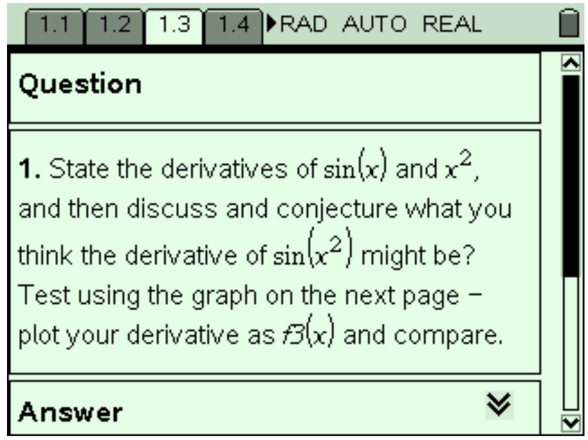
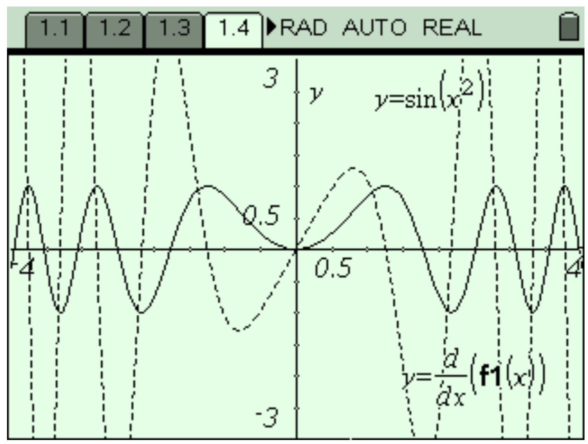
Introducing the Differential Calculus: Composite Functions

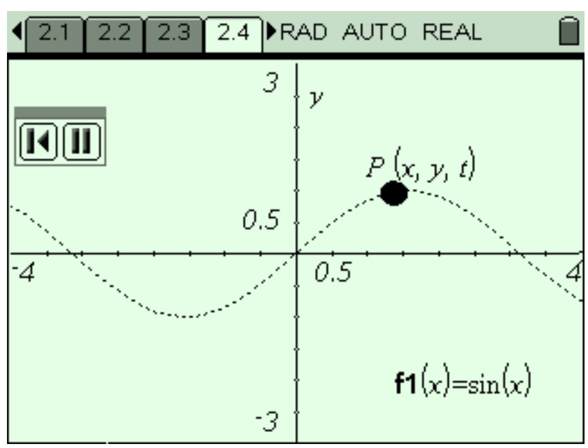
ID: XXXX

Name ______________________

Class ______________________







In this activity, we explore ways to differentiate harder functions. The focus here is on functions which can be expressed as the composition of two functions: ***y = f(g(x)).*** Open the file *CalcActXX_Composite_Functions_EN.tns* on your handheld and follow along with your teacher to work through the activity. Use this document as a reference and to record your answers.

EXERCISES

1. State the derivatives of ***sin(x)*** and ***x^2*** and then discuss and conjecture what you think the derivative of ***sin(x^2)*** might be? Test your conjecture using the graph page provided – plot your derivative and compare to the example shown.

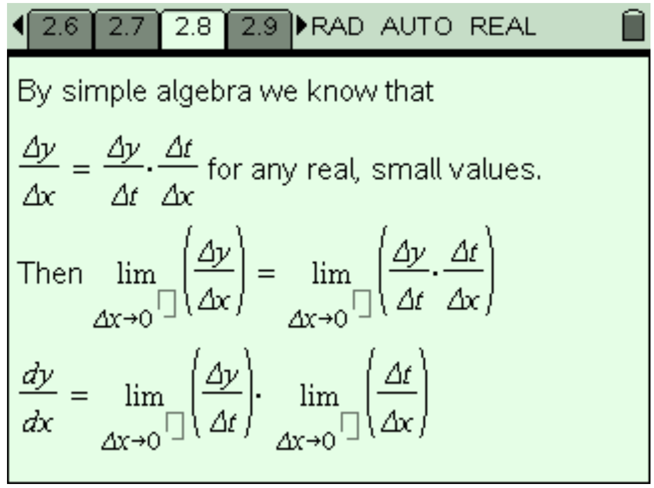
2. If we use a "chain rule" such that ***dy/dx = dy/du * du/dx*** this suggests that the derivatives (which are actually limits from first principles work that we have done) can behave very much like fractions (at least in some cases). Is this reasonable?

3. Carefully explain the relationship between the variables ***x, y*** and ***t*** in the graph shown.

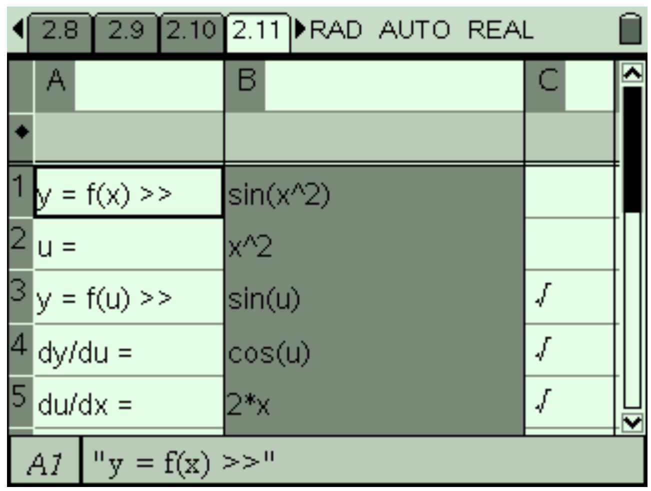
4. If ***y = f1(x), y = u(t)*** and ***t = v(x)***, eliminate the variable ***t*** to define a composite function for ***y***.

5. Since we know that ***x, y*** and ***t*** are related, then we can safely assume (for continuous and differentiable functions, at least) that as $\Delta x \to 0$ then $\Delta t \to 0$. Use this to simplify the result:

$$\frac{dy}{dx} = \lim_{\Delta x \to 0}\left(\frac{\Delta y}{\Delta t}\right) \cdot \lim_{\Delta x \to 0}\left(\frac{\Delta t}{\Delta x}\right)$$



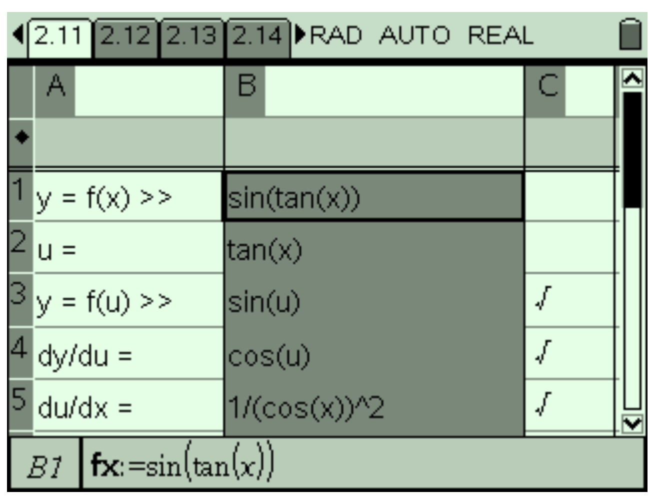
6. Use this "chain rule" to find the derivative of ***y = sin(x²)***. Check your result using the spreadsheet provided – enter your function into B1, then your choice of ***u*** for substitution into B2. Substitute your value for ***u*** into the original function (B3) and then enter the derivative of the function with respect to ***u*** and the derivative of ***u*** with respect to ***x*** are placed next, and finally the resulting derivative of the function. Each step is check algebraically to show if you have made an error at any point in your working. This format is a suitable model for a complete worked solution for such problems.



7. Now use the same method to find the derivative of ***sin(x)²***.

8. $\frac{d}{dx}(\ln(x^2))$

9. $\frac{d}{dx}(\sin(\tan(x)))$



10. Clearly describe this method for a composite function, ***y = f(g(x))***.

SUGGESTED SOLUTIONS

1. The derivative of ***sin(x)*** is ***cos(x),*** and that for x^2 is ***2x***. The required derivative could be any comination of these, such as ***cos(x2)*** or ***2x*cos(x²).***

2. The result is reasonable, since derivatives result from taking the limits of fractions (the gradient between any two points is a fraction). But it needs to be proved.

3. The position of the point P is defined by three numbers: - the path it follows is defined by ***x*** and ***y,*** but where it exists on that path is given by the variable ***t***.

4. Since ***y = u(t)*** and ***t = v(x)*** then ***y = u(v(x)) = f(x)***.

5. $\frac{dy}{dx} = \lim_{\Delta t \to 0}\left(\frac{\Delta y}{\Delta t}\right) \cdot \lim_{\Delta x \to 0}\left(\frac{\Delta t}{\Delta x}\right) = \frac{dy}{dt} \cdot \frac{dt}{dx}$ as required.

6. For ***y = sin(x²)***: Let ***u = x²,*** then ***du/dx = 2x*** and ***dy/du = cos(u)***.

   By the chain rule, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$, thus $\frac{dy}{dx} = \cos(u) * 2x = 2x\cos(x^2)$

7. For ***y = sin(x)²***: Let **u = sin(x)**, then ***du/dx = cos(x)*** and ***dy/du = 2u.***

   By the chain rule, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$, thus $\frac{dy}{dx} = 2u * \cos(x) = 2\sin(x)\cos(x)$

8. $\frac{d}{dx}(\ln(x^2))$: Let ***u = x²*** then ***du/dx = 2x*** and ***dy/du = 1/u***

   By the chain rule, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$, thus $\frac{dy}{dx} = \frac{1}{u} \cdot 2x = \frac{2}{x}$

9. $\frac{d}{dx}(\sin(\tan(x)))$: Let ***u = tan(x)*** then ***du/dx = sec²(x)*** and ***dy/du = cos(u)***

   By the chain rule, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$, thus $\frac{dy}{dx} = \cos(u) \cdot \sec^2(x) = \frac{\cos(\tan(x))}{\cos(x)^2}$

10. In general, $\frac{d}{dx}(f(g(x))$: Let ***u = g(x)*** then ***du/dx = g'(x)*** and ***dy/du = f'(u)***

   By the chain rule, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$, thus $\frac{dy}{dx} = f'(u) \cdot g'(x) = f'(g(x) \cdot g'(x)$