Introducing the Differential Calculus
The Quotient Rule

ID: XXXX

Time required
45 minutes

Activity Overview

In this activity, students explore ways to differentiate harder functions. The focus here is on functions which can be expressed as a quotient of two simpler functions. The approach taken here is largely symbolic and makes full use of the computer algebra facilities of TI-Nspire CAS. Prepared programs and an algebraic spreadsheet are also utilized for skill development and consolidation.

Concepts

- *Quotient rule for differentiation, product rule for differentiation.*

Teacher Preparation

This investigation offers opportunities for review and consolidation of key concepts related to the methods of differentiation of simple functions and the product rule, both covered in preceding activities. Opportunities are then provided for skill development and practise of the method of taking derivatives of quotients. As such, care should be taken to provide ample time for ALL students to engage actively with the requirements of the task, allowing some who may have missed aspects of earlier work the opportunity to build new and deeper understanding.

- *This activity can serve to consolidate earlier work on the product rule. It offers a suitable introduction to derivatives of more difficult functions.*
- *Begin by reviewing the method of differentiation of by product rule, both graphically and symbolically, and methods of differentiation of the standard function forms.*
- *The screenshots on pages X–X (top) demonstrate expected student results. Refer to the screenshots on page X (bottom) for a preview of the student .tns file.*
- ***To download the .tns file, go to http://education.ti.com/exchange and enter "XXXX" in the search box.***

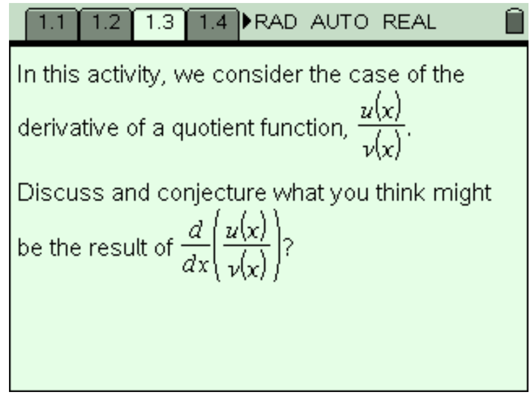
Classroom Management

- *This activity is intended to be **teacher led.** You should seat your students in pairs so they can work cooperatively on their handhelds. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds, although the majority of the ideas and concepts are only presented in **this** document; be sure to cover all the material necessary for students' total comprehension.*
- *Students can either record their answers on the handheld or you may wish to have the class record their answers on a separate sheet of paper.*

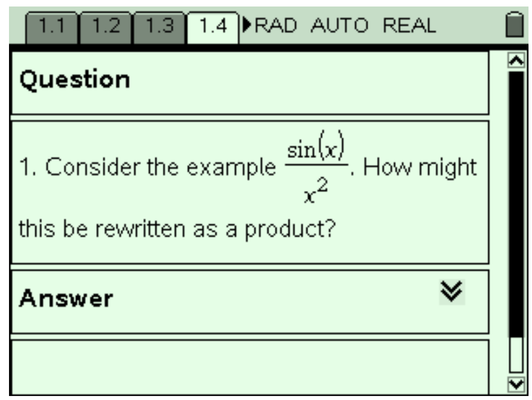
TI-Nspire™ Applications

Calculator Lists & Spreadsheet, Notes and Programming.

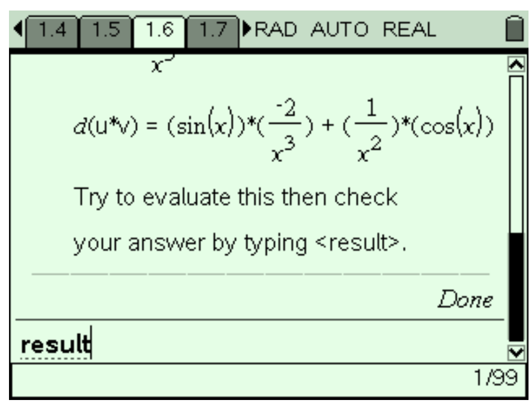
Step 1: Begin with discussion and review of both the product rule and of the derivatives of standard function forms. Ensure that students are comfortable with these and then challenge them to consider the more difficult form of quotient functions.



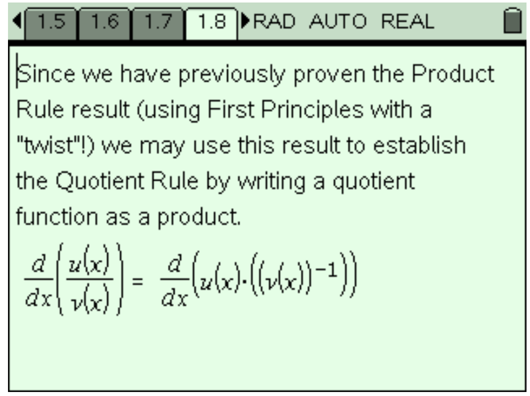
Step 2: The approach taken here is to treat the quotient function as a special case of the product, and so to use preceding work on the product rule to establish this new differentiation method.



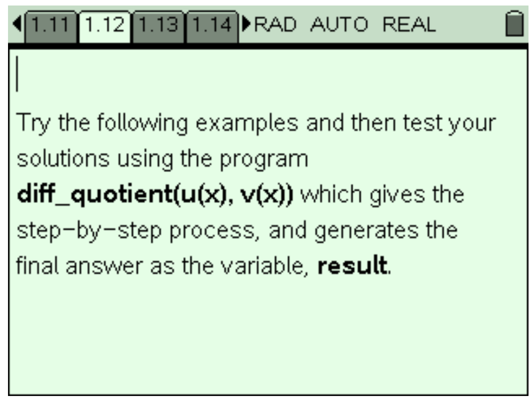
Step 3: Once again, an algebraic spreadsheet and prepared programs are provided to scaffold students in their investigation and consolidation of the required skills: the **diff_product(fn1, fn2)** program gives a step-by-step working of the product rule for **fn1*fn2**, while we lead into using **diff_quotient(fn1, fn2)** to verify and then support our new method.



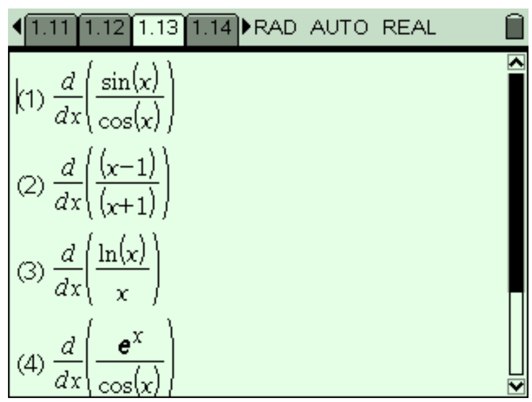
Step 4: Rather than going back to first principles again to establish this result (and this approach is possible, if quite difficult), students are introduced to the fundamental method of proof which involves the use of previously established results. In this case, having proved the product rule, we are free to use it to establish (and prove) a new rule for quotients.

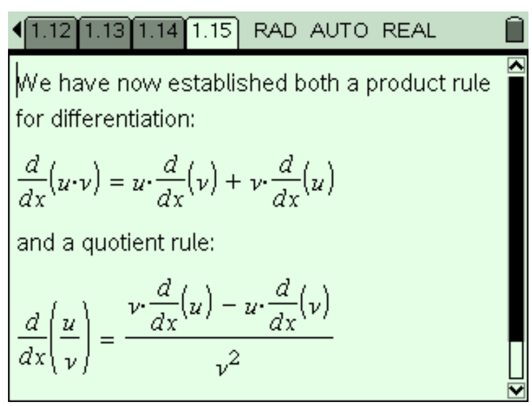


Step 5: Once again, students should attempt the algebraic derivation of the quotient rule themselves, and then use the supplied **algebraic spreadsheet** and the prepared program, **diff_quotient**, to verify their process, and to observe a model for future use of this rule.



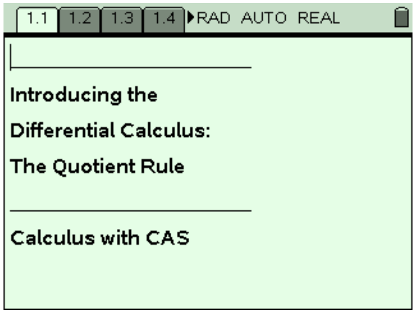
Step 6 The consolidation stage now offers students examples for them to attempt, supported by access to the program and the CAS facilities of TI-Nspire CAS.

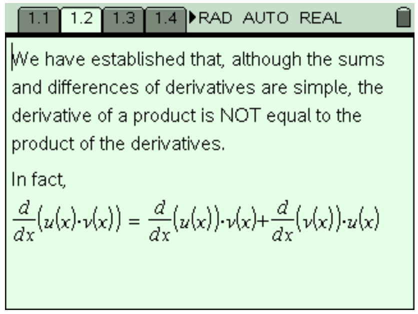


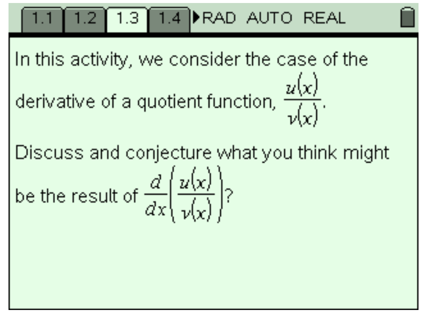


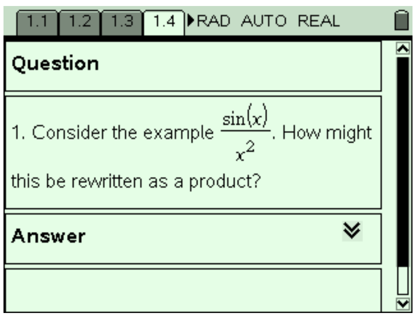
Introducing the Differential Calculus: The Quotient Rule– ID: XXXX

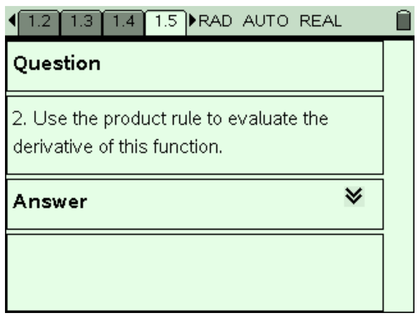
(Student)TI-Nspire File: *CalcActXX_Quotient_Rule_EN.tns*

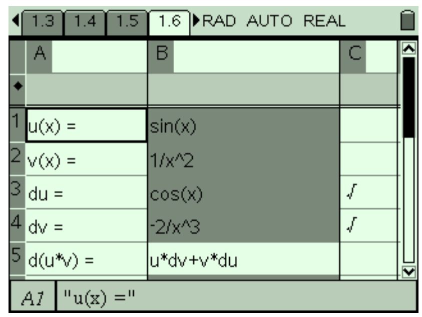


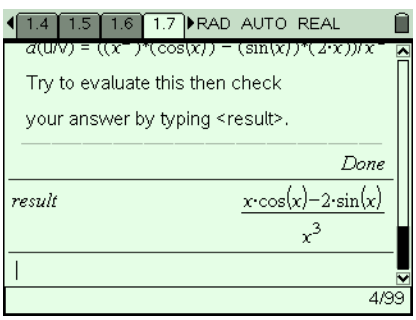


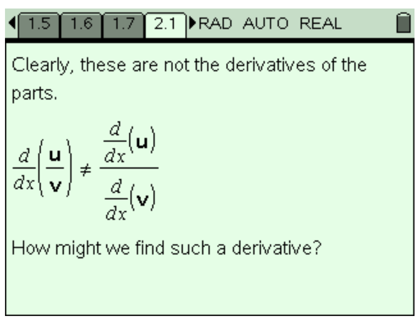


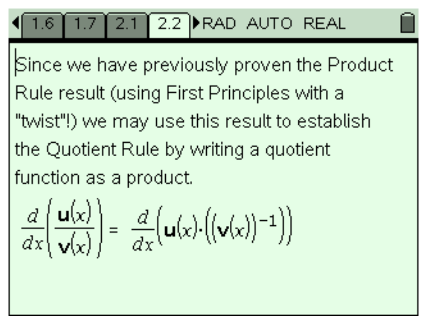


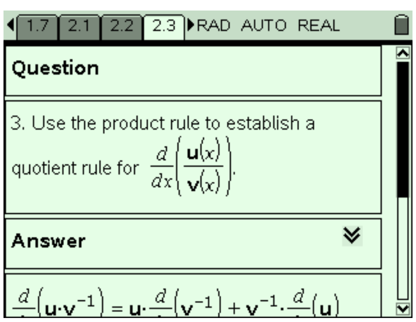


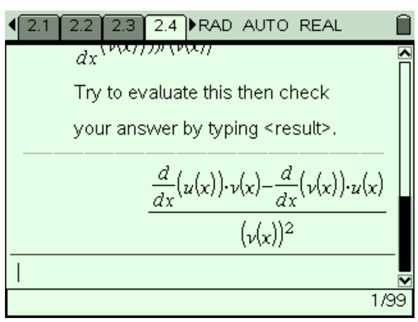


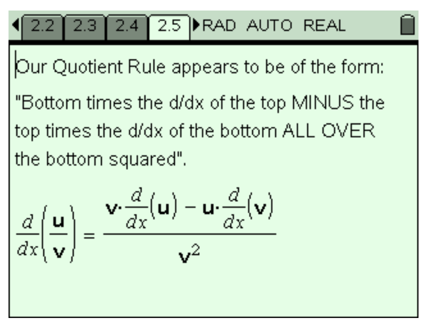


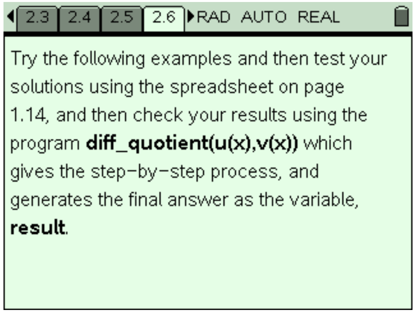


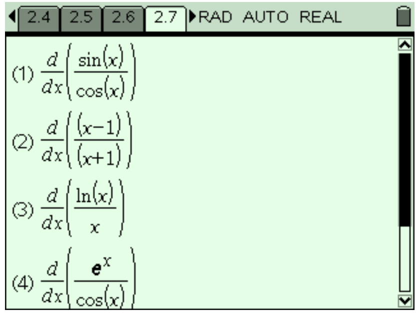


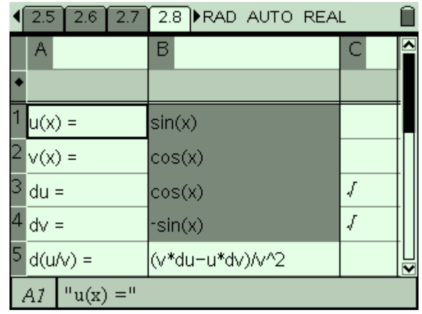


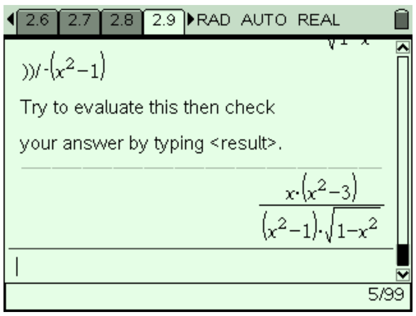


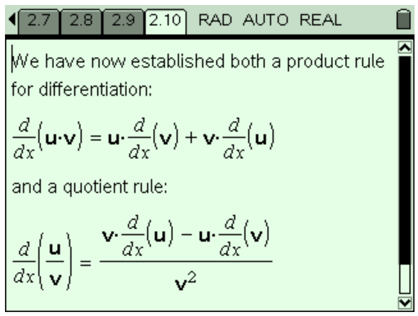



2.3 2.4 2.5 2.6 RAD AUTO REAL
Try the following examples and then test your solutions using the spreadsheet on page 1.14, and then check your results using the program **diff_quotient(u(x),v(x))** which gives the step-by-step process, and generates the final answer as the variable, **result**.


2.4 2.5 2.6 2.7 RAD AUTO REAL
(1) $\frac{d}{dx}\left(\frac{\sin(x)}{\cos(x)}\right)$
(2) $\frac{d}{dx}\left(\frac{(x-1)}{(x+1)}\right)$
(3) $\frac{d}{dx}\left(\frac{\ln(x)}{x}\right)$
(4) $\frac{d}{dx}\left(\frac{e^x}{\cos(x)}\right)$


2.5 2.6 2.7 2.8 RAD AUTO REAL
A
B
C
1 u(x) = sin(x)
2 v(x) = cos(x)
3 du = cos(x)
4 dv = -sin(x)
5 d(u/v) = (v*du−u*dv)/v^2
A1 "u(x) ="


2.6 2.7 2.8 2.9 RAD AUTO REAL
))/·(x^2-1)
Try to evaluate this then check
your answer by typing <result>.
$\frac{x\cdot(x^2-3)}{(x^2-1)\cdot\sqrt{1-x^2}}$
5/99


2.7 2.8 2.9 2.10 RAD AUTO REAL
We have now established both a product rule for differentiation:
$\frac{d}{dx}(\mathbf{u}\cdot\mathbf{v}) = \mathbf{u}\cdot\frac{d}{dx}(\mathbf{v}) + \mathbf{v}\cdot\frac{d}{dx}(\mathbf{u})$
and a quotient rule:
$\frac{d}{dx}\left(\frac{\mathbf{u}}{\mathbf{v}}\right) = \frac{\mathbf{v}\cdot\frac{d}{dx}(\mathbf{u}) - \mathbf{u}\cdot\frac{d}{dx}(\mathbf{v})}{\mathbf{v}^2}$