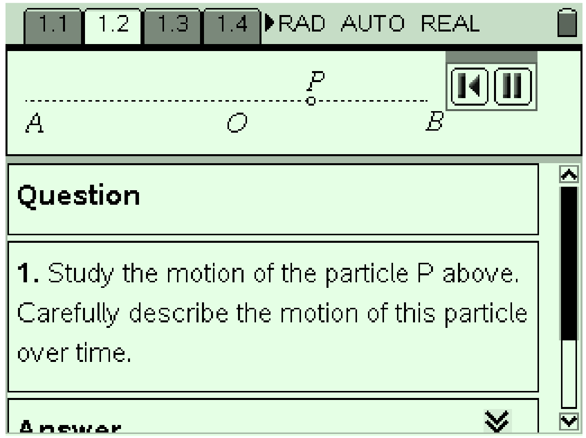
Applications of Calculus: Simple Harmonic Motion ID: XXXX

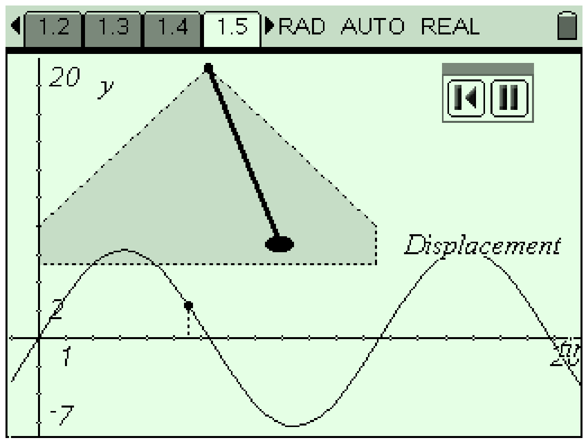
Name ____________________

Class ____________________

In this activity, we explore the application of both differentiation and integration to the real world as applied to motion on a swing.

Open the file *CalcActXX_Simple_Harmonic_Motion_EN.tns* on your handheld and follow along with your teacher to work through the activity. Use this document as a reference and to record your answers.





EXERCISES

1. Study the motion of the particle **P** shown. Carefully describe the motion of this particle over time.

2. What general function could be given to describe the position ***x*** of **P** at any time ***t*** seconds?

3. When on a swing, focus upon your speed: when are you moving fastest? When do you stop?

4. Now focus upon the force acting upon you – once you start swinging, there is only one force acting: gravity, pulling downwards. When is this action greatest? When least?

5. Motion is defined to be simple harmonic if the acceleration is directly proportional to displacement from the origin. Clearly explain what this statement means.

6. Use the **displacement** equation $x = a \cdot \sin(n \cdot t)$ to derive a formula for **velocity.**

7. From this equation, derive a formula for **acceleration** in terms of **time**.

8. Now substitute the formula for **displacement** into this **acceleration** formula.

9. In your own words, clearly explain how this formula relates to our motion on a swing.

10. Referring to the graphs of motion you have seen, carefully describe the critical points of this motion in terms of displacement, velocity and acceleration.

EXTENSIONS

- Ex1 Find other examples of simple harmonic motion and try to analyze these in the same way as we have for the child on a swing.

- Ex2 Carefully study the following, then explain and justify each statement:

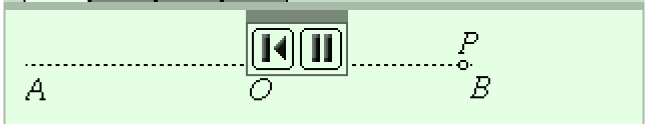
  - $a = \frac{d^2}{dt^2}(x) = \frac{dv}{dt}$
  - $a = v \cdot \frac{dv}{dx}$

- Ex3 Show that $\int v \cdot \frac{dv}{dx} dx = \frac{1}{2}v^2$ Use these results to derive the SHM formulas.

- Ex4 When might it be appropriate to express the time/displacement form using cosine instead of sine?

SUGGESTED SOLUTIONS

1.2 1.3 1.4 1.5 RAD AUTO REAL



1. The motion is cyclic – initially negative (moving towards the origin, O) then positive. Velocity reaches a maximum value in the middle and is zero at end-points A and B.

1.2 1.3 1.4 1.5 RAD AUTO REAL

Answer

2. Let OA = OB = a (**amplitude**) and assume the particle traces out n cycles per second

$\left(period = \frac{2\pi}{time\ per\ cycle}\right)$.

Then $x = a\cdot\sin(n\cdot t)$.

1.3 1.4 1.5 1.6 RAD AUTO REAL

3. Focus first upon your speed: when are you moving fastest? When do you stop?

Answer

3. You move fastest as you pass through the centre of the swing's path, and you stop (briefly) at each end of the path – just as our point P!

1.5 1.6 1.7 1.8 RAD AUTO REAL

force acting: gravity, pulling downwards. When is this action greatest? When least?

Answer

4. Gravity will be greatest when you are furthest from the ground – at each end of the path. It will be least when you are at your lowest point, in the middle.

1.9 1.10 1.11 1.12 RAD AUTO REAL

Answer

5. If the origin of motion is taken to be the centre of the swing's path (where acceleration/force is least) then the further you move away from that position, the greater the acceleration/force acting upon you trying to return you to that position.

1.10 1.11 1.12 1.13 RAD AUTO REAL

6. Use the displacement equation to derive a formula for velocity.

Answer

6. If $x = a\cdot\sin(n\cdot t)$ then

$v = \frac{dx}{dt} = a\cdot n\cdot\cos(n\cdot t)$

1.11 1.12 1.13 **1.14** RAD AUTO REAL

7. From this equation, derive a formula for acceleration in relation to time.

Answer

7. $a=\frac{d^2}{dt^2}(x) = \frac{d}{dt}(a\cdot n\cdot\cos(n\cdot t)) ==$
$-a\cdot n^2\cdot\sin(n\cdot t)$

1.12 1.13 1.14 **1.15** RAD AUTO REAL

displacement into this acceleration formula.

Answer

8. Given $x = a\cdot\sin(n\cdot t)$ and
$a = -a\cdot n^2\cdot\sin(n\cdot t)$ then
$a = -n^2\cdot a\cdot\sin(n\cdot t)$
$a = -n^2\cdot x$

1.13 1.14 1.15 **1.16** RAD AUTO REAL

Answer

9. The force/acceleration acting upon a child on a swing acts in a negative direction to the motion, proportional to the square of the period and the displacement from the origin – in other words, the further you are from the "rest position" the more force there is from gravity to return you there!

1.14 1.15 1.16 **1.17** RAD AUTO REAL

Answer

10. At point O, displacement and acceleration are zero, while velocity has a maximum value;

At the end points, velocity is zero, while displacement and acceleration attain their maximum values.

1.16 1.17 2.1 **2.2** RAD AUTO REAL

Answer

Ex1. Most musical instruments (eg guitar string when plucked, reed in a wind instrument – hence the name simple *harmonic* motion);

The tides and even a cork or object moving up and down with the tides.

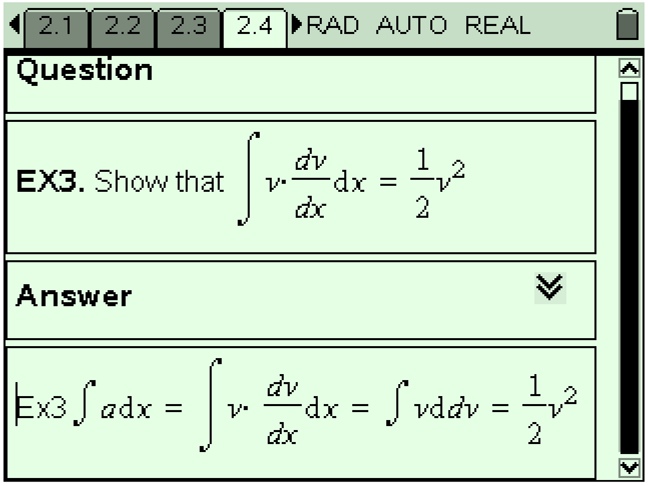
1.17 2.1 2.2 **2.3** RAD AUTO REAL

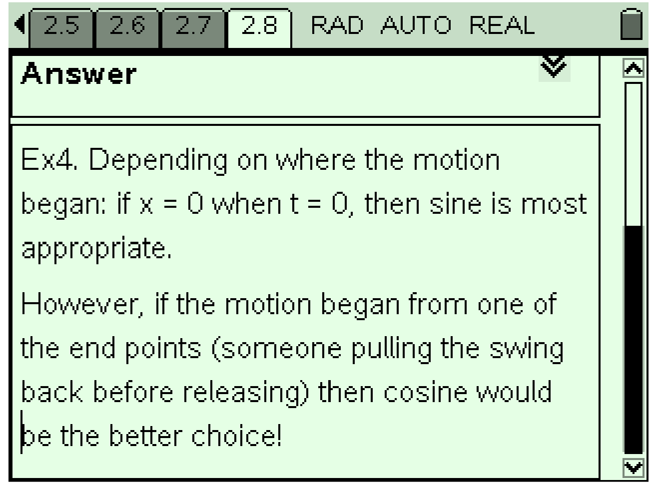
$a = \frac{\quad}{dt^2}(x) = \frac{\quad}{dt}$ and $a = v\cdot\frac{\quad}{dx}$

Answer

Ex2. $a = \frac{d^2}{dt^2}(x) = \frac{d}{dt}\frac{dx}{dt} = \frac{dv}{dt}$

$a = \frac{dv}{dx}\cdot\frac{dx}{dt} = v\cdot\frac{dv}{dx}$


2.1 2.2 2.3 2.4 RAD AUTO REAL
Question
EX3. Show that $\int v \cdot \frac{dv}{dx} dx = \frac{1}{2}v^2$
Answer
Ex3 $\int a\,dx = \int v \cdot \frac{dv}{dx} dx = \int v\,dv = \frac{1}{2}v^2$


2.5 2.6 2.7 2.8 RAD AUTO REAL
Answer
Ex4. Depending on where the motion began: if x = 0 when t = 0, then sine is most appropriate.
However, if the motion began from one of the end points (someone pulling the swing back before releasing) then cosine would be the better choice!