QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

IMPROVING ON EXPECTATIONS: PRELIMINARY RESULTS FROM USING NETWORK-SUPPORTED FUNCTION-BASED ALGEBRA

Walter Stroup, Lupita Carmona & Sarah M. Davis
The University of Texas at Austin, USA
wstroup@mail.utexas.edu

This paper reports preliminary analyses comparing results on the state-administered 8^{th} Grade and 9^{th} Grade algebra Texas Assessment of Knowledge and Skills (TAKS) for a treatment and a control group. The treatment group consisted of 127 students from algebra classes at a highly diverse school in central Texas taught by two relatively new teachers using a network-supported function-based algebra (NFBA) approach as integrated with the ongoing use of an existing school-wide algebra curriculum. The control group was comprised of 99 students taught by two more-senior teachers in the same school using only the school-wide algebra curriculum. The intervention consisted of implementing 20-25 class days worth of NFBA materials over an eleven-week period in the spring of 2005. Because the students were not randomly assigned to the classes, the study is a quasi-experimental design. Using a two sample paired t-Test for means, statistically significant results for the treatment group (p-value one tail = 0.000335 $> \alpha = 0.05$) were obtained. We can conclude the NFBA intervention was effective in improving outcomes related to learning the algebra objectives assessed on the 9^{th} Grade TAKS.

1.0 Introduction

To date, the multiple-strands based approach to curricula promoted by the National Council of Teachers of Mathematics (1989, 2000) has not displaced the single-strand Algebra I course as gatekeeper in the educational system of the United States. If anything, the standard, "stand-alone", Algebra I course is now even more central at many levels, including in state curricula (e.g., minimum course requirements and exit exams) and in nationally administered tests (e.g., the new SAT tests). As a result, improving student outcomes related to the content of the traditional Algebra I curriculum is, perhaps, the single most strongly felt need relative to secondary mathematics education. Given the raised expectations regarding introductory algebra, we look to ask if there are ways of systematically improving on expected student outcomes in ways that move beyond the current overemphasis on addressing performance shortcomings with remediation? Our study looks to move in this direction. As illustrated by the results for our control group, past student performance on state-administered tests tends to be predictive of future testing outcomes. In our effort to identify approaches that are likely to improve expected student outcomes, not maintain them, we compared paired 8^{th} and 9^{th} Grade TAKS results for the students in our study and asked the question: Do the students in our treatment group outperformed their peers in the control group on the algebra objectives tested on the state administered, ninth-grade, Texas Assessment of Knowledge and Skills (TAKS)? Did the network-supported function-based algebra intervention have the effect of improving on expected student outcomes?

Our intervention centered on the use of function-based algebra as supported by generative activity design in a next-generation classroom network technology (i.e., the TI-Navigator 2.0 network combined with classroom sets of TI-84 Plus calculators). We call this approach network-supported function based algebra (NFBA). After providing some background for our study we report our results. Because the students were not randomly assigned, the study is based on a quasi-experimental design. Using a two sample paired t-Test for means, statistically significant results in outcomes for the treatment group (p-value one tail = 0.000335 $> \alpha = 0.05$) were obtained.

2.0 Background

There are three strands of analysis that are brought together in framing our study: (1) using function-based algebra (FBA) in a way that speaks more directly to the *structural* aspects of a standard introductory algebra curriculum, (2) situating this version of a function-based approach relative to *generative activity design* as supported by the capabilities of next-generation classroom networks (Stroup, Ares & Hurford, 2005) and (3) explaining our use of performance on previous high stakes mathematics testing to evaluate the effectiveness of the algebra-specific interventions implemented for this study.

Function-Based Algebra Revisited – Emphasizing Mathematical Structure

In ways that highlight the idea of function, affordable technologies like the graphing calculator have long been recognized to have the potential to substantively alter the organization of teaching and learning algebra concepts. Indeed, a number of approaches to pursuing function-based algebra (FBA) are discussed in the research literature (for an overview see Kaput, 1995). Many of these approaches were developed as part of an ambitious, and still ongoing, effort to fundamentally reorganize school-based mathematics to focus on *modeling*. For curricula, this would mean that the formal set-theoretical approaches to defining function that had come to be associated with the "new math" movement would be downplayed and largely replaced by an approach highlighting how functions can be used to model co-variation – i.e., how one variable is related to, or co-varies with, another variable. Computing technologies like the graphing calculator were to support significant engagement with, and movement between, *representations of functions* in symbolic, tabular and graphical forms. Indeed a technology-supported engagement with these "multiple representations" of functional dependencies, especially as situated in motivating "real world" contexts, has come to typify both *what function-based algebra is* and *why function-based algebra it is expected to be effective* with learners.

In the United States this modeling-based approach to FBA informed the development of the "standards-based" mathematics curricula funded by the National Science Foundation and then incorporated into various levels of "systemic reform" initiative also supported by NSF. These systemic reform initiatives, anticipating the language associated with the more recent No Child Left Behind legislation, were to "raise the bar" and "close the gaps" in student performance. The significance of this modeling-focused alignment notwithstanding (e.g., the State Systemic Initiative in Texas played a considerable role in the State-wide adoption of graphing technologies for algebra instruction and assessment), in day-to-day practice a modeling-focused approach to FBA has fallen well short of displacing much of what still constitutes the core of traditional algebra instruction. In part, the feedback from educators seems to be that as powerful as "real world applications" might be in motivating some students, the "bottom line" is that abstractions and formalisms are what continue to be emphasized on standardized exams and thus are what teachers feel considerable pressure to engage. Among school-based educators who are feeling enormous pressure to improve testing outcomes, modeling-based FBA is simply not seen as sufficiently helpful in addressing the core "structural" topics of a standard algebra curriculum. In framing our study, however, it is important to underscore that this perceived shortcoming is *not* a limitation in the potential power of using a function-based approach, but is only a limitation in a particular implementation of FBA that is, itself, principally motivated by the goal of making modeling the overarching focus of school-based mathematics (and far less by improving outcomes related to learning introductory algebra). For our study we take the strong position that while emphasizing modeling should continue to be important, a function-based approach also has enormous potential to improve student understanding of the structural aspects of introductory algebra. To make this point both with teachers and in our materials development, we have found it helpful to advance the following deliberately provocative, but still sincere, claim: *When viewed through the lens of a larger sense of what FBA can be, nearly 70% of a standard introductory algebra curriculum centers on only three big topics*. These three big topics are: *equivalence* (of functions), *equals* (as one kind of comparison of functions), and a systematic engagement with aspects of *the linear function*. This approach, as it is to be investigated in this study, builds on ideas associated with FBA introduced by Schwartz and Yerushalmy (1992) (see also Kline, 1945).

A major strength of this more structurally-focused, function-based algebra is that it allows for a consistent interpretations of both *equivalence* and *equals* in ways that students can use to understand the seeming ambush of “rules for simplifying” and “rules for solving” typically presented early-on in a standard algebra curriculum. If the expression $x + x + 3$ is *equivalent* to the expression $2x + 3$, then the function $f(x) = x + x + 3$ and the (simplified) function $g(x) = 2x + 3$, when assigned to Y1 and Y2 on the calculator, will have graphs that are everywhere coincident. They will also have paired values in the tables that are, for any values in the domain, the same. Students will say “the graphs” are “on top of each other.” This “everywhere the same-ness” associated with equivalence then will be readily distinguished from *equals*, as just one kind of comparison of functions. Equals comes to be associated with the value(s) of the independent variable where the given functions intersect (and > is associated with where one function is “above” another; < where one is “below”). The students will understand from looking at the graphs that the function $f(x) = 2x$ and the function $g(x) = x + 3$ are clearly *not* equivalent (they are not everywhere the same). But there is one value of x where these functions will pair this x with the same y-value (the students will say there is one place where the functions are “equal” or “at the same value”). Graphically, equals is represented as the intersection in a way that is quite general and that readily extends beyond comparisons of linear functions (e.g., $-x^2 + 2x + 8 = x^2 – 4x + 4$).

This distinction between equivalence and equals is helpful because in a standard, non-function based, algebra curriculum rules for simplifying expressions and rules for solving systems of equations are introduced very near each other and, not surprisingly, often become confounded. In addition students will feel like they have no ready way of checking their results, other than asking the teacher. In marked contrast, using a function-based approach, as supported by the use of a combined graphing, tabular and symbolic technology like a graphing calculator, students can readily “see” the difference between these ideas and can use these insights to make sense of results from “grouping like terms” as distinct from “doing the same thing to both sides”. This then allows the students to test their own results, using the technology, for either simplifying or solving. For simplifying they can ask themselves if the resulting simplified function is everywhere “the same” as the given function? For solving systems of linear equations they can ask did their attempts to “do the same thing” to the linear functions on both sides of the equation preserve the solution set (i.e., the x-value at the intersection)? Having students be able to distinguish and make sense of these two core topics in a standard algebra curriculum is significant and illustrates the power of FBA to help with structural aspects of a standard Algebra I curriculum. These ideas were emphasized in the materials we developed.

Of course, a modeling-oriented approach to FBA can be helpful in supporting student understanding of the third of the big three topics: a systematic engagement with aspects of the linear function. But herein we want to continue to illustrate some elements of a less modeling-centric engagement with FBA. As a result, we will illustrate implementing aspects of studying linear functions using generative activity design as supported by new network technologies. The effectiveness of this structural approach to FBA, without network capabilities, has begun to be established (cf., Brawner, 2001). We now move on to consider the role new network technologies can have in further enhancing function-based algebra.

Supporting Generative Design with TI-Navigator 2.0™

Briefly, generative design (cf. Stroup, Ares & Hurford, 2005) centers on taking tasks that typically converge to one outcome, e.g., “simplify 2x + 3x”, and turning them into tasks where students can create a space of responses, e.g., “create functions that are the same as f(x) = 5x.” The same “content” is engaged for these two examples, but with generative design a “space” of diverse ways for students to participate is opened up, and the teacher, based on the responses, can get a “snapshot” of current student understanding (so, for example, if none of the functions the students create to be same as f(x) = 5x involve the use of negative terms, the teacher can see in real time that students may not be confident with negative terms and can use this information to adjust the direction of the class). To illustrate how generative design and NFBA can help with the third of the three core topics in a standard algebra curriculum, we’ll briefly sketch some of the activities we used in our intervention.

The Navigator 2.0™ system allows students to move an individual point around on his/her calculator screen and also have the movement of this point, along with the points from all the other students, be projected in front of the class. In one introductory activity students are asked to “move to a place on the calculator screen where the y-value is two times the x-value”. There are many places the students can move to in satisfying this rule, and this is what makes the task generative. Often the majority of the points are located in the first quadrant and this gives the teacher some sense of where the students are in terms of confidence with negative x- and y-values. This exploration of a rule for pairing points does describe a function and this approach to creating functions is not dependent on co-variation (indeed, should the teacher want to discuss it, this activity can be used to highlight a set-theoretic approach to defining a function). After observing that “a line” forms in the upfront space, all the points then can be sent back to the students’ calculators and can act as “targets” for creating different functions on their calculators (in Y1= , Y2 =, etc.) that include (“go through”) these points. Then the students can send up what they consider their “most interesting” functions. A space of often quite interesting equivalent expressions is thereby created and shared in the upfront-space. To further explore ideas related to linear functions, students also can be given a rule like “move to a place where your x-value plus your y-value add up to 2.” Again a “line” forms but now when the points are sent back to calculators, the students are pushed to explore ideas related to moving from a linear function in standard form (i.e., x and y summing to 2) to the same function being expressed in slope-intercept form (the form the students must use on the calculators in order to send a function through the points). Again, these and many other structural ideas found in a standard algebra curriculum can be explored using network-supported function-based algebra.

Improving on Expectations

As is mentioned earlier, the intent of the No Child Left Behind Legislation in the United

States is to “raise the bar” of what is expected of all students and to “close the gaps” in performance of currently underserved populations. The effort is to be forward looking as higher expectations and measurable progress are to present a tight system of positive feedback in driving demonstrable improvement in educational outcome. Even in a time of heightened political partisanship in the United States, this vision is still seen as compelling and potentially unifying. But as systems theorists (cf., Senge, 1994) are quick to remind us, a challenge in implementing major structural reforms is ensuring that the intended dynamics meant to both characterize and drive the change – in this case positive forms of feedback between raised expectations and measurable outcomes – are not themselves overwhelmed by unanticipated and unintended consequences of what may be well-intending implementation. Relative to learning algebra, one widely used strategy is to preserve the current approaches to teaching algebra and then address shortcomings in student outcomes with remediation. The problem is that remediation, almost by definition, is an inherently *backward* looking and *corrective* strategy. Its role is to fix what is seen as broken, not to drive forward progress. Relative to mathematics education, with more and more effort at each grade level (especially in underperforming schools but also in lower “tracks” in higher performing schools) spent on correcting for past or anticipated shortcomings (e.g., “reviewing” material not mastered from previous years, funding remediation classes during the school year and/or in the summer, or spending considerable class time practicing test-taking skills) attention to proactive strategies (strategies that improve on expected outcomes) is being compromised. From a structural point of view a *positive* feedback loop – like that between raised expectations and measurable progress found at the heart of the NCLB legislation – needs practical *forward looking* and *forward acting* strategies to be effective.

To make the case for NFBA being an example of one such strategy, we look to compare our treatment group outcomes on the 9th Grade TAKS algebra objectives relative to what might be expected based on previous performances on the 8th Grade TAKS.

3.0 The Study

Research Question: Does the network-supported function-based approach outlined above improve the performance of the treatment group in statistically significant ways relative to the performance of control group peers?

The Sample

The study participants were 226 students from a diverse high school in central Texas. All the students were enrolled in “non-repeater” (non-remedial) sections of Algebra I and nearly all the students were in ninth grade. Two relatively junior teachers were assigned by the department chair to the experimental group and two more-experienced teachers were assigned to the control group: 127 students were in the treatment group and 99 students were in the control group.

Activities

In their Algebra I class, the treatment groups used a NFBA over nine weeks of instruction in the spring of 2005. The treatment and control groups kept their curricula on the same topics but the experimental group used the NFBA materials, on average, approximately two days a week.

Methods

The raw 8th and 9th grade scores for the State-administered TAKS tests were obtained for the students participating in the study. The 8th grade TAKS was taken before the intervention and the 9th grade TAKS scores for the algebra objectives were collected after the intervention.

Analyses

The raw scores on the 8th grade TAKS and the algebra items on the 9th grade TAKS were converted to percent-correct results. Table 1 and Figure 1 show the comparison of the means for the 8th and 9th grade TAKS for the treatment and control groups.

	Treatment	*Control*
8th GRADE TAKS SCORES	53.8	56.4
9th GRADE TAKS SCORES (Algebra Items)	57.9	56.1

Table 1. Mean TAKS Score Results for Treatment and Control Groups

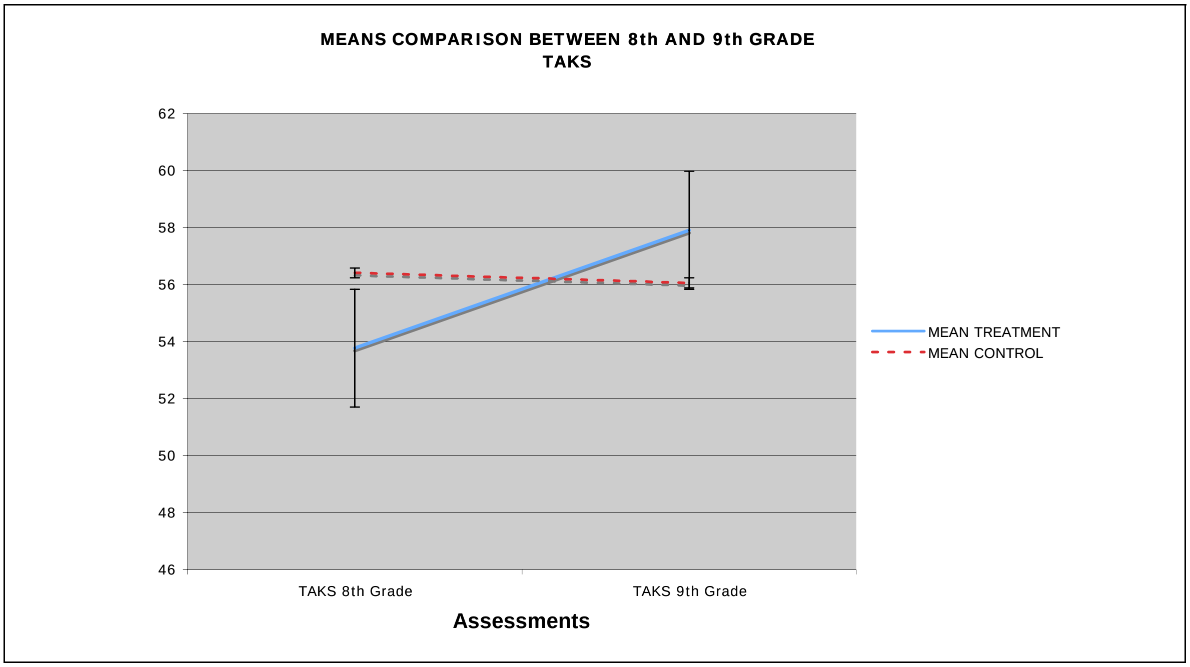

Figure 1. Means Comparison of TAKS Results

We implemented *two* approaches to study changes attributable to the intervention: (1) comparing the student performances between the treatment and control groups first before the intervention (8th Grade TAKS) and then after the intervention (9th Grade TAKS for Algebra Items); (2) comparing the *paired* student performances before and after the intervention for the control group and then the treatment group.

First approach: Comparison Between Treatment and Control Group Results First Before and then After the Intervention

Using this first approach no statistical difference was found between treatment and control groups' results either before or after the intervention. The graph in Figure 1, however, suggests a need for additional analyses. On the graph it is clear that, although no statistically significant differences were found using the given methods, the treatment group started off about 2% lower than the control group on the average 8th grade TAKS scores. Then after the intervention the plot of the 9th grade results shows that the students in the control group maintained almost the same average on the 9th grade TAKS score (the dotted line is almost completely horizontal, showing no change) whereas the treatment group's graph shows appreciable improvement, approximately

4%. This suggests the possibility of comparing *paired* scores before and after intervention, for the control and the treatment groups separately, using a two-sample paired t-Test for the means.

Second approach: Comparison of Paired TAKS Scores Before and After Intervention for the Control Group and then for the Treatment Group

We performed a two sample paired t-test for means for the *control group* to look for changes in TAKS scores before and after intervention. As might be suspected from examining graph for the control group in Figure 1, the results of the t-test show no evidence that the means for the control group before and after the intervention are different (p-value one tail = 0.402 > α = 0.05). As a result we can conclude that the students in the control group maintained consistent averages for the 8th grade and 9th grade algebra TAKS scores. There was no statistically significant improvement. This result is consistent with the sense that absent changes in practice, performance in one year is likely to predictive of performance in the next.

When we implemented a two sample paired t-test for the means for the treatment group, the results (p-value one tail = 0.000335 > α = 0.05) provided strong evidence of differences in means before and after intervention. This suggests the students in the treatment group improved significantly in paired results on the 8th and 9th Grade TAKS. Considering that the treatment and control groups were comparable, that no improvement was shown for the paired 8th and 9th grade TAKS scores in the control group, and that improvement was shown for the paired 8th and 9th grade TAKS scores in the treatment group, we have strong evidence to say that this improvement in TAKS scores was an effect of the intervention. Network-supported function-based algebra does appear to have been proactively effective in improving student outcomes.

References

Brawner, B. F. (2001). *A function-based approach to algebra: Its effects on the achievement and understanding of academically-disadvantaged students*. Unpublished doctoral dissertation, The University of Texas at Austin.

Kaput, J.J. (1995). A research base supporting long term algebra reform? In D.T.Owens, M.K.Reed, & G.M.Millsaps (Eds.), *Proceedings of the Seventeenth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (vol. 1, pp. 71–94). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education. (ERIC Document Reproduction Service No. ED 389 539).

Klein, F. (1945). *Elementary mathematics from an advanced stand-point: Arithmetic, algebra, analysis*. New York: Dover.

Schwartz, J. & Yerushalmy, M. (1992). Getting students to function in and with algebra. In G. Harel & E. Dubinsky (Eds.), *The concept of function: Aspects of epistemology and pedagogy*. MAA Notes, 25, (pp.261 - 289), Washington, DC: Mathematical Association of America.

Senge, P. M. (1994). *The fifth discipline: The art and practice of the learning organization*. New York: Doubleday.

Stroup, W. M., Ares, N., & Hurford, A. (2005). A Dialectic Analysis of Generativity: Issues of Network Supported Design in Mathematics and Science. *Journal of Mathematical Thinking and Learning, 7*(3), 181–206.