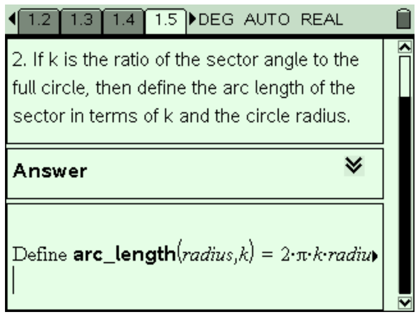
(Student)TI-Nspire File: *GeoActXX_Cones_and_Witches_Hats_EN.tns*

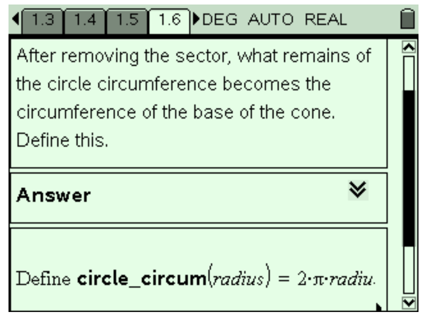


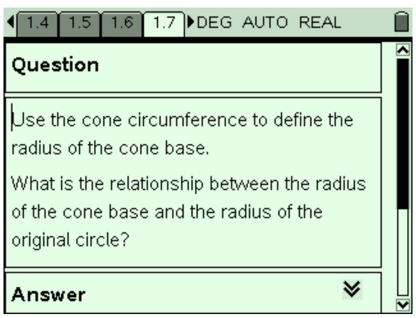


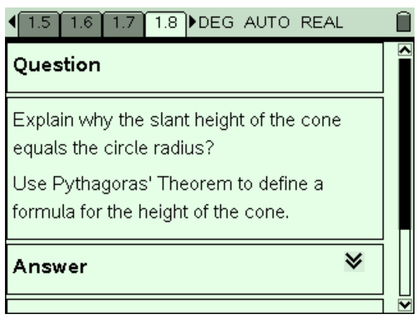


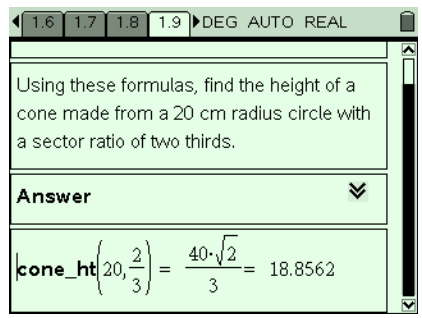


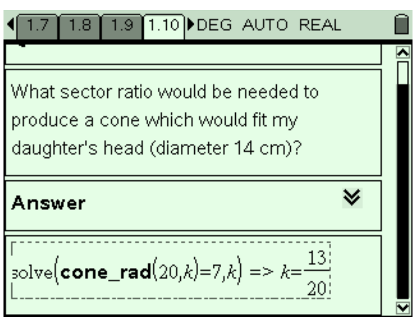


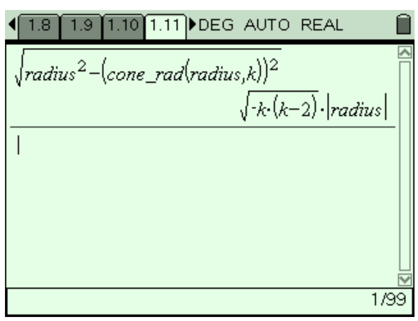


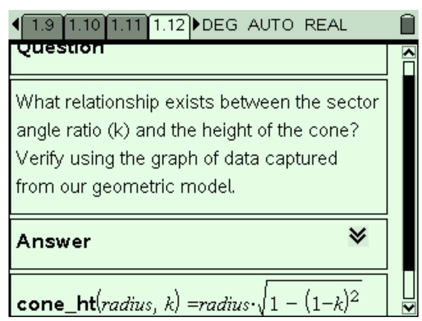


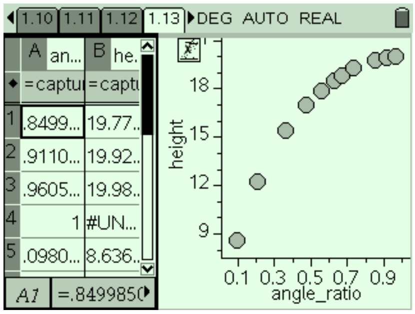


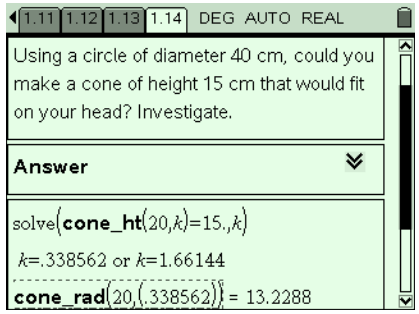









1.10 1.11 1.12 1.13 DEG AUTO REAL
A an... B he.
=captu =captu
1 .8499... 19.77..
2 .9110... 19.92..
3 .9605... 19.98..
4 1 #UN...
5 .0980... 8.636..
A1 =.8499850
height
18
15
12
9
0.1 0.3 0.5 0.7 0.9
angle_ratio


1.11 1.12 1.13 1.14 DEG AUTO REAL
Using a circle of diameter 40 cm, could you make a cone of height 15 cm that would fit on your head? Investigate.
Answer
solve(cone_ht(20,k)=15.,k)
k=.338562 or k=1.66144
cone_rad(20,(.338562)) = 13.2288

