

Introducing the Differential Calculus

The Quotient Rule

Time required

45 minutes

ID: XXXX

Activity Overview

In this activity, students explore ways to differentiate harder functions. The focus here is on functions which can be expressed as a quotient of two simpler functions. The approach taken here is largely symbolic and makes full use of the computer algebra facilities of TI-Nspire CAS. Prepared programs and an algebraic spreadsheet are also utilized for skill development and consolidation.

Concepts

- Quotient rule for differentiation, product rule for differentiation.
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Teacher Preparation

This investigation offers opportunities for review and consolidation of key concepts related to the methods of differentiation of simple functions and the product rule, both covered in preceding activities. Opportunities are then provided for skill development and practise of the method of taking derivatives of quotients. As such, care should be taken to provide ample time for ALL students to engage actively with the requirements of the task, allowing some who may have missed aspects of earlier work the opportunity to build new and deeper understanding.

- This activity can serve to consolidate earlier work on the product rule. It offers a suitable introduction to derivatives of more difficult functions.
- Begin by reviewing the method of differentiation of by product rule, both graphically and symbolically, and methods of differentiation of the standard function forms.
- The screenshots on pages X–X (top) demonstrate expected student results. Refer to the screenshots on page X (bottom) for a preview of the student .tns file.
- **To download the .tns file, go to <http://education.ti.com/exchange> and enter "XXXX" in the search box.**

Classroom Management

- This activity is intended to be **teacher led**. You should seat your students in pairs so they can work cooperatively on their handhelds. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds, although the majority of the ideas and concepts are only presented in **this** document; be sure to cover all the material necessary for students' total comprehension.
- Students can either record their answers on the handheld or you may wish to have the class record their answers on a separate sheet of paper.

TI-Nspire™ Applications

Calculator Lists & Spreadsheet, Notes and Programming.

Step 1: Begin with discussion and review of both the product rule and of the derivatives of standard function forms. Ensure that students are comfortable with these and then challenge them to consider the more difficult form of quotient functions.

1.1 1.2 1.3 1.4 ▸RAD AUTO REAL

In this activity, we consider the case of the derivative of a quotient function, $\frac{u(x)}{v(x)}$.

Discuss and conjecture what you think might be the result of $\frac{d}{dx}\left(\frac{u(x)}{v(x)}\right)$?

Step 2: The approach taken here is to treat the quotient function as a special case of the product, and so to use preceding work on the product rule to establish this new differentiation method.

1.1 1.2 1.3 1.4 ▸RAD AUTO REAL

Question

1. Consider the example $\frac{\sin(x)}{x^2}$. How might this be rewritten as a product?

Answer ▾

Step 3: Once again, an algebraic spreadsheet and prepared programs are provided to scaffold students in their investigation and consolidation of the required skills: the **diff_product(fn1, fn2)** program gives a step-by-step working of the product rule for **fn1*fn2**, while we lead into using **diff_quotient(fn1, fn2)** to verify and then support our new method.

1.4 1.5 1.6 1.7 ▸RAD AUTO REAL

x^2

$$d(u*v) = (\sin(x)) * \left(\frac{-2}{x^3}\right) + \left(\frac{1}{x^2}\right) * (\cos(x))$$

Try to evaluate this then check your answer by typing <result>.

Done

result

1/99

Step 4: Rather than going back to first principles again to establish this result (and this approach is possible, if quite difficult), students are introduced to the fundamental method of proof which involves the use of previously established results. In this case, having proved the product rule, we are free to use it to establish (and prove) a new rule for quotients.

1.5 1.6 1.7 1.8 RAD AUTO REAL

Since we have previously proven the Product Rule result (using First Principles with a "twist!") we may use this result to establish the Quotient Rule by writing a quotient function as a product.

$$\frac{d}{dx} \left(\frac{u(x)}{v(x)} \right) = \frac{d}{dx} \left(u(x) \cdot (v(x))^{-1} \right)$$

Step 5: Once again, students should attempt the algebraic derivation of the quotient rule themselves, and then use the supplied **algebraic spreadsheet** and the prepared program, **diff_quotient**, to verify their process, and to observe a model for future use of this rule.

1.11 1.12 1.13 1.14 RAD AUTO REAL

Try the following examples and then test your solutions using the program **diff_quotient(u(x), v(x))** which gives the step-by-step process, and generates the final answer as the variable, **result**.

Step 6 The consolidation stage now offers students examples for them to attempt, supported by access to the program and the CAS facilities of TI-Nspire CAS.

1.11 1.12 1.13 1.14 RAD AUTO REAL

(1) $\frac{d}{dx} \left(\frac{\sin(x)}{\cos(x)} \right)$

(2) $\frac{d}{dx} \left(\frac{x-1}{x+1} \right)$

(3) $\frac{d}{dx} \left(\frac{\ln(x)}{x} \right)$

(4) $\frac{d}{dx} \left(\frac{e^x}{\cos(x)} \right)$

1.12 1.13 1.14 1.15 RAD AUTO REAL

We have now established both a product rule for differentiation:

$$\frac{d}{dx} (u \cdot v) = u \cdot \frac{d}{dx} (v) + v \cdot \frac{d}{dx} (u)$$

and a quotient rule:

$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \cdot \frac{d}{dx} (u) - u \cdot \frac{d}{dx} (v)}{v^2}$$

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(Student)TI-Nspire File: CalcActXX_Quotient_Rule_EN.tns

1.1 1.2 1.3 1.4 ▶RAD AUTO REAL

**Introducing the Differential Calculus:
The Quotient Rule**

Calculus with CAS

1.1 1.2 1.3 1.4 ▶RAD AUTO REAL

We have established that, although the sums and differences of derivatives are simple, the derivative of a product is NOT equal to the product of the derivatives.

In fact,

$$\frac{d}{dx}(u(x) \cdot v(x)) = \frac{d}{dx}(u(x)) \cdot v(x) + \frac{d}{dx}(v(x)) \cdot u(x)$$

1.1 1.2 1.3 1.4 ▶RAD AUTO REAL

In this activity, we consider the case of the derivative of a quotient function, $\frac{u(x)}{v(x)}$.

Discuss and conjecture what you think might be the result of $\frac{d}{dx}\left(\frac{u(x)}{v(x)}\right)$?

1.1 1.2 1.3 1.4 ▶RAD AUTO REAL

Question

1. Consider the example $\frac{\sin(x)}{x^2}$. How might this be rewritten as a product?

Answer ⌵

1.2 1.3 1.4 1.5 ▶RAD AUTO REAL

Question

2. Use the product rule to evaluate the derivative of this function.

Answer ⌵

1.3 1.4 1.5 1.6 ▶RAD AUTO REAL

A	B	C
1	u(x) = sin(x)	
2	v(x) = 1/x^2	
3	du = cos(x)	f
4	dv = -2/x^3	f
5	d(u*v) = u*dv+v*du	
A1	"u(x) ="	

1.4 1.5 1.6 1.7 ▶RAD AUTO REAL

$d(uv) = ((x^2) * (\cos(x)) - (\sin(x)) * (2*x)) / x^3$

Try to evaluate this then check your answer by typing <result>.

Done

result $\frac{x \cdot \cos(x) - 2 \cdot \sin(x)}{x^3}$

4/99

1.5 1.6 1.7 2.1 ▶RAD AUTO REAL

Clearly, these are not the derivatives of the parts.

$$\frac{d}{dx}\left(\frac{u}{v}\right) \neq \frac{d}{dx}(u) \cdot \frac{d}{dx}(v)$$

How might we find such a derivative?

1.6 1.7 2.1 2.2 ▶RAD AUTO REAL

Since we have previously proven the Product Rule result (using First Principles with a "twist!") we may use this result to establish the Quotient Rule by writing a quotient function as a product.

$$\frac{d}{dx}\left(\frac{u(x)}{v(x)}\right) = \frac{d}{dx}\left(u(x) \cdot (v(x))^{-1}\right)$$

1.7 2.1 2.2 2.3 ▶RAD AUTO REAL

Question

3. Use the product rule to establish a quotient rule for $\frac{d}{dx}\left(\frac{u(x)}{v(x)}\right)$.

Answer ⌵

$$\frac{d}{dx}(u \cdot v^{-1}) = u \cdot \frac{d}{dx}(v^{-1}) + v^{-1} \cdot \frac{d}{dx}(u)$$

2.1 2.2 2.3 2.4 ▶RAD AUTO REAL

$\frac{d}{dx}(u(x) \cdot (v(x))^{-1})$

Try to evaluate this then check your answer by typing <result>.

$$\frac{\frac{d}{dx}(u(x)) \cdot v(x) - \frac{d}{dx}(v(x)) \cdot u(x)}{(v(x))^2}$$

1/99

2.2 2.3 2.4 2.5 ▶RAD AUTO REAL

Our Quotient Rule appears to be of the form:

"Bottom times the d/dx of the top MINUS the top times the d/dx of the bottom ALL OVER the bottom squared".

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot \frac{d}{dx}(u) - u \cdot \frac{d}{dx}(v)}{v^2}$$

2.3 2.4 2.5 2.6 ▸ RAD AUTO REAL

Try the following examples and then test your solutions using the spreadsheet on page 1.14, and then check your results using the program **diff_quotient(u(x),v(x))** which gives the step-by-step process, and generates the final answer as the variable, **result**.

2.4 2.5 2.6 2.7 ▸ RAD AUTO REAL

(1) $\frac{d}{dx} \left(\frac{\sin(x)}{\cos(x)} \right)$
 (2) $\frac{d}{dx} \left(\frac{x-1}{x+1} \right)$
 (3) $\frac{d}{dx} \left(\frac{\ln(x)}{x} \right)$
 (4) $\frac{d}{dx} \left(\frac{e^x}{\cos(x)} \right)$

2.5 2.6 2.7 2.8 ▸ RAD AUTO REAL

	A	B	C
1	u(x) =	sin(x)	
2	v(x) =	cos(x)	
3	du =	cos(x)	✓
4	dv =	-sin(x)	✓
5	d(u/v) =	(v*du-u*dv)/v^2	
A1	"u(x) ="		

2.6 2.7 2.8 2.9 ▸ RAD AUTO REAL

$\gg) \cdot (x^2 - 1)$
 Try to evaluate this then check your answer by typing <result>.

$$\frac{x \cdot (x^2 - 3)}{(x^2 - 1) \cdot \sqrt{1 - x^2}}$$

5/99

2.7 2.8 2.9 2.10 ▸ RAD AUTO REAL

We have now established both a product rule for differentiation:

$$\frac{d}{dx}(\mathbf{u} \cdot \mathbf{v}) = \mathbf{u} \cdot \frac{d}{dx}(\mathbf{v}) + \mathbf{v} \cdot \frac{d}{dx}(\mathbf{u})$$

and a quotient rule:

$$\frac{d}{dx} \left(\frac{\mathbf{u}}{\mathbf{v}} \right) = \frac{\mathbf{v} \cdot \frac{d}{dx}(\mathbf{u}) - \mathbf{u} \cdot \frac{d}{dx}(\mathbf{v})}{\mathbf{v}^2}$$