Applications of Calculus: Simple Harmonic Motion

ID: XXXX

Time required
45 minutes

Activity Overview

In this activity, students are introduced to the important topic of simple harmonic motion in terms of the motion of a child on a swing. Using multiple representations to support understanding, students derive the defining formulas – first, beginning with the trigonometric relationship between time and displacement, and differentiating up to the form for acceleration, then by integration from acceleration back to displacement. This activity provides an introduction to differential equations.

Concepts

- *Differentiation and Integration of standard forms, substitution methods for integration.*

Teacher Preparation

This investigation offers opportunities for review and consolidation of key concepts related to differentiation and integration of trigonometric functions. It provides a firm link between the theory and applications of the Calculus. As such, care should be taken to provide ample time for ALL students to engage actively with the requirements of the task, allowing some who may have missed aspects of earlier work the opportunity to build new and deeper understanding.

- *This activity can serve to consolidate earlier work on differentiation and integration. It offers a suitable introduction to differential equations.*
- *Begin by reviewing the method of differentiation of trigonometric functions, and methods of integration of the standard function forms.*
- *The screenshots on pages X–X (top) demonstrate expected student results. Refer to the screenshots on page X (bottom) for a preview of the student .tns file.*
- ***To download the .tns file, go to http://education.ti.com/exchange and enter "XXXX" in the search box.***

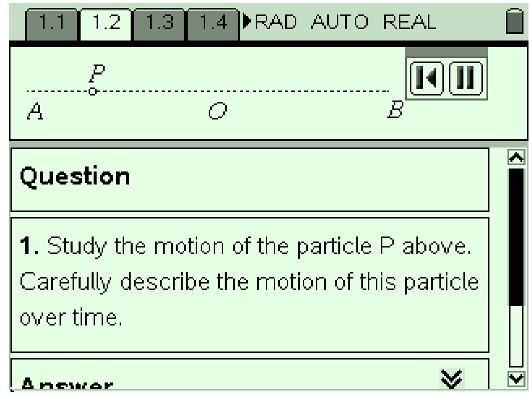
Classroom Management

- *This activity is intended to be **teacher led.** You should seat your students in pairs so they can work cooperatively on their handhelds. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds, although the majority of the ideas and concepts are only presented in **this** document; be sure to cover all the material necessary for students' total comprehension.*
- *Students can either record their answers on the handheld or you may wish to have the class record their answers on a separate sheet of paper.*

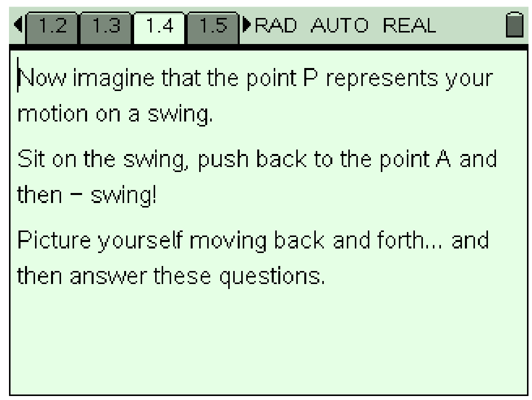
TI-Nspire™ Applications

Graphs & Geometry, Calculator, Notes.

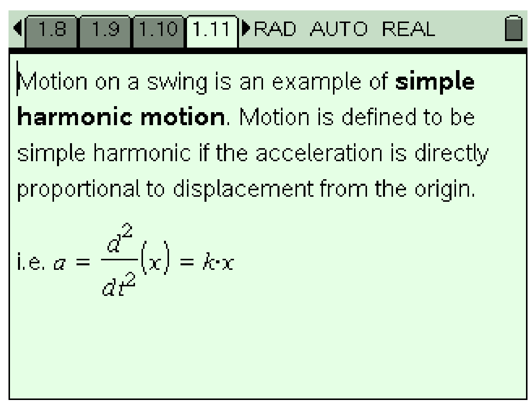
Step 1: Begin with discussion and review of both the differentiation and integration of standard forms and of the trigonometric functions in particular. Ensure that students are comfortable with these and then challenge them to apply what they know about these processes to real-world situations, with a particular focus upon rates of change.



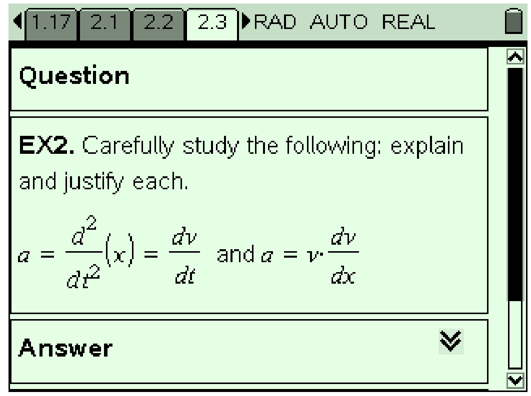
Step 2: Motion on a swing should be something that all students can relate to. It offers a suitable context for a closer examination of rates of change of displacement leading to velocity, and rate of change of velocity leading to acceleration.



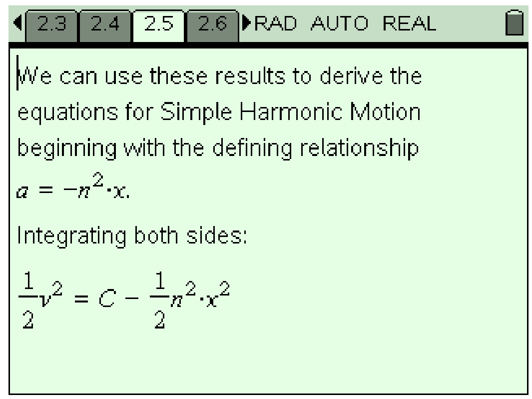
Step 3: Particular care should be taken to build understanding of this concept of acceleration – while students will relate readily to displacement and velocity, acceleration is best introduced in terms of force (being careful to distinguish between them!) Once established, this forms the basis for deriving the forms for simple harmonic motion, beginning with the standard relationship between time and displacement, leading to velocity and finally to acceleration. Substituting back leads readily to the defining equation in terms of acceleration and displacement.

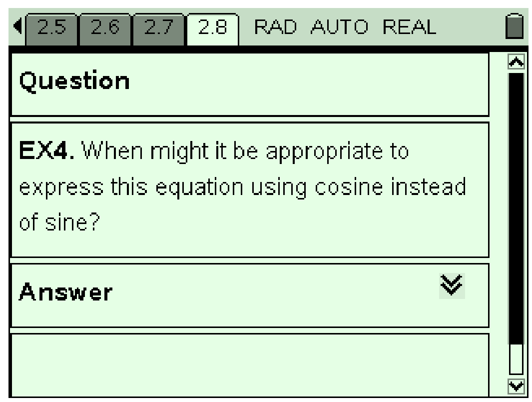


Step 4: The second part of this activity offers a series of extensions, beginning with the challenge for students to describe other examples of simple harmonic motion. They should then be introduced to the variety of forms for acceleration, and attempt to justify these from their knowledge of rates of change.



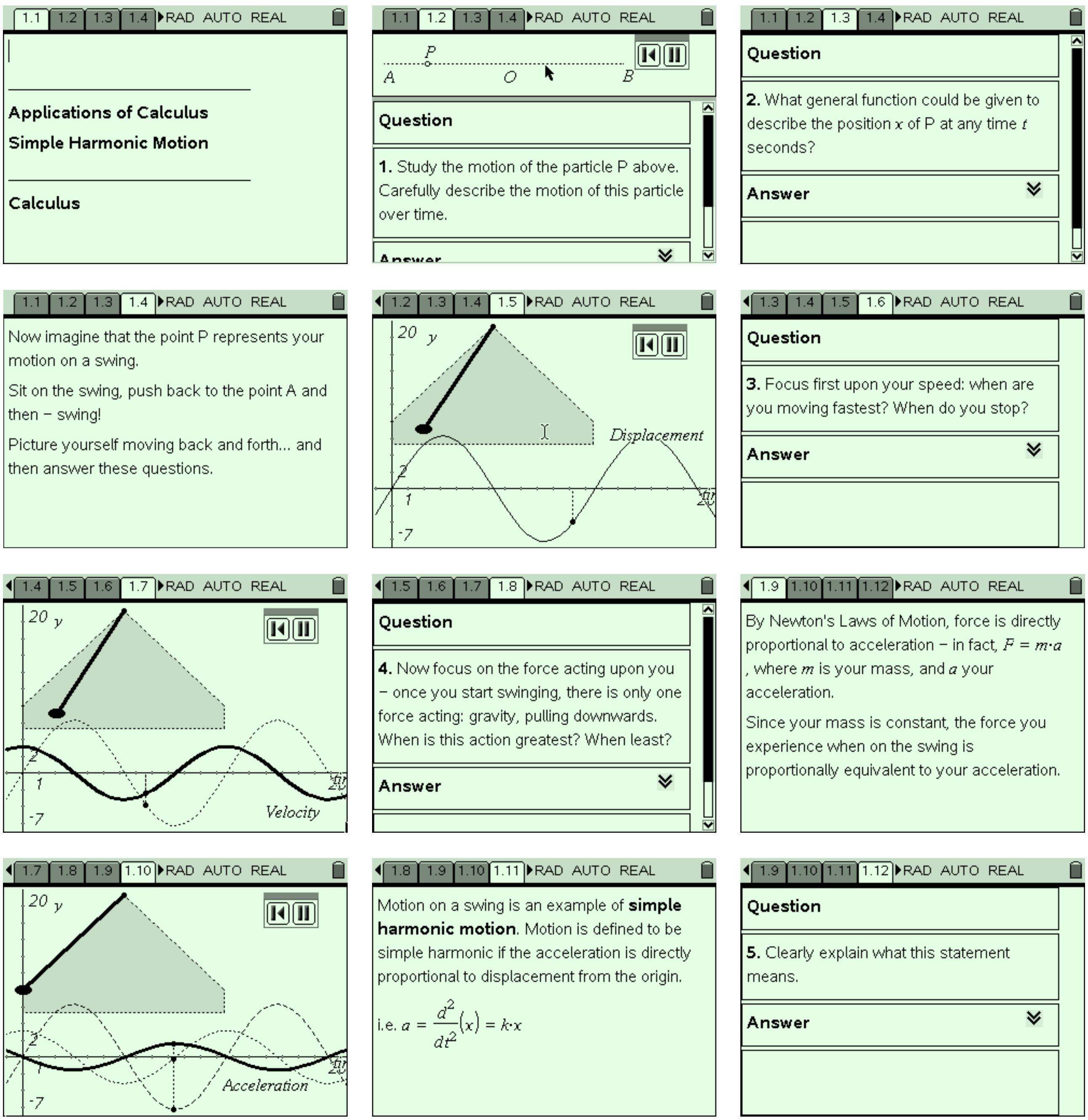
Step 5: What follows supports students in deriving the simple harmonic forms by integrating from the acceleration equation. This involves some substitution of critical values drawn from the physical example of the child on the swing and, finally, integration of inverse trigonometric function forms. When deriving this form, students should be encouraged to discuss the use of the sine or the cosine form – each appropriate depending upon the physical conditions. This should lead to consideration of the **phase shift** as related to the starting point of the motion.





Applications of Calculus: Simple Harmonic Motion – ID: XXXX

(Student)TI-Nspire File: *CalcActXX_Simple_Harmonic_Motion_EN.tns*



1.10 1.11 1.12 1.13 RAD AUTO REAL

Question

6. Use the displacement equation to derive a formula for velocity.

Answer

1.11 1.12 1.13 1.14 RAD AUTO REAL

Question

7. From this equation, derive a formula for acceleration in relation to time.

Answer

1.12 1.13 1.14 1.15 RAD AUTO REAL

Question

8. Now substitute the formula for displacement into this acceleration formula.

Answer

1.13 1.14 1.15 1.16 RAD AUTO REAL

Question

9. In your own words, clearly describe how this formula relates to our motion on a swing.

Answer

1.14 1.15 1.16 1.17 RAD AUTO REAL

Question

10. Referring to the graphs of motion you have just seen, carefully describe the critical points of this motion in terms of displacement, velocity and acceleration.

Answer

1.15 1.16 1.17 2.1 RAD AUTO REAL

Simple Harmonic Motion

Extension Activities

1.16 1.17 2.1 2.2 RAD AUTO REAL

Question

EX1. Find other examples of simple harmonic motion, and try to analyze these in the same way as we have for the child on the swing.

Answer

1.17 2.1 2.2 2.3 RAD AUTO REAL

Question

EX2. Carefully study the following: explain and justify each.

$a = \frac{d^2}{dt^2}(x) = \frac{dv}{dt}$ and $a = v \cdot \frac{dv}{dx}$

Answer

2.1 2.2 2.3 2.4 RAD AUTO REAL

Question

EX3. Show that $\int v \cdot \frac{dv}{dx} dx = \frac{1}{2} v^2$

Answer

2.2 2.3 2.4 2.5 RAD AUTO REAL

We can use these results to derive the equations for Simple Harmonic Motion beginning with the defining relationship $a = -n^2 \cdot x$.

Integrating both sides:

$\frac{1}{2} v^2 = C - \frac{1}{2} n^2 \cdot x^2$

2.3 2.4 2.5 2.6 RAD AUTO REAL

When $x = a$, $v = 0$ => $C = \frac{1}{2} n^2 \cdot a^2$

=> $v^2 = n^2 \cdot (a^2 - x^2)$

=> $v = |n| \cdot \sqrt{a^2 - x^2} = \frac{dx}{dt}$

=> $\frac{dt}{dx} = \frac{1}{|n| \cdot \sqrt{a^2 - x^2}}$

2.4 2.5 2.6 2.7 RAD AUTO REAL

Integrate both sides wrt x:

$t = \frac{1}{|n|} \sin^{-1}\left(\frac{x}{a}\right) + C$

When t = 0, x = 0 => C = 0

$t = \frac{1}{|n|} \sin^{-1}\left(\frac{x}{a}\right)$

=> $x = a \cdot \sin(n \cdot t)$ as required.

2.5 2.6 2.7 2.8 RAD AUTO REAL

Question

EX4. When might it be appropriate to express this equation using cosine instead of sine?

Answer