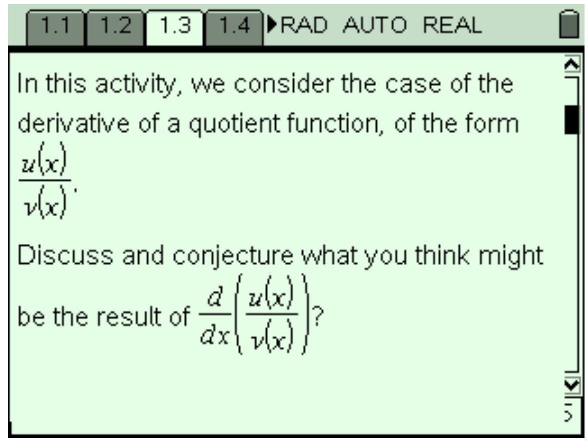
Introducing the Differential Calculus: The Quotient Rule

ID: XXXX

Name ______________________

Class ______________________

In this activity, we explore ways to differentiate harder functions. The focus here is on functions which can be expressed as a quotient of two simpler functions.



Open the file *CalcActXX_Quotient_Rule_EN.tns* on your handheld and follow along with your teacher to work through the activity. Use this document as a reference and to record your answers.

EXERCISES

1. Consider the example $\frac{\sin(x)}{x^2}$. How might this be expressed as a **product**?

2. Use the **product rule** to evaluate the derivative of this function.

3. Use the product rule to establish a quotient rule for $\frac{\partial}{\partial x}(\frac{u(x)}{v(x)})$

4. Try the following examples and then check your method and results using the supplied spreadsheet and the program **diff_quotient**:

   a. $\frac{\partial}{\partial x}(\frac{\sin(x)}{\cos(x)})$ b. $\frac{\partial}{\partial x}(\frac{x-1}{x+1})$ c. $\frac{\partial}{\partial x}(\frac{\ln(x)}{x})$

   d. $\frac{\partial}{\partial x}(\frac{e^x}{\cos(x)})$ e. $\frac{\partial}{\partial x}(\frac{1+x^2}{\sqrt{1-x^2}})$

SUGGESTED SOLUTIONS

1. $\frac{\sin(x)}{x^2} = \sin(x) * x^{-2}$

2. $\frac{\partial}{\partial x}(\frac{\sin(x)}{x^2}) = \frac{\partial}{\partial x}(\sin(x) * x^{-2}) = \sin(x) * \frac{\partial}{\partial x}(x^{-2}) + x^{-2} * \frac{\partial}{\partial x}(\sin(x)$

$\frac{\partial}{\partial x}(\frac{\sin(x)}{x^2}) = \frac{\cos(x)}{x^2} - \frac{2\sin(x)}{x^3}$

3. $\frac{\partial}{\partial x}(\frac{u(x)}{v(x)}) = \frac{\partial}{\partial x}(u(x) * [v(x)^{-1}]) = u(x) * \frac{\partial}{\partial x} v(x)^{-1} + v(x) * \frac{\partial}{\partial x}(u(x)$

$\frac{\partial}{\partial x}(\frac{u(x)}{v(x)}) = u(x) * \frac{-1}{v^2} * \frac{\partial v}{\partial x} + v(x) * \frac{\partial u}{\partial x} = \frac{v(x) * \frac{\partial u}{\partial x} - u(x) * \frac{\partial v}{\partial x}}{v^2}$

[NOTE: differentiation by substitution required here.]

4.

a. $\frac{\partial}{\partial x}(\frac{\sin(x)}{\cos(x)}) = \frac{\cos(x) * \cos(x) - \sin(x) * [-\sin(x)]}{\cos^2(x)} = \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \sec^2(x)$

b. $\frac{\partial}{\partial x}(\frac{x-1}{x+1}) = \frac{(x+1)*1 - (x-1)*1}{(x+1)^2} = \frac{2}{(x+1)^2}$

c. $\frac{\partial}{\partial x}(\frac{\ln(x)}{x}) = \frac{x * \frac{1}{x} - \ln(x) * 1}{x^2} = \frac{1 - \ln(x)}{x^2}$

d. $\frac{\partial}{\partial x}(\frac{e^x}{\cos(x)}) = \frac{\cos(x) * e^x - e^x * [-\sin(x)]}{\cos^2(x)} = \frac{e^x * [\sin(x) + \cos(x)]}{\cos^2(x)}$

e. $\frac{\partial}{\partial x}(\frac{1+x^2}{\sqrt{1-x^2}}) = \frac{\sqrt{1-x^2} * 2x - (1+x^2) * \frac{1}{2\sqrt{1-x^2}}}{1-x^2} = \frac{x * (x^2 - 3)}{(x^2 - 1) * \sqrt{1-x^2}}$

