TI-*nspire*™

Applications of Calculus:
Projectile Motion ID: xxxx

Name _____

Class _____

In this activity, we explore the application of differential equations to the real world as applied to projectile motion.

Open the file *CalcActXX_Projectile_Motion_EN.tns* on your handheld and follow along with your teacher to work through the activity. Use this document as a reference and to record your answers.

EXERCISES

1. Imagine lying on your back and watching the path of a ball as it moves above you – discuss and describe the motion you observe.

2. Now imagine standing behind the thrower, directly in line with the path of the motion, What do you see?

3. From the instant that the ball leaves the thrower's hand, what forces act upon it?

4. Clearly explain how displacement (*x*), velocity (*v*) and acceleration (*a*) are related.

5. Express the horizontal and vertical components of the velocity in terms of the initial velocity (V_0) and the initial angle (α) .

6. The defining equations for projectile motions are the two components of acceleration: $A_x = 0$ and $A_y = -g$ (where g is the gravity constant, 9.8 m/s^2 or 32 ft/s^2). These are differential equations. Integrate with respect to time to derive the velocity equations.
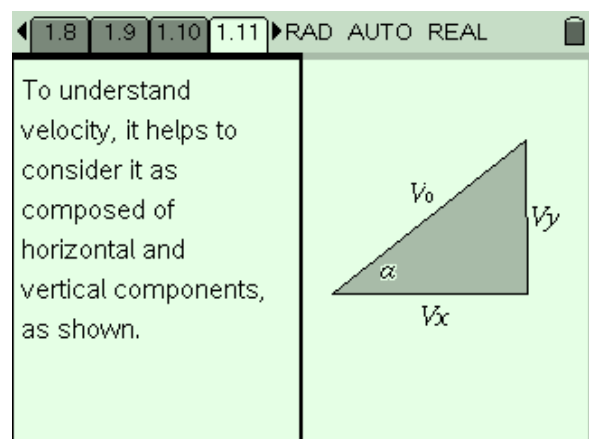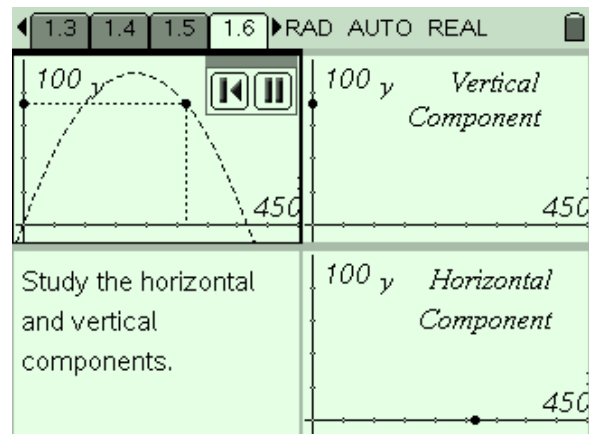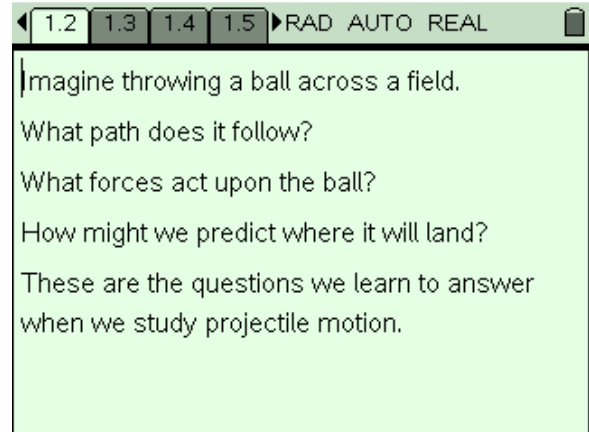
7. Now use these velocity equations to derive the displacement components.

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8. Finally, eliminate **t** and derive the trajectory path in **x** and **y**.

9. Interpret these results in terms of our ball throw. What assumptions have we made?

10. I throw a ball from a height of 1.6 meters, with an initial velocity of 30 meters per second at an angle of 50 degrees to the ground. Find the trajectory equation, the maximum distance travelled by the ball, and the maximum height of the throw. (Use 9.8 m/s^2 for gravity).

11. A cannon fires a shot from a position 1 meter above the ground at an angle of 35 degrees with an initial velocity of 120 meters per second. How far will the shot travel? (Use 9.8 m/s^2 for gravity).

EXTENSIONS

- Ex1   Is there another angle at which I could aim the cannon to fire a shot exactly the same distance?

- Ex2   At what angle and with what initial velocity should I shoot from a position 4 meters from the hoop in order to score? (I will be shooting from a height of 1.7 meters and assume the hoop is 3 meters high).

- Ex3   Prove your result algebraically.
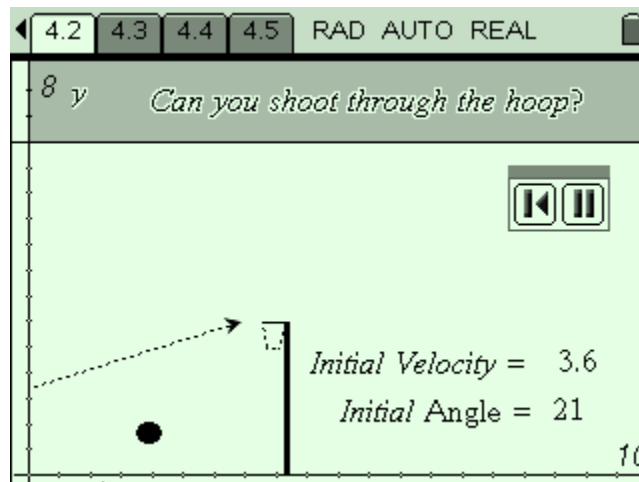
SUGGESTED SOLUTIONS

1. The ball appears to move overhead at a steady rate, moving at constant velocity.

2. Exactly the same as throwing something straight up from your hand and then catching it - the moment it leaves your hand, it begins to slow until it eventually stops and then begins to accelerate back down again. So, too, the ball goes up, slowing until it reaches its highest point and then falling down again, speeding up until it hits the ground.

3. Once the ball leaves my hand, the only force acting is gravity, pulling it downwards. So the horizontal force is zero, the vertical force is the mass of the ball multiplied by the force of gravity acting downwards.

4. Velocity is the rate of change of displacement, with respect to time, i.e. $v = \dfrac{dx}{dt}$

   Acceleration is the rate of change of velocity with respect to time. Ie

   $$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2}{dt^2}(x)$$

5. $Vx = V_0 * \cos(\alpha)$ and $Vy = V_0 * \sin(\alpha)$

6. $Vx = \int(ax\,dt) = C1$ but when $t = 0$, $Vx = V_0 * \cos(\alpha)$

   $Vy = \int(ay,t) = C2 - g*t$

   When $t = 0$, $Vy = V_0 * \sin(\alpha) = C$ hence $Vy = V_0 * \sin(\alpha) - g*t$

7. $x = \int(Vx,t) = V_0 * \cos(\alpha) * t + C3$

   When $t = 0$, $x = x_0 = C3 \gg x = V_0 * \cos(\alpha) * t + x_0$

   $y = \int(Vy,t) = V_0 * \sin(\alpha) * t - \frac{1}{2} * g * t^2 + C4$

   When $t = 0$, $y = y_0 = C4 \gg y = V_0 * \sin(\alpha) * t - \frac{1}{2} * g * t^2 + y_0$

8. From $x = V_0 * \cos(\alpha) * t + x_0 \gg t = (x - x_0)/V_0 * \cos(\alpha)$

   Substituting for t: $y = \dfrac{V_0 \cdot \sin(\alpha) \cdot (x - x_0)}{V_0 \cdot \cos(\alpha)} - \dfrac{1}{2} \cdot g \cdot \dfrac{x - x_0}{V_0 \cdot \cos(\alpha)^2} + y_0$

   $\gg y = \tan(\alpha) \cdot (x - x_0) - \dfrac{1}{2} \cdot g \cdot \dfrac{x - x_0}{V_0 \cdot \cos(\alpha)^2} + y_0$

9. The equation is quadratic in x with starting point (x_0, y_0) as given.

   For a CAS treatment of maximum distance and maximum height:

Define $proj(x) = \tan(\alpha) \cdot (x - x0) - \dfrac{1}{2} \cdot g \cdot \left(\dfrac{x - x0}{v0 \cdot \cos(\alpha)}\right)^2 + y0$ *Done*

$solve(proj(x) = 0, x)$

$x = \dfrac{-\left(\sqrt{2 \cdot g \cdot y0 + v0^2 \cdot (\sin(\alpha))^2} \cdot v0 \cdot \cos(\alpha) - g \cdot x0 - v0^2 \cdot \sin(\alpha) \cdot \cos(\alpha)\right)}{g}$ or $x = \dfrac{\sqrt{2 \cdot g \cdot y0 + v0^2 \cdot (\sin(\alpha))^2} \cdot v}{\,}$

$solve\left(\dfrac{d}{dx}(proj(x)) = 0, x\right)$ $x = \dfrac{g \cdot x0 + v0^2 \cdot \sin(\alpha) \cdot \cos(\alpha)}{g}$ or $\dfrac{1}{v0^2 \cdot (\cos(\alpha))^2} = 0$

$proj\left(\dfrac{g \cdot x0 + v0^2 \cdot \sin(\alpha) \cdot \cos(\alpha)}{g}\right)$ $\dfrac{v0^2 \cdot (\sin(\alpha))^2}{2 \cdot g} + y0$

   We have ignored air and wind resistance and assumed that the mass of the object is not relevant.

10. The ball travels 91.76 meters.
    The ball reaches a height of 28.5 meters.
    The ball lands after 4.76 seconds.

$-.013177 \cdot x^2 + \dfrac{x}{\tan\left(\dfrac{2 \cdot \pi}{9}\right)} + 1.6$

Eliminate t from these equations to derive
the trajectory equation in x and y.
Check your result by typing <projectile_fn>
Maximum distance is stored as <maxdist>
Maximum height is stored as <maxheight>
Maximum time is stored as <maxtime>

                                                            Done

$projectile\_fn$ $-.013177 \cdot x^2 + \dfrac{x}{\tan\left(\dfrac{2 \cdot \pi}{9}\right)} + 1.6$

$maxdist$	91.7647
$maxheight$	28.546
$maxtime$	4.75869

11. The shot will land 1382.2 meters away.

Extension

Ex1   Yes, the complement of the angle will land the same distance away. In this case, an angle of 55 degrees will have the same effect.

Ex2   One option would involve an angle of 45 degrees and an initial velocity of 7.5 m/s.

Ex3   To prove that my figures work, I would need to show that the trajectory path for this shot will pass close to the point (4,3)

	A	B	C
◆			
7	Horizonta…		
8	Ax =	0	
9	Vx =	5.3033	✓
10	x =	5.3033008588991*t	✓
11	>> t =	.18856180831641*x	✓
12	Vertical		
13	Ay =	-9.8	✓
14	Vy =	5.3033008588991−9.8*t	✓
15	y =	-4.9*t^2+5.3033008588991*t…	✓
16			
17	Eliminate …	y=-.17422222222222*x^2+.…	✓
18			
19	MaxDispl…	x=7.11182763297	✓
20	MaxHeigh…	y=3.1349489795917	✓
21	MaxTime=..	t=1.3410190789074	✓
22			
23		y=2.9124444444444	