# Introducing the Integral Calculus: Integration by Substitution

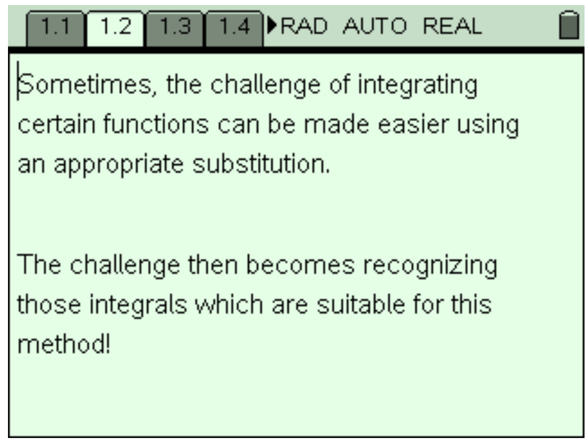
**ID: XXXX**

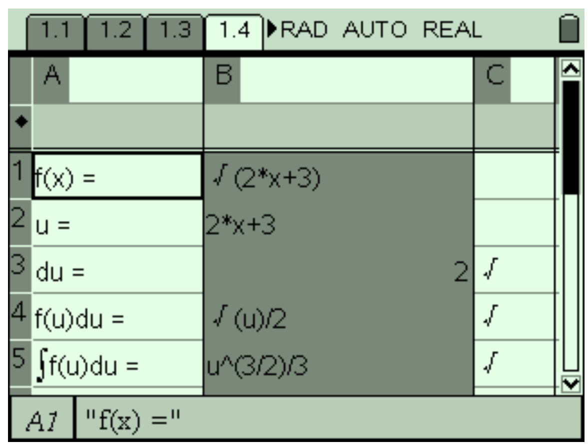
**Name** ______________________

**Class** ______________________

*In this activity, we explore ways to integrate harder functions using substitution methods which allow these to be transformed into one of the standard forms.*

Open the file *CalcActXX_Integration_By_Substitution_EN.tns* on your handheld and follow along with your teacher to work through the activity. Use this document as a reference and to record your answers.




## EXERCISES

1. Consider, for example, $\int \sqrt{2x+3}\,dx$. Suppose we let ***u* = *2x+3***. Can you evaluate the integral? Check using the spreadsheet provided.

2. Now try $\int \sin(x)\cdot\cos(x)\,dx$ [Let ***u* = *sin(x)***].

3. Now try that same integral using ***u* = *cos(x)***. What do you observe?

4. In fact, we could even approach $\int \sin(x)\cdot\cos(x)\,dx$ another way, since $\sin(x)*\cos(x) = \frac{1}{2}\sin(2x)$.

5. Now try $\int \frac{x+1}{x^2+2x+3}\,dx$

6. $\int \sin(x)*e^{\cos(x)}\,dx$

7. $\int \frac{x}{4x^2+1}\,dx$

8. Can you see what all these integrals have in common that makes them suitable for this method?

9. $\int \tan(x)\,dx$

10. $\int \cos^3(x)$

## SUGGESTED SOLUTIONS

1.3 1.4 2.1 2.2 RAD AUTO REAL

**Answer**

1. If u = 2x+3 then du = 2dx >>

$$\int \sqrt{2x+3}\,dx = \frac{1}{2}\int u^{\frac{1}{2}}\,du = \frac{1}{3}u^{\frac{3}{2}}+C$$

$$\int \sqrt{2x+3}\,dx = \frac{1}{3}(2x+3)\cdot\sqrt{2x+3} + C$$

1.3 1.4 2.1 2.2 RAD AUTO REAL

[Let $u = \sin(x)$]

**Answer**

2. If $u = \sin(x)$ then **du** $= \cos(x)\cdot dx$ >>

$$\int \sin(x)\cdot\cos(x)dx = \int u\,du = \frac{1}{2}u^2 + C_1$$

$$\int \sin(x)\cdot\cos(x)dx = \frac{1}{2}\sin(x)^2 + C_1$$

1.4 2.1 2.2 2.3 RAD AUTO REAL

3. If $u = \cos(x)$ then **du** $= -\sin(x)\cdot dx$

$$\int \sin(x)\cdot\cos dx = \int -u\,du = \frac{-1}{2}u^2+C_2$$

$$\int \sin(x)\cdot\cos dx = \frac{1}{2}\cos(x)^2+C_2$$

Now $\frac{1}{2}\cos(x)^2 \neq \frac{1}{2}\sin(x)^2$ but they ARE only a constant apart!

2.1 2.2 2.3 2.4 RAD AUTO REAL

4. Let $u = 2x$, then **du** $= 2dx$

$$\frac{1}{2}\int \sin(u)\cdot\frac{1}{2}du = \frac{1}{4}\cdot -\cos(u) + C_3$$

$$\int \sin(x)\cdot\cos(x)dx = \frac{-1}{4}\cos(2x) + C_3$$

which is yet another form which is only a constant value away!

2.2 2.3 2.4 3.1 RAD AUTO REAL

**Answer**

5. Let $u = x^2+2x+3$, then **du** $= (2x+2)\cdot dx$

$$\int \frac{x+1}{x^2+2x+3}dx = \frac{1}{2}\int \frac{1}{u}du = \frac{1}{2}\ln(u)+C$$

$$\int \frac{x+1}{x^2+2x+3}dx = \frac{1}{2}\ln(x^2+2x+3) + C$$

2.4 3.1 3.2 4.1 RAD AUTO REAL

**6.** $\int \sin(x)\cdot e^{\cos(x)}dx$

**Answer**

6. Let $u = \cos(x)$, then **du** $= -\sin(x)\cdot dx$

$$\int \sin(x)\cdot e^{\cos(x)}dx = \int -e^{u}du = -e^{u}+C$$

$$\int \sin(x)\cdot e^{\cos(x)}dx = -e^{\cos(x)} + C$$

| | |
|---|---|
| 3.2 4.1 4.2 5.1 RAD AUTO REAL<br>**Answer**<br>7. Let $u = 4x^2+1$, then **du** $= 8x \cdot dx$<br>$\int \frac{x}{4x^2+1}dx = \frac{1}{8}\int \frac{1}{u}du = \frac{1}{8}\ln(u)+C$<br>$\int \frac{x}{4x^2+1}dx = \frac{1}{8}\ln(4x^2+1) + C$ | 4.2 5.1 5.2 5.3 RAD AUTO REAL<br>for this method?<br>**Answer**<br>Each contains the derivative of the substitution element: if "u" is the substitution, then du/dx exists as part of the expression, or at least a constant multiple of it. |
| 5.1 5.2 5.3 6.1 RAD AUTO REAL<br>**Answer**<br>9. $\int \tan(x)dx = \int \frac{\sin(x)}{\cos(x)}dx$<br>Let $u = \cos(x)$ then **du** $= \sin(x)\cdot dx$<br>$\int \frac{\sin(x)}{\cos(x)}dx = \int \frac{1}{u}du = \ln(u) + C$<br>$\int \tan(x)dx = \ln(\cos(x)) + C$ | 5.2 5.3 6.1 7.1 RAD AUTO REAL<br>10. $\int \cos(x)^3 dx = \int \cos(x)\cdot\cos(x)^2 dx =$<br>$\int (1-\sin(x)^2)\cdot\cos(x)dx$<br>Let $u = \sin(x)$ then **du** $= \cos(x)\cdot dx$<br>$\int \cos(x)^3 dx = \int 1-u^2 du = u-\frac{u^3}{3} + C$<br>$\int \cos(x)^3 dx = \frac{3\sin(x) - \sin(x)^3}{3} + C$ |