Applications of Calculus: Torricelli's Law

ID: XXXX

Time required
45 minutes

Activity Overview

In this activity, students develop and use differential equations which support their learning of both Torricelli's Law for falling liquids and projectile motion. Using multiple representations to support understanding, students derive the defining formulas – first, beginning with the fundamental form for the velocity of escaping liquid through to the trajectory path of that fluid. This activity provides an introduction to differential equations.

Concepts

- *Differential equations, projectile motion.*

Teacher Preparation

This investigation offers opportunities for review and consolidation of key concepts related to differential equations. It provides a firm link between the theory and applications of the Calculus and offers opportunities for students to consolidate their skills (including using a prepared program). As such, care should be taken to provide ample time for ALL students to engage actively with the requirements of the task, allowing some who may have missed aspects of earlier work the opportunity to build new and deeper understanding.

- *This activity can serve to consolidate earlier work on integration. It offers a suitable introduction to differential equations.*
- *Begin by reviewing the method of integration of trigonometric functions (including inverse trigonometric functions), and methods of integration of the standard function forms.*
- *The screenshots on pages X–X (top) demonstrate expected student results. Refer to the screenshots on page X (bottom) for a preview of the student .tns file.*
- ***To download the .tns file, go to http://education.ti.com/exchange and enter "XXXX" in the search box.***

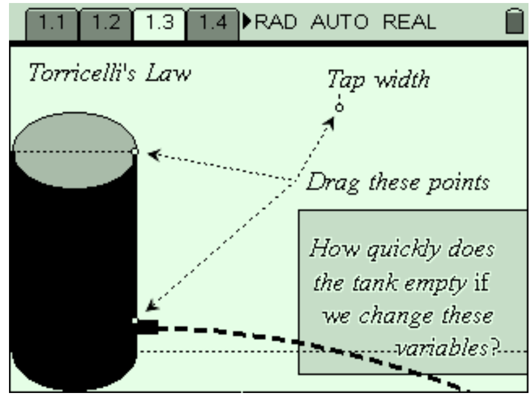
Classroom Management

- *This activity is intended to be **teacher led.** You should seat your students in pairs so they can work cooperatively on their handhelds. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds, although the majority of the ideas and concepts are only presented in **this** document; be sure to cover all the material necessary for students' total comprehension.*
- *Students can either record their answers on the handheld or you may wish to have the class record their answers on a separate sheet of paper.*

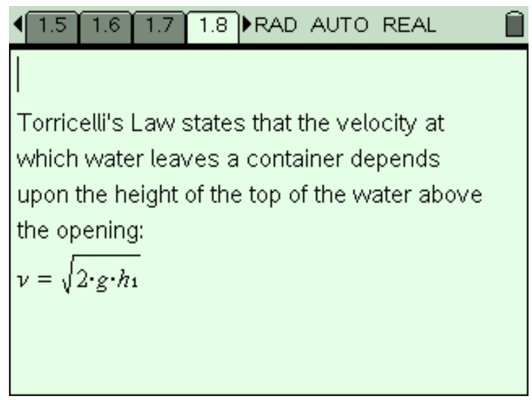
TI-Nspire™ Applications

Notes, Graphs & Geometry, Calculator, Programming.

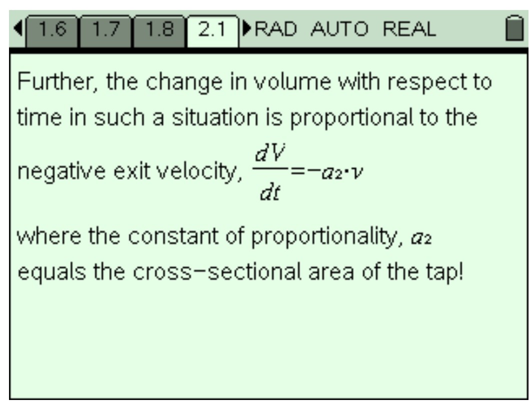
Step 1: Torricelli's Law (1643) describes the relationship between the velocity of fluid leaving a container under the force of gravity and the height of the fluid. In this activity we derive and explore these relationships using differential equations. Initially students begin with a dynamic model which allows them to change the key variables – height of fluid, height of the tap and the tap width.



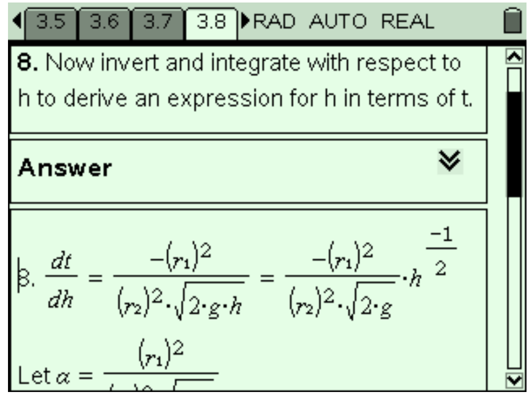
Step 2: Using the principle of conservation of energy, students should discuss the potential energy possessed by water at the top of the tank, the kinetic energy as it leaves through the tap, and then equate these to derive the common form for Torricelli's Law, as shown.



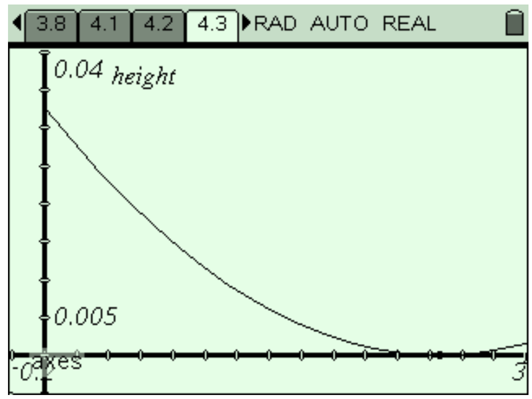
Step 3: This may then be linked to a differential equation for the change in volume with respect to time, which is clearly linked to the change in height of the fluid, and to both the speed at which the fluid escapes, and the diameter of the tap. Students should be given oppotunities to discuss and become comfortable with these ideas, before proceeding with the application of the chain rule which follows. They should be encouraged to be systematic in identifying all the relevant parameters and functions which apply to such a system, and how these relate to each other.



Step 4: Through application of the chain rule and idenitfying the relationship between volume of a cylinder and height of the container, students should be able to build the relationship between time and height for this system, leading to a formula for height with respect to time.

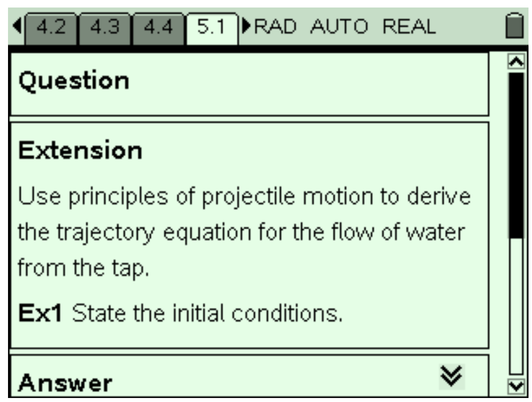


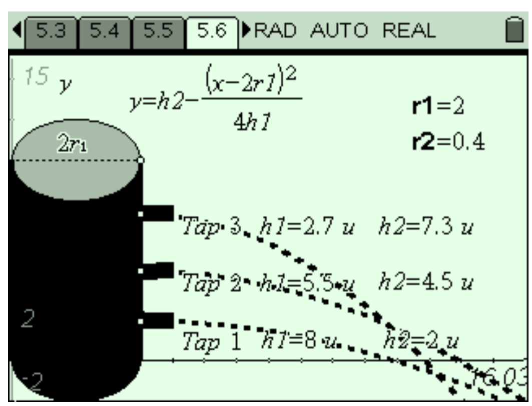
Step 5: It is critical that students are able to apply what they have learned to the real-world situation it describes, and the graphical representation of the height/time formula is an important contributor to this understanding. Students must be encouraged to clearly describe in their own words how the various components interact in this system, particularly the rates of change that are involved.



Extension

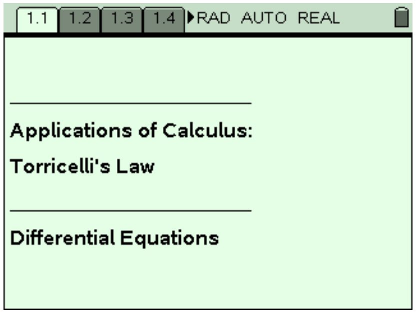
The extension activities involve derivation of the trajectory path for the escaping water using the principles of projectile motion – once again, students are led, step by step, through the process in order to support them in developing such solutions in the future. They are then scaffolded through the opportunity to check their solution using a prepared program, **projectile(initial_x, initial_y, initial_velocity, initial_angle, gravity_constant)**. This may be applied to the three different taps given in the model on page 5.6, allowing students the chance to consolidate their application of this important process.

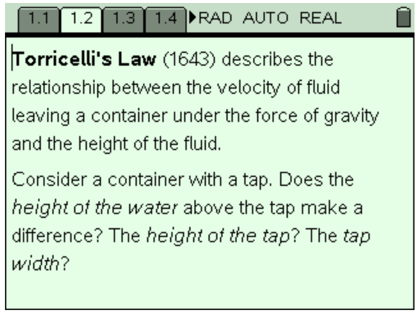


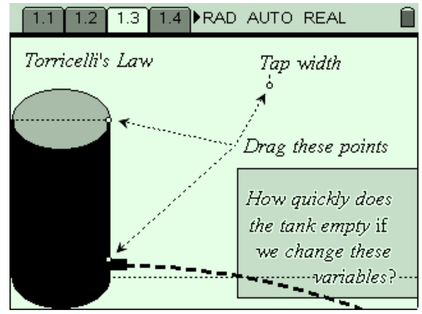


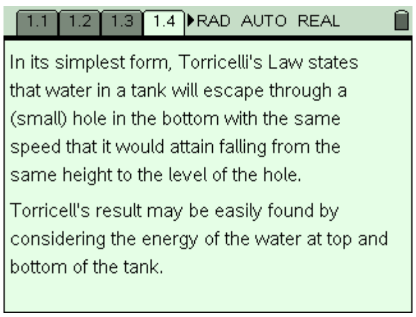
Applications of Calculus: Simple Harmonic Motion – ID: XXXX

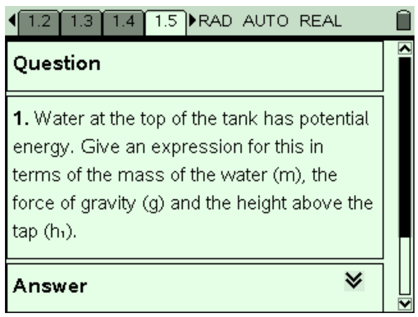
(Student)TI-Nspire File: *CalcActXX_Torricelli's_Law_EN.tns*

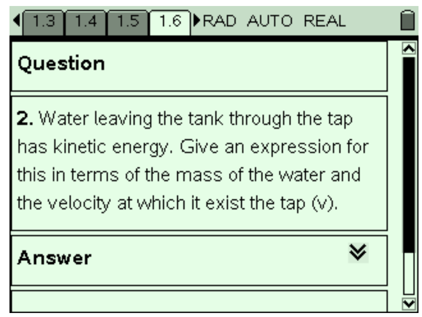


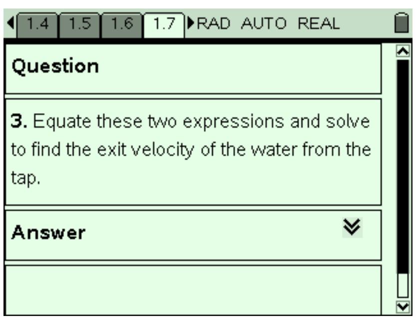


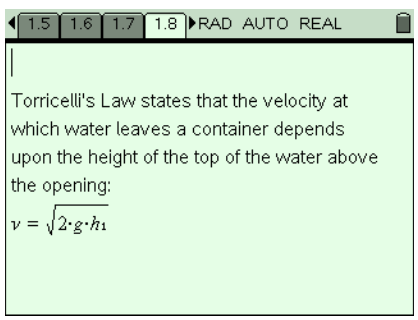


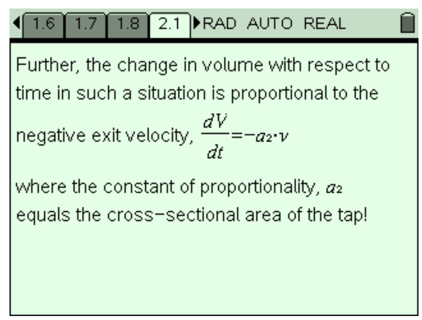


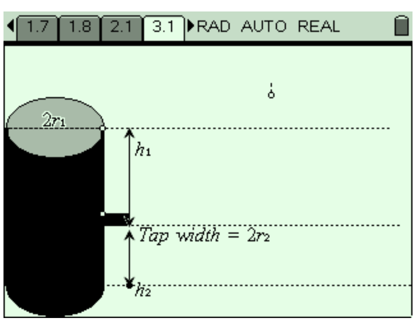


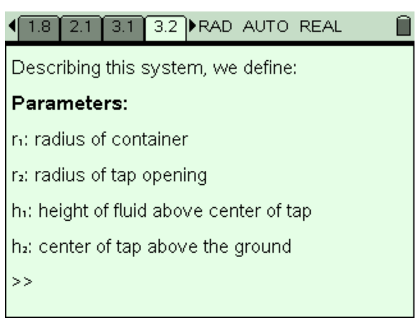


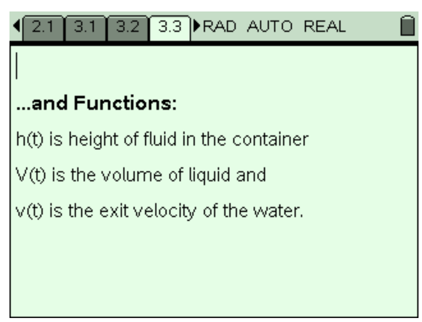












3.1 3.2 3.3 3.4 RAD AUTO REAL

Question

4. Using the chain rule, find an expression for $\frac{dV}{dt}$ in terms of $\frac{dh}{dt}$.

Answer

3.2 3.3 3.4 3.5 RAD AUTO REAL

Question

5. Simplify this expression using the relationship between volume and height of a cylinder.

Answer

3.3 3.4 3.5 3.6 RAD AUTO REAL

Question

6. Put these together to find an expression for $\frac{dh}{dt}$ in terms of v, the exit velocity.

Answer

3.4 3.5 3.6 3.7 RAD AUTO REAL

Question

7. But we have an expression for v in terms of h! Put these together.

Answer

3.5 3.6 3.7 3.8 RAD AUTO REAL

Question

8. Now invert and integrate with respect to h to derive an expression for h in terms of t.

Answer

3.6 3.7 3.8 4.1 RAD AUTO REAL

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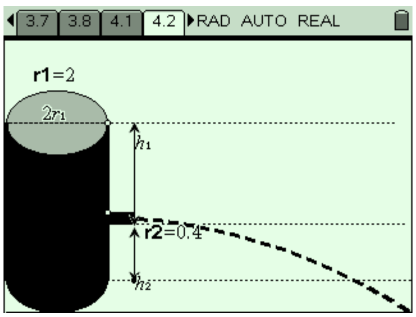
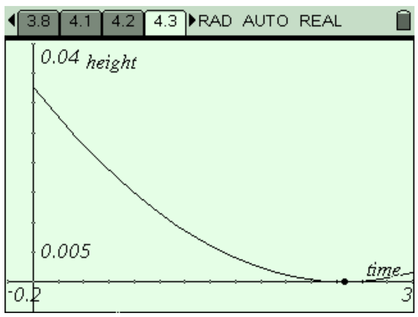




4.1 4.2 4.3 4.4 RAD AUTO REAL

Question

9. Interpret this result in terms of our physical situation.

Answer

4.2 4.3 4.4 5.1 RAD AUTO REAL

Question

Extension

Use principles of projectile motion to derive the trajectory equation for the flow of water from the tap.

Ex1 State the initial conditions.

Answer

4.3 4.4 5.1 5.2 RAD AUTO REAL

Question

Ex2 Derive the horizontal equations of motion.

Answer

4.4 5.1 5.2 5.3 RAD AUTO REAL

Question

Ex3 Derive the vertical equations of motion.

Answer

5.1 5.2 5.3 5.4 RAD AUTO REAL

Question

Ex4 Combine horizontal and vertical components to form the trajectory equation.

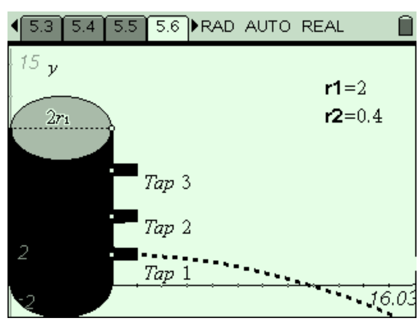
Answer

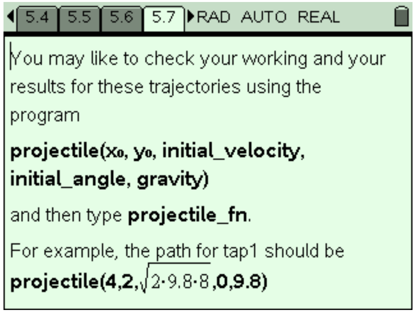
5.2 5.3 5.4 5.5 RAD AUTO REAL

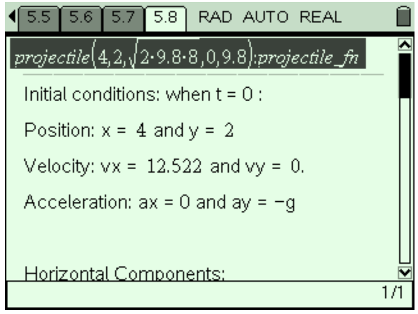
Question

Ex5 Using the model on the next page, by measurement and calculation find and plot the trajectory paths for water flowing from taps 1, 2 and 3.

Answer




5.4 5.5 5.6 5.7 RAD AUTO REAL
You may like to check your working and your results for these trajectories using the program
projectile(x0, y0, initial_velocity, initial_angle, gravity)
and then type projectile_fn.
For example, the path for tap1 should be
projectile(4,2,√2·9.8·8,0,9.8)


5.5 5.6 5.7 5.8 RAD AUTO REAL
projectile(4,2,√2·9.8·8,0,9.8):projectile_fn
Initial conditions: when t = 0 :
Position: x = 4 and y = 2
Velocity: vx = 12.522 and vy = 0.
Acceleration: ax = 0 and ay = −g
Horizontal Components:
1/1