Introducing the Integral Calculus: Integration by Substitution

ID: XXXX

Time required
45 minutes

Activity Overview

In this activity, we explore methods for computing integrals of functions not in one of the standard forms The focus here is upon the use of substitution to transform the given integral into a standard form. The approach taken is largely symbolic and makes full use of the computer algebra facilities of TI-Nspire CAS. Prepared algebraic spreadsheets are utilized for skill development and consolidation.

Concepts

- *Integration of standard forms, substitution methods for integration.*

Teacher Preparation

This investigation offers opportunities for review and consolidation of key concepts related to methods of substitution and integration of composite functions. Opportunities are provided for skill development and practice of the method of taking integrals of suitable functions. As such, care should be taken to provide ample time for ALL students to engage actively with the requirements of the task, allowing some who may have missed aspects of earlier work the opportunity to build new and deeper understanding.

- *This activity can serve to consolidate earlier work on integration. It offers a suitable introduction to integration by substitution.*
- *Begin by reviewing the method of differentiation of composite functions (the "chain rule"), and methods of integration of the standard function forms.*
- *The screenshots on pages X–X (top) demonstrate expected student results. Refer to the screenshots on page X (bottom) for a preview of the student .tns file.*
- ***To download the .tns file, go to http://education.ti.com/exchange and enter "XXXX" in the search box.***

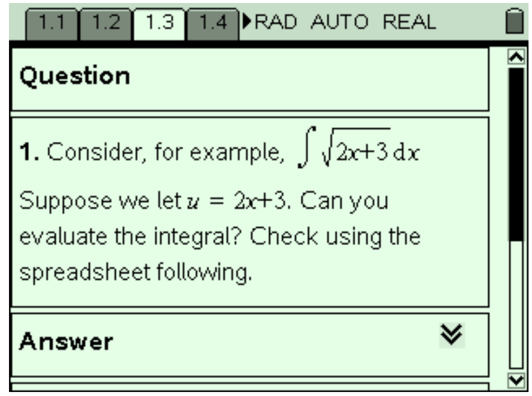
Classroom Management

- *This activity is intended to be **teacher led.** You should seat your students in pairs so they can work cooperatively on their handhelds. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds, although the majority of the ideas and concepts are only presented in **this** document; be sure to cover all the material necessary for students' total comprehension.*
- *Students can either record their answers on the handheld or you may wish to have the class record their answers on a separate sheet of paper.*

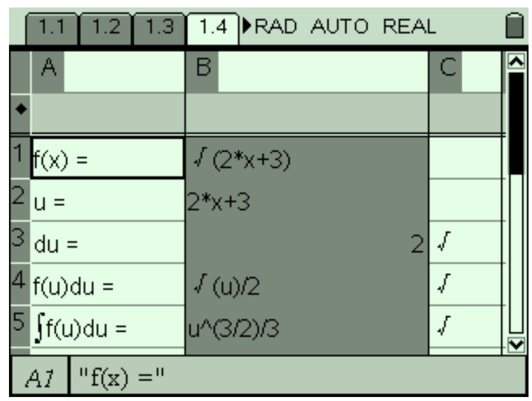
TI-Nspire™ Applications

Calculator, Notes, Lists & Spreadsheet.

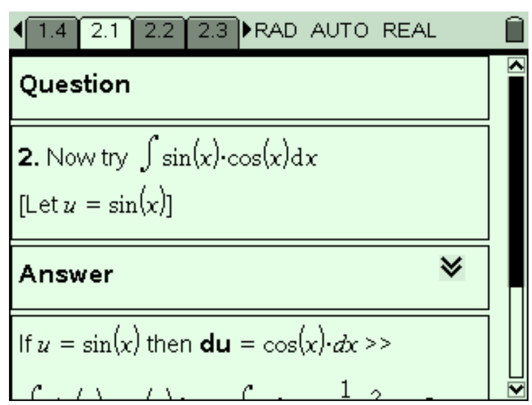
Step 1: Begin with discussion and review of both the chain rule for differentiation of composite functions and of the integrals of standard function forms. Ensure that students are comfortable with these and then challenge them to consider more difficult forms – in this case, composite functions of the form *y = f(g(x))* which may be suitable for integration by substitution methods.



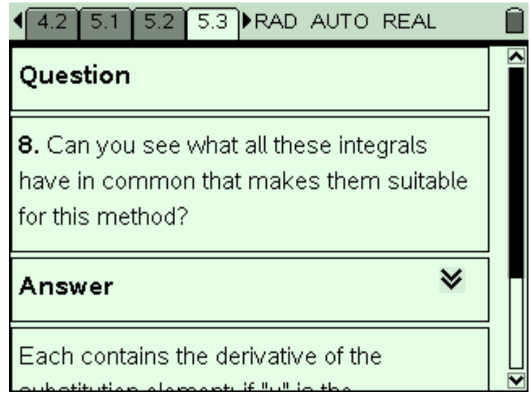
Step 2: Students are scaffolded in their application of integration by substitution through the availability of an algebraic spreadsheet, set up for this purpose. The function to be integrated is entered into B1, then the choice of substitution, ***u***, into B2. Students enter the derivative, ***du/dx*** into B3, and then substitute ***u*du*** for ***f(x)*dx.*** They can then integrate with respect to ***u***, and finally replace to give the result in terms of ***x***. Each step is checked for algebraic equivalence.



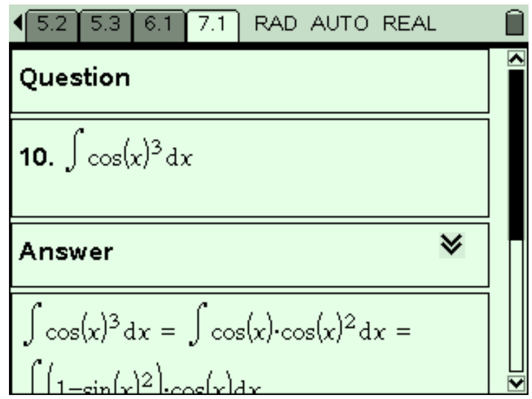
Step 3: A series of carefully chosen questions follow, with the substitution specified for the first few, and the spreadsheet available for support. The integral for ***sin(x)*cos(x)*** is developed in three different ways, and then other function types follow.



Step 4: After working through several different examples, students are challenged to identify the common feature: that each of the functions given in some way includes the derivative of the function to be substituted. This realization is critical for students to understand that this method will not work for all functions, but only for certain well-chosen forms.

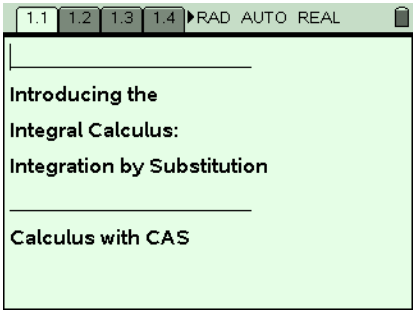


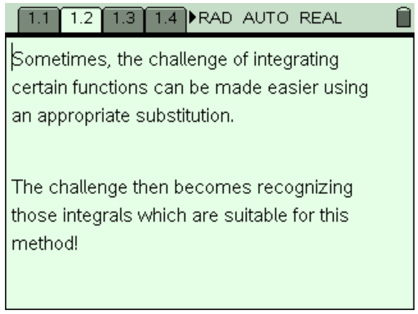
Step 5: The last two questions have been selected to offer a little more challenge and require some insight and rearrangement using their knowledge of trigonometric identities and relationships.

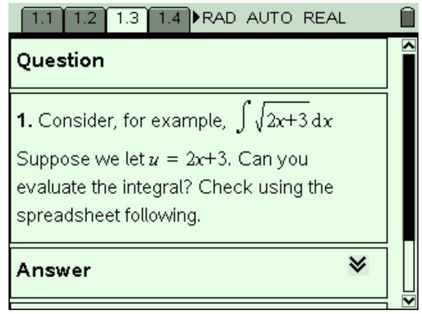


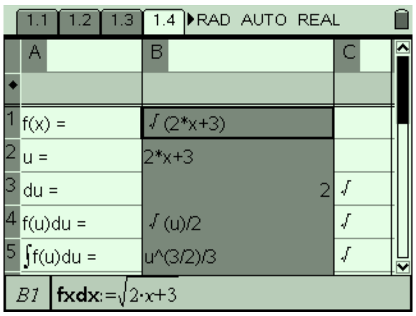
Introducing the Integral Calculus: Integration by Substitution – ID: XXXX

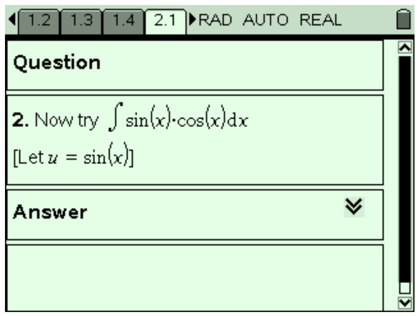
(Student)TI-Nspire File: *CalcActXX_Integration_By_Substitution_EN.tns*

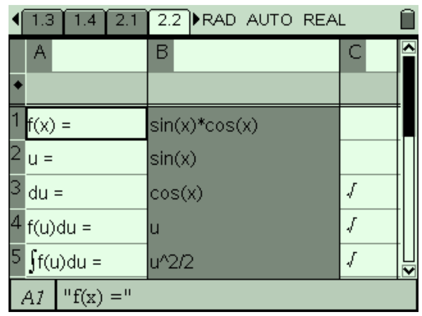


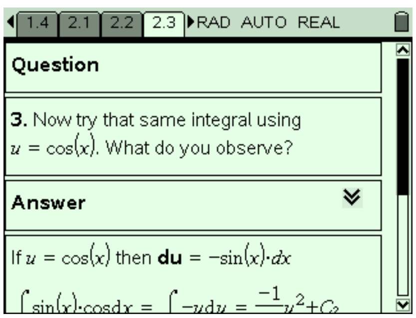


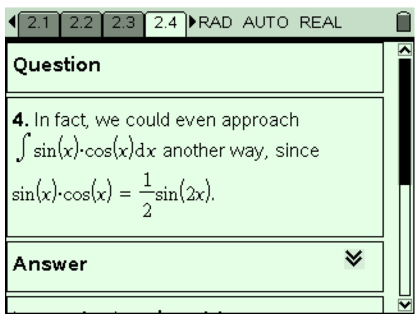


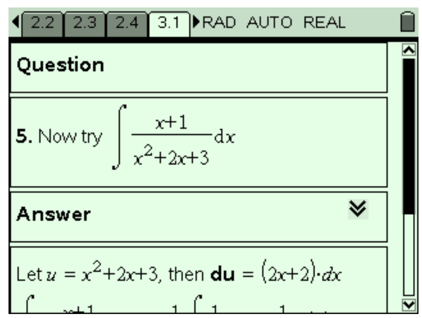


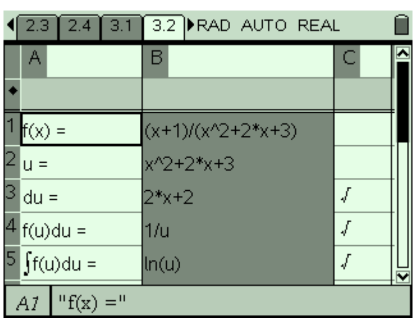


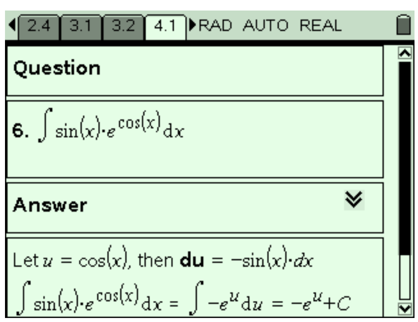


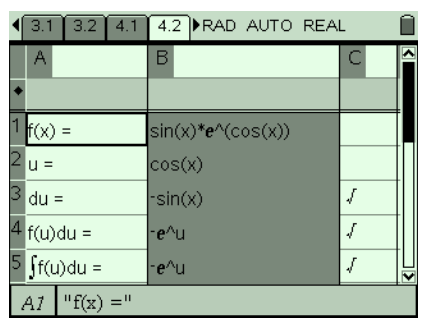


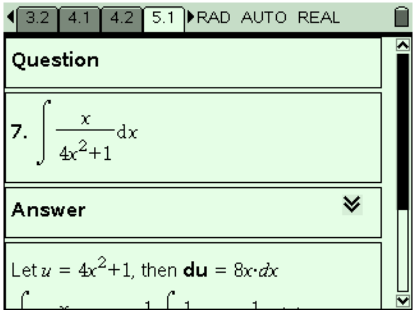


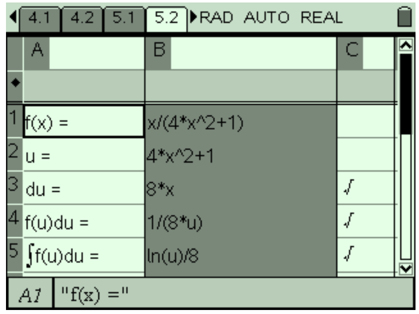


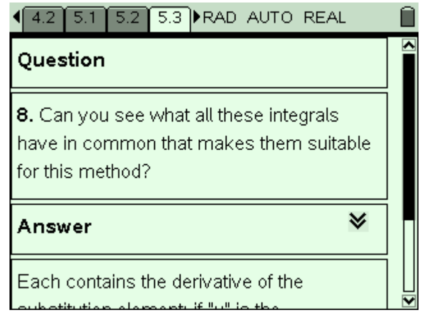


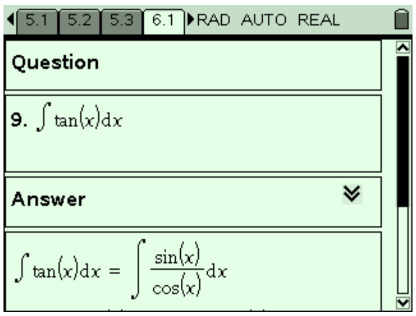


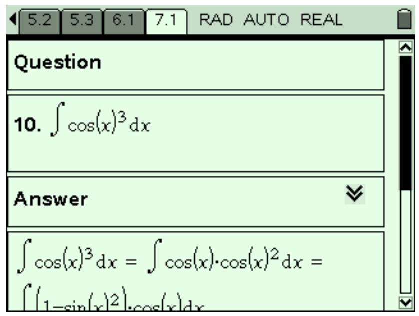



3.2 4.1 4.2 5.1 RAD AUTO REAL
Question
7. $\int \frac{x}{4x^2+1} dx$
Answer
Let $u = 4x^2+1$, then **du** = $8x \cdot dx$


4.1 4.2 5.1 5.2 RAD AUTO REAL
A
B
C
1 f(x) = x/(4*x^2+1)
2 u = 4*x^2+1
3 du = 8*x
4 f(u)du = 1/(8*u)
5 ∫f(u)du = ln(u)/8
A1 "f(x) ="


4.2 5.1 5.2 5.3 RAD AUTO REAL
Question
8. Can you see what all these integrals have in common that makes them suitable for this method?
Answer
Each contains the derivative of the


5.1 5.2 5.3 6.1 RAD AUTO REAL
Question
9. $\int \tan(x) dx$
Answer
$\int \tan(x) dx = \int \frac{\sin(x)}{\cos(x)} dx$


5.2 5.3 6.1 7.1 RAD AUTO REAL
Question
10. $\int \cos(x)^3 dx$
Answer
$\int \cos(x)^3 dx = \int \cos(x) \cdot \cos(x)^2 dx =$