Introducing the Differential Calculus
Composite Functions

ID: XXXX

Time required
45 minutes

Activity Overview

In this activity, we explore ways to differentiate some more difficult functions. The focus here is on functions which can be expressed as a composition of two functions. The approach taken here is largely symbolic and makes full use of the computer algebra facilities of TI-Nspire CAS. Prepared algebraic spreadsheets are utilized for skill development and consolidation.

Concepts

- *Differentiation of standard forms, fundamentals of derivatives, chain rule.*

Teacher Preparation

This investigation offers opportunities for review and consolidation of key concepts related to the methods of first principles and differentiation of simple functions. Opportunities are provided for skill development and practice of the method of taking derivatives of composite functions. As such, care should be taken to provide ample time for ALL students to engage actively with the requirements of the task, allowing some who may have missed aspects of earlier work the opportunity to build new and deeper understanding.

- *This activity can serve to consolidate earlier work on differentiation. It offers a suitable introduction to derivatives of more difficult functions.*
- *Begin by reviewing the method of differentiation of first principles, both graphically and symbolically, and methods of differentiation of the standard function forms.*
- *The screenshots on pages X–X (top) demonstrate expected student results. Refer to the screenshots on page X (bottom) for a preview of the student .tns file.*
- ***To download the .tns file, go to http://education.ti.com/exchange and enter "XXXX" in the search box.***

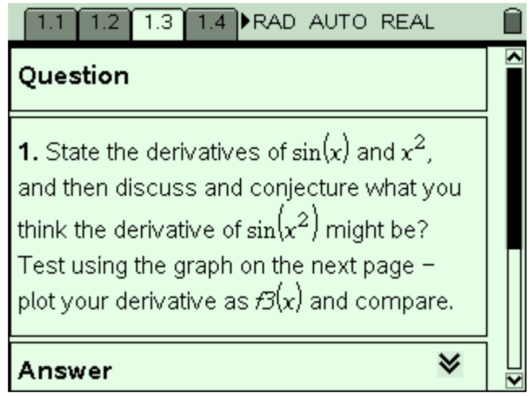
Classroom Management

- *This activity is intended to be **teacher led.** You should seat your students in pairs so they can work cooperatively on their handhelds. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds, although the majority of the ideas and concepts are only presented in **this** document; be sure to cover all the material necessary for students' total comprehension.*
- *Students can either record their answers on the handheld or you may wish to have the class record their answers on a separate sheet of paper.*

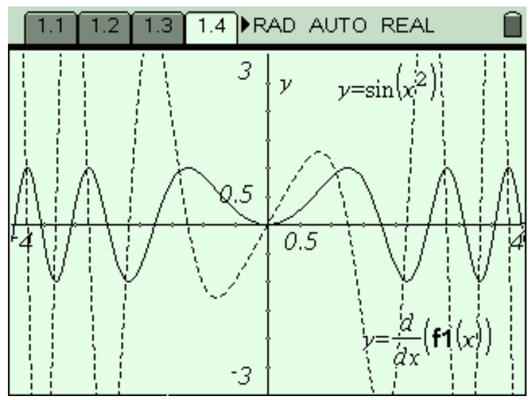
TI-Nspire™ Applications

Calculator, Notes, Lists & Spreadsheet and Graphs & Geometry.

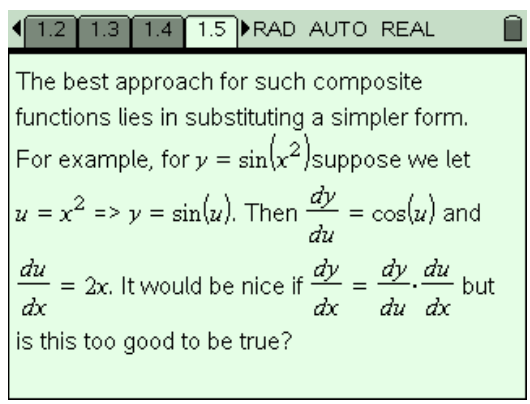
Step 1: Begin with discussion and review of both first principles methods and of the derivatives of standard function forms. Ensure that students are comfortable with these and then challenge them to consider more difficult forms – in this case, composite functions of the form $y = f(g(x))$.



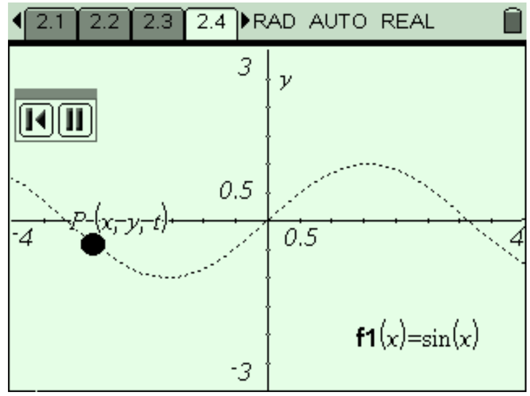
Step 2: A graphical approach is quite appropriate here as a means by which students may make and test conjectures – the device shows the function and the graph of its derivative – students try their own ideas and readily see whether these are correct.



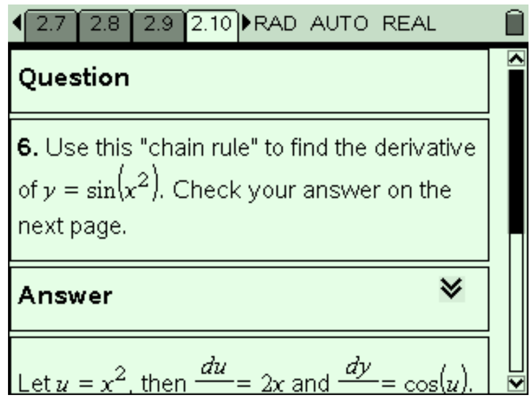
Step 3: Students will probably have little trouble coming up with the chain rule, nor accepting that it is valid, but they should be encouraged to question and check through the justification of this method – for this they need to go back to fundamental ideas of limits and whether these can be treated "like fractions".



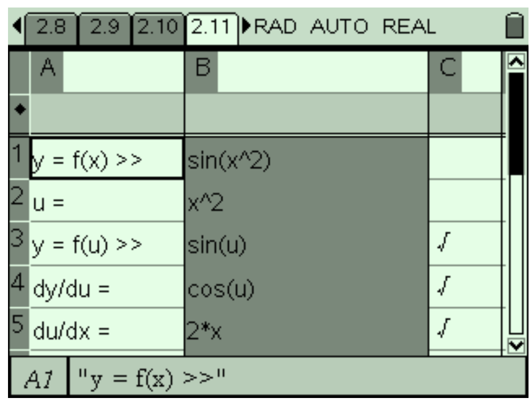
Step 4: As part of this investigation, the introduction of parametric forms – especially in the sense of adding a “time variable” to the graphing of a function – sets the scene for much later work. It would be well to allow students time for a thorough treatment of this concept. They should also be expected to put these understandings into their own words.



Step 5: Having derived the chain rule for composite functions, it remains only for students to take the time to consolidate its application, and worked examples are provided.

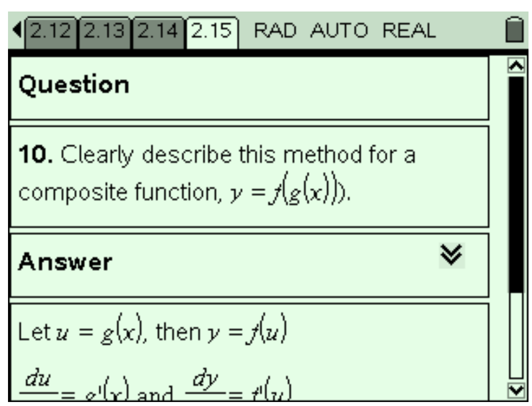


Step 6 An **algebraic spreadsheet** is available to support students in working through the process: students supply each step, which is checked for algebraic equivalence.



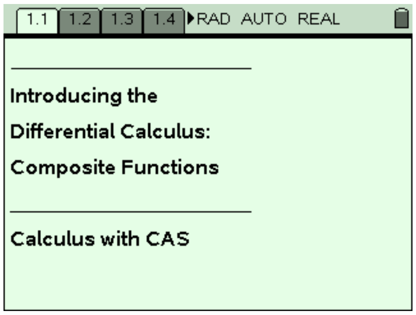
This scaffolding tool may be used as much or as little as desired, noting that it does offer a model for a well-structured worked solution for such questions.

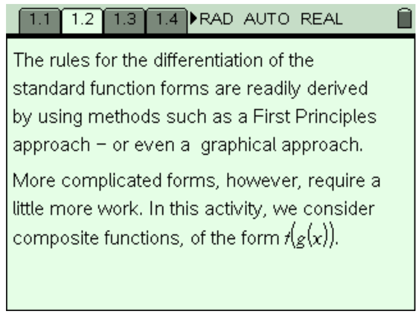
Finally, after working through several examples, students should be encouraged to apply these ideas to the general case.

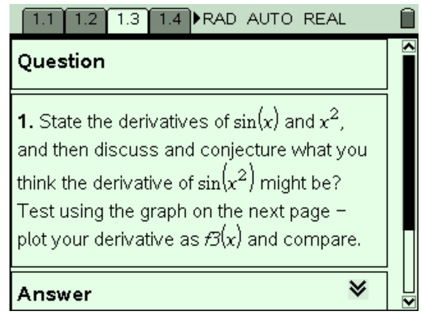


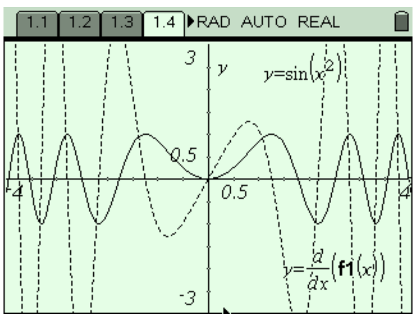
Introducing the Differential Calculus: Composite Functions – ID: XXXX

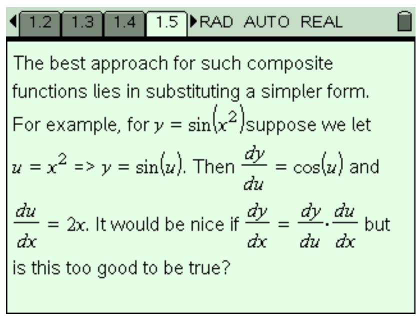
(Student)TI-Nspire File: *CalcActXX_Composite_Functions_EN.tns*

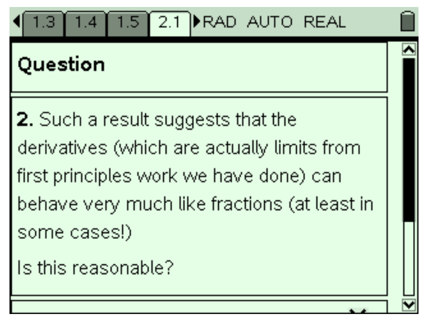


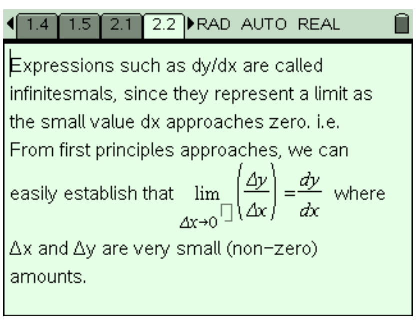


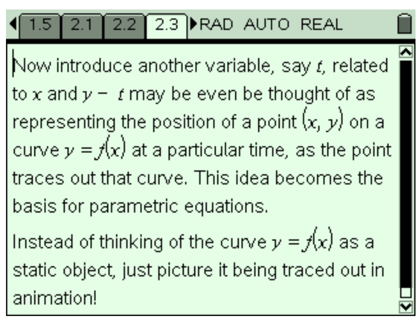


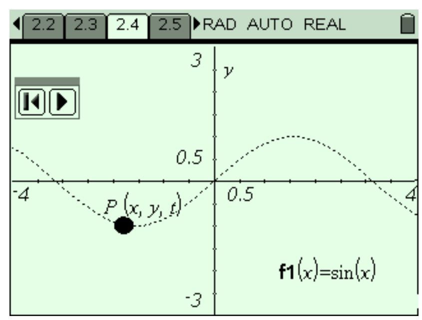


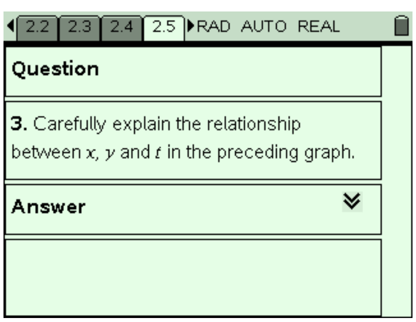


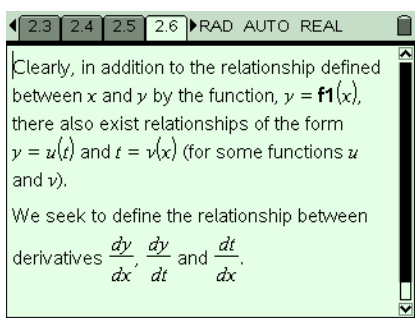


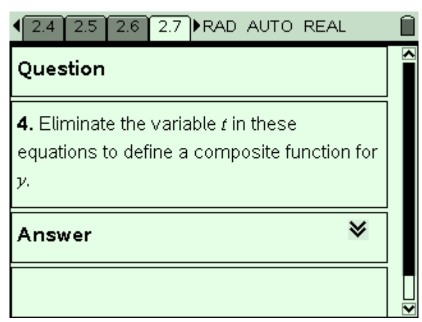


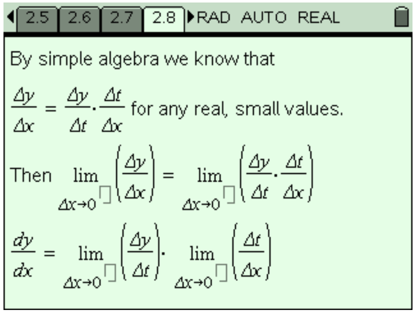


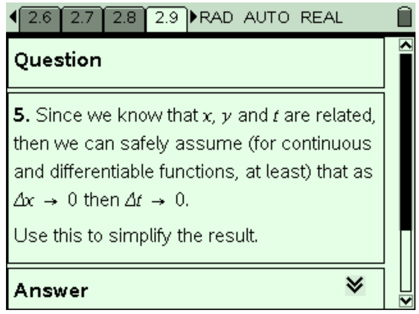


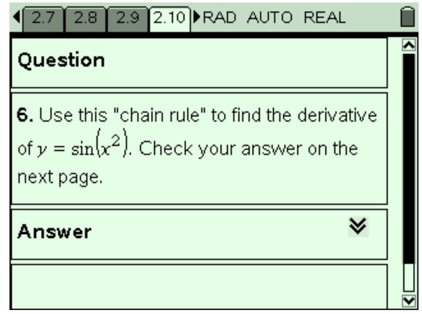


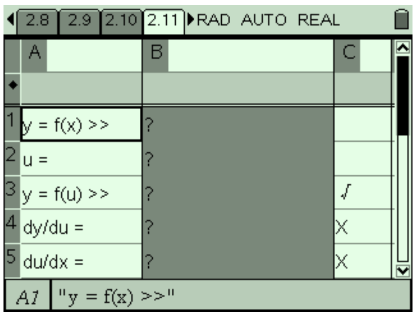


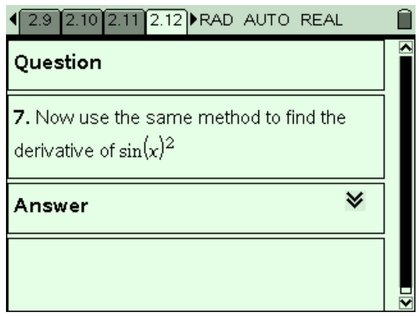


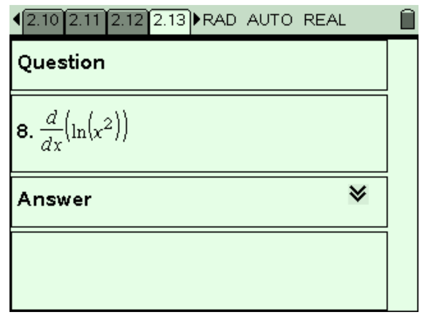

2.5 2.6 2.7 2.8 RAD AUTO REAL
By simple algebra we know that
$\frac{\Delta y}{\Delta x} = \frac{\Delta y}{\Delta t} \cdot \frac{\Delta t}{\Delta x}$ for any real, small values.
Then $\lim_{\Delta x \to 0} \left(\frac{\Delta y}{\Delta x}\right) = \lim_{\Delta x \to 0} \left(\frac{\Delta y}{\Delta t} \cdot \frac{\Delta t}{\Delta x}\right)$
$\frac{dy}{dx} = \lim_{\Delta x \to 0} \left(\frac{\Delta y}{\Delta t}\right) \cdot \lim_{\Delta x \to 0} \left(\frac{\Delta t}{\Delta x}\right)$

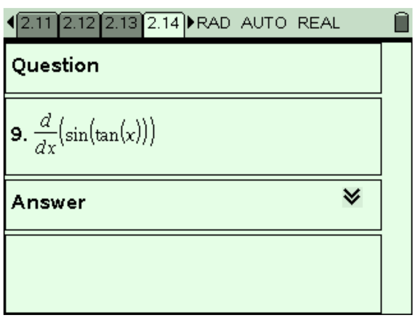

2.6 2.7 2.8 2.9 RAD AUTO REAL
Question
5. Since we know that x, y and t are related, then we can safely assume (for continuous and differentiable functions, at least) that as $\Delta x \to 0$ then $\Delta t \to 0$.
Use this to simplify the result.
Answer

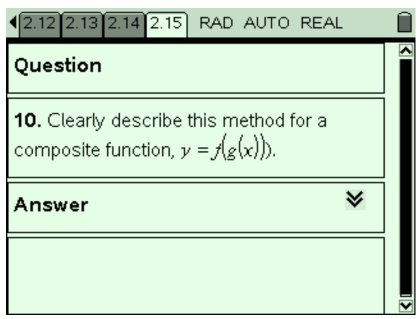

2.7 2.8 2.9 2.10 RAD AUTO REAL
Question
6. Use this "chain rule" to find the derivative of $y = \sin(x^2)$. Check your answer on the next page.
Answer


2.8 2.9 2.10 2.11 RAD AUTO REAL
A
B
C
1 y = f(x) >> ?
2 u = ?
3 y = f(u) >> ? √
4 dy/du = ? X
5 du/dx = ? X
A1 "y = f(x) >>"


2.9 2.10 2.11 2.12 RAD AUTO REAL
Question
7. Now use the same method to find the derivative of $\sin(x)^2$
Answer


2.10 2.11 2.12 2.13 RAD AUTO REAL
Question
8. $\frac{d}{dx}(\ln(x^2))$
Answer


2.11 2.12 2.13 2.14 RAD AUTO REAL
Question
9. $\frac{d}{dx}(\sin(\tan(x)))$
Answer


2.12 2.13 2.14 2.15 RAD AUTO REAL
Question
10. Clearly describe this method for a composite function, $y = f(g(x))$.
Answer

