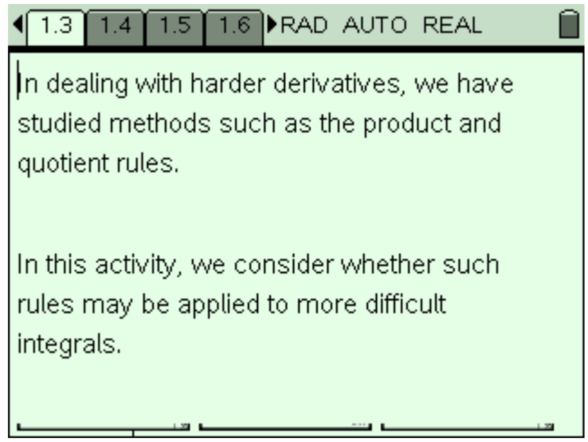
Introducing the Integral Calculus: Integration by Parts

ID: XXXX

Name ______________________

Class ______________________

In previous activities, we have explored the differential calculus through investigations of the methods of first principles, the product and quotient rules. In this activity the product rule becomes the basis for an integration method for more difficult integrals.



Open the file *CalcActXX_Integration_by_Parts_EN.tns* on your handheld and follow along with your teacher to work through the activity. Use this document as a reference and to record your answers.

EXERCISES

1. State the product rule for a function of the form ***u(x)*v(x)***.

2. Apply the product rule to the function ***sin(x)*ln(x)***.

3. In an examination, a student makes the following statement:

$$\int \frac{\partial}{\partial x}(f(x)dx = \frac{\partial}{\partial x}(\int f(x)dx) = f(x)$$

Do you agree?

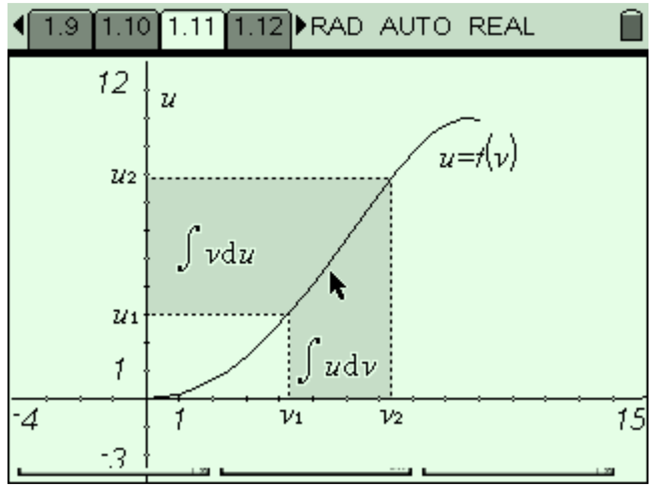
4. State the result of taking the integral of the left-hand side of the product rule,

$$\int (\frac{\partial}{\partial x}(u(x) * v(x))dx$$

5. And the right-hand side? $\int (u(x) * \frac{\partial v}{\partial x} + v(x) * \frac{\partial u}{\partial x})dx$

6. Carefully explain the relationship between the areas shown on the graph and the relationship derived:

   $$\int_{v_1}^{v_2} u * dv = u * v - \int_{u_1}^{u_2} v * du$$



7. Use the method of **integration by parts** to compute the integral of ***ln(x)***.
   (Check your result by running the program **intbyparts(ln(x),1)** and typing **result)**.

8. Find $\int \cos(\ln(x))dx$

9. Substitute the previous result for ***cos(ln(x))*** into the integration by parts result for ***sin(ln(x))***.

10. Now try the following using Integration by Parts, then check your answers using the **intbyparts()** program and typing **result**.

    a. $\int \tan^{-1}(x)dx$ b. $\int x^2 * e^x \cdot dx$ c. $\int x * \tan^{-1}(x)dx$

    d. $\int x * \cos(2x+1)dx$

11. **(Extension 1)** Does it matter in which order ***u(x)*** and ***v(x)*** are selected for the method of integration by parts?

12. **(Extension 2)** Is there likely to be an integration rule based upon the quotient rule just as Integration by Parts was based upon the product rule?

SUGGESTED SOLUTIONS

1. $\frac{\partial}{\partial x}(u * v) = u * \frac{\partial v}{\partial x} + v * \frac{\partial u}{\partial x}$

2. $\frac{\partial}{\partial x}(\sin(x) * \ln(x)) = \sin(x) * \frac{1}{x} - \ln(x) * \cos(x)$

3. No. While $\frac{\partial}{\partial x}(\int f(x)dx) = f(x)$ is true, $\int \frac{\partial}{\partial x}(f(x)dx = f(x) + C$

4. $\int (\frac{\partial}{\partial x}(u(x) * v(x))dx = u(x) * v(x) + C_1$

5. $v(x) * \int u(x) \cdot dx + u(x) * \int v(x) \cdot dx$

6. The area between the curve ***u = f(v)*** and the ***v***-axis (between the specified limits, i.e. $\int_{v_1}^{v_2} u \cdot dv$) can be found by taking the area of the large rectangle ($u_2 \cdot v_2$) minus the smaller rectangle ($u_1 \cdot v_1$), and then subtracting the area of the region between the curve and ***u***-axis, between the specified limits ($\int_{u_1}^{u_2} v \cdot du$).

7. **∫(ln(x)*(1)) dx)** using Integration by Parts
   ∫(u*dv) = u*v - ∫v*du
   u(x) = ln(x)
   du = 1/x dx
   dv = 1 dx
   v(x) = x
   ∫(ln(x))*(1) dx) = ln(x)*x - ∫(x*(1/x)) dx
   ∫(ln(x)) dx = x*ln(x)-x

8. $\int \cos(\ln(x)) \cdot dx = x * \cos(\ln(x)) + \int \sin(\ln(x)) \cdot dx$

9. $\int \sin(\ln(x)) * 1 \cdot dx = x * \sin(\ln(x)) - 2[x * \cos(\ln(x)) + \int \sin(\ln(x)) \cdot dx]$
   $\int \sin(\ln(x)) \cdot dx = \frac{x}{2} * [\sin(\ln(x)) - \cos(\ln(x))]$

10. (a) ***∫((□$\tan^{-1}(x)$□)*(□1□) dx) using Integration by Parts***

∫(u*dv) = u*v - ∫v*du

u(x) = □$\tan^{-1}(x)$

du =□((1)/(x^2+1))□ dx

dv = □1□ dx

v(x) = □x

∫(□$\tan^{-1}(x)$□)*(□1□) dx) = (□$\tan^{-1}(x)$□)*(□x□) - ∫(□x□)*(□((1)/(x^2+1))□) dx

= x*$\tan^{-1}(x)$ – ½*ln(x^2+1) + C

(b) ***∫((□e^x□)*(□x^2□) dx) using Integration by Parts***

∫(u*dv) = u*v - ∫v*du

u(x) = □x^2

du =□2x□ dx

dv = □e^x□ dx

v(x) = □e^x

∫(□x^2□)*(□e^x□) dx) = (□x^2□)*(□e^x□) - ∫(□e^x□)*(□2x□) dx

Repeat for ***∫(□e^x□)*(□2x□) dx***

∫(□e^x□)*(□2x□) dx = 2x*e^x - 2∫(□e^xdx = 2x*e^x – 2e^x

Hence, ∫(□x^2□)*(□e^x□) dx) = □x^2□* e^x□ - 2x*e^x + 2e^x = e^x*[x^2 – 2x + 2]

(c) ***∫((□$\tan^{-1}(x)$□)*(□x□) dx) using Integration by Parts***

∫(u*dv) = u*v - ∫v*du

u(x) = □$\tan^{-1}(x)$

du =□1/(x^2 + 1))□ dx

dv = □x□ dx

v(x) = □x^2/2

∫(□$\tan^{-1}(x)$□)*(□x□) dx) = □$\tan^{-1}(x)$)*□x^2/2□ - ∫(□x^2/2 *(1/(x^2+1)) dx

$$\int \tan^{-1}(x) * x \cdot dx = \frac{x^2 * \tan^{-1}(x)}{2} - \frac{1}{2} * \int \frac{x^2}{x^2+1} \cdot dx$$

$$\int \tan^{-1}(x) * x \cdot dx = \frac{x^2 * \tan^{-1}(x)}{2} - \frac{1}{2} * \int \frac{x^2+1}{x^2+1} - \frac{1}{x^2+1} \cdot dx$$

$$\int \tan^{-1}(x) * x \cdot dx = \frac{x^2 * \tan^{-1}(x)}{2} - \frac{1}{2} * (x - \tan^{-1}(x)) = (\frac{x^2+1}{2}) * \tan^{-1}(x) - \frac{x}{2}$$

(d) ***∫(□cos(2x+1)□)*□x□) dx) using Integration by Parts***

∫(u*dv) = u*v - ∫v*du

u(x) = □x

du =□1□ dx

dv = □cos(2x+1)□ dx

v(x) = □sin(2x+1)/2

∫□x□*□cos(2x+1) dx) = □x□*□sin(2x+1)/2 - ∫(sin(2x+1)/2□)*(□1□) dx

= x*sin(2x+1)/2 – [-cos(2x+1)/4] = ¼*[2x sin(2x+1) + cos(2x+1)]

11. Often it does not matter, if both functions are readily differentiable and integrable, but in cases where one may not be reduced by the process, it can make a difference. Try the example questions from question 10.

12. No, since the quotient rule formula does not allow us to easily rearrange the parts - the denominator v^(2) causes problems.