INTEGRATING TECHNOLOGY IN SENIOR MATHEMATICS

Non-Calculus Mathematics

Stephen Arnold

Dr Stephen Arnold has been a teacher for 25 years, and has been actively using classroom technology for over 15 years. In addition to teaching mathematics, science and technology in a range of New South Wales high schools, Stephen has been involved in teacher education at James Cook University and the University of Newcastle. For the past five years, Stephen has been applying what he has learned by developing new ways of integrating technology into the classroom.

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Maximising the Benefits of Graphic Calculators
For Non-Calculus Mathematics

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INTRODUCTION

Recently I taught a senior non-calculus Mathematics class populated by those students who have always found their mathematics less than a satisfying educational experience. These are the students that the Board of Studies would claim should not be doing mathematics at all for their final examination. And yet there they are – and few would drop this subject at the end of Year 11, no matter what I recommended to them or to their parents. This situation, I believe, is repeated in many schools across the nation: students who previously may have been well-suited to a less challenging course, now facing a subject that is in many respects way beyond them.

What are our responsibilities as teachers in this regard? I counselled the students (and their parents), pointing out that they need not be doing mathematics at all, that this course is a very challenging one, even for many students who had performed well previously… even that they should be presenting their best subjects for their HSC – and this is unlikely to be one of those. And yet, for a variety of reasons, they sat there on the other side of those desks and looked to me to prepare them as best I could for the examination at the end of year 12.

I decided to do something I had not done before – to take full and unashamed advantage of the available technology; in this case, Texas Instruments TI-83 graphic calculators, which these students had owned since Year 9, but which had not been utilised to their full potential up to this point. And even though I have been using the technology for a long time now, I was surprised at many of the advantages that this technology offered my students. I have spoken often to teachers about technology potentially offering a more level playing field for all students – over those two years I began to see what this could mean in reality.

The General Mathematics course for the New South Wales Higher School Certificate is the first senior mathematics course in this state which is explicit in its requirement for the use of technology in support of effective teaching and learning of the knowledge, skills and competencies required at this level. The syllabus assumes the use of spreadsheets, graphic calculators and other technological tools, deliberately incorporating within each unit outline a statement specifying appropriate technological applications for that content area.

The significant importance accorded to the use of technology in fulfilling the requirements for this course is further demonstrated in allowing student access to graphic calculators for their Higher School Certificate examination. While the intention of the Board of Studies was clear in that students without access to such tools would not be disadvantaged in their examination, it is the key premise of this book that there <u>are</u> clear and substantial benefits for students who learn with these powerful mathematical tools.

Within the context of classroom teaching and learning, the evidence is clear worldwide that the learning experiences offered through appropriate and thoughtful use of graphic calculators and other technological tools are significantly enhanced. Student attitudes are inevitably found to be more positive as they take an active responsibility for their own learning, empowered by access to supportive devices, which scaffold their learning.

Within the examination room, students who have learned how to access the many powerful statistical, algebraic and financial features available on graphic calculators cannot help but be advantaged over peers using only scientific calculators.

This book supports teachers in making best use of the many appropriate features offered by graphic calculators as they apply to many senior Mathematics courses. In addition to the powerful capabilities available on all standard graphic calculators, students and schools with access to the Texas Instruments TI-83 Plus calculators have a further advantage in the growing range of software Apps, which have been developed for these devices. These extraordinary software tools are designed in close association with teachers, ensuring that they are appropriate and significant in their support of the learning of mathematics. While these Apps may not be used in external examinations in some states, their use in teaching and learning offers much to further advantage those with access to them. From full-featured spreadsheets to interactive geometry, interactive learning sequences and simulation tools, graphic calculators have become something much more than they were! While access to computer-based tools for learning should still be a part of the teaching and learning of such a course, the availability of Apps reduces the demand on already overcrowded school computer facilities, while making such powerful instructional aids available to students in class and at home.

Finally, it should be noted that the range of mathematical skills and experiences brought to such courses has become much greater than in the past. Students range in ability from those who performed successfully at an advanced level to those who struggled at a much less challenging level. For those less able to cope with the mathematical demands of such a course, then, access to graphic calculators potentially does offer a more level playing field, scaffolding students well beyond the simple numerical support offered by scientific calculators. As this book demonstrates, there are many simple ways in which students can use these tools intelligently in order to achieve at a level substantially higher than would have been possible without such assistance.

If your students have access to graphic calculators, then this book shows ways to maximize the benefits of their use, both for classroom and examination.

Consider the possibilities!

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Content Areas	**Applicable Apps**	**Page**

ALGEBRAIC MODELLING

1. FINANCIAL MATHEMATICS

The graphic calculator is well-supported in its financial capabilities. With the addition of a full spreadsheet, there is little that cannot be done in this regard. This section begins with an introductory overview of the TI-83 Plus capabilities, using a financial question as the springboard. It assumes little or no prior knowledge of the calculator's functioning and is a suitable first activity.

Learning Activities

Learning Activities	Content Reference	App
1. Looking around the TI-83 Plus	Investing money	
2. Comparing Interest	Investing money	
3. Budgets by spreadsheet	Earning money	CellSheet™
4. Staying interested	Investing money	CellSheet™
5. Sharing the wealth	Investing in Shares	CellSheet™
6. Loans and other expenses	Annuities/Loans	Finance
7. A balancing act	Annuities/Loans	Finance
8. Some nice things to know	Credit/borrowing	Finance

# LEARNING ACTIVITY 1: LOOKING AROUND THE TI-83 PLUS

AIM   To provide an introduction to some of the main features of the graphing calculator.

METHOD   The same financial problem will be approached in four different ways, each highlighting a different feature of the TI-83 Plus.

PROCESS   The problem: *If I invest $5 000 at a rate of 5% pa, how long before my investment doubles?*

SETTING UP

Since we are working with money, we might begin by pressing the **MODE** key (next to the yellow **2ⁿᵈ** key) and setting the mode to 2 decimal places as shown (Use the blue arrow keys to move down to the second row and across to the number **2**, then press **ENTER**). This will automatically present all results correct to 2 decimal places and avoid any rounding errors. Press **CLEAR** or **2ⁿᵈ–MODE (QUIT)** to return to the home screen.
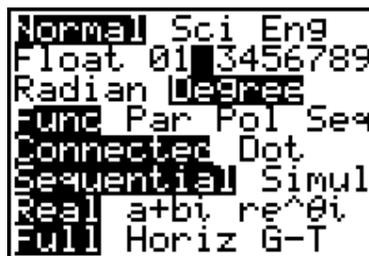
1. **Method 1: Home Screen Calculations (1)**

   Enter the Principal (or Present Value), $5 000. Press **ENTER**.

   Press **x 1.05 ENTER** to multiply the last answer by 1.05 and so increase it by 5%. Press **ENTER** again and the last action (**ans x 1.05**) will be repeated, each time increasing the previous amount by 5%, as shown. Keep pressing **ENTER** until the amount passes $10 000.00. Did you remember to count the number of years it took?
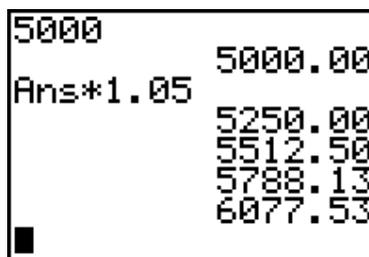
2. **Method 2: Home Screen Calculations (2)**

   This method stores values into variable names (using the **STO** key) and then uses the compound interest formula to evaluate the result.

   Enter **5000 STO-> P** (use **ALPHA – 8** for **P**); **5/100 STO-> R**. Use 10 years as our first guess, and enter **10 STO-> N.**
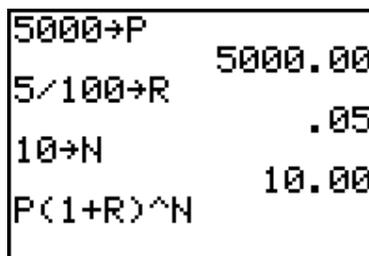
   Use the **ALPHA** key to enter the compound interest formula as it is given on the formula sheet. Press **ENTER**. The result is too small, so try some other values for **N** until you find the number of years for your investment to exceed $10 000. (**NOTE**: To recall a previous entry – such as the compound interest formula after entering a new value for **N** – press **2ⁿᵈ–ENTER** repeatedly to cycle back through all previous entries).

3. Method 3: Table of Values

The **Y=** key is the gateway to a new calculator environment. This is where we go when we wish to work with graphs and algebra. It is also the way to access the table of values. Press **Y=** and enter **P(1 + R)^X** into **Y1**. (**NOTE** we use **X** instead of **N** here, since this is the part we wish to find, the *variable*).
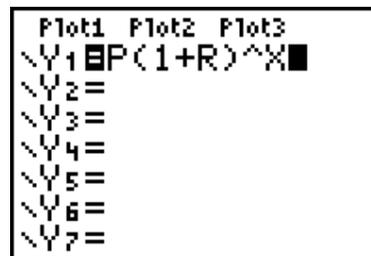
Now use the **TABLE** menu (**2ⁿᵈ-GRAPH**) to access the table of values, which shows the values for each year. Scroll down the years using the down arrow until you find when the investment first exceeds $10 000. This occurs between the 14th and 15th years.

The result may be found even more accurately using the **Table Settings** menu (**2ⁿᵈ-WINDOW**). As shown, set the table to start at 14 years and to increase, not by 1 year, but by 0.1 years.

The table now shows that the investment reaches $10 000 sometime between 14.2 and 14.3 years. You may continue to close in on your answer until you reach as accurate a result as you wish. In this case, that would appear to be just below 14.21 years (**NOTE: 0.21 x 12 = 2.52**, indicating about 2 and a half months.)

4. Method 4: Finance App

If you are using a TI-83 Plus, press the blue **APPS** key and choose the first App on the list: **1:Finance**. (If you are using a TI-83, then the **Finance** menu may be found above the x^{-1} key.) On the 83Plus, **Finance** is the only App which may not be removed: it is built into the calculator's operating system!
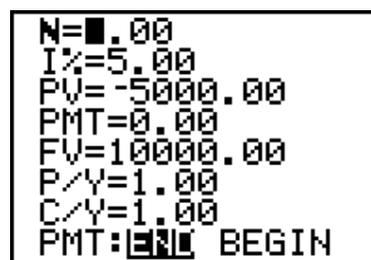
Choose the first option at the following screen (**1:TVM Solver…**) TVM stands for Time Value of Money, and provides an inbuilt financial calculator.
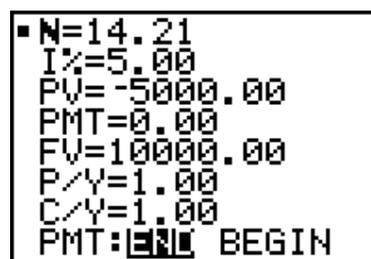
As shown, type **0** for **N** (*number of* payments), enter **5** for **I%** (*interest rate*), **-5000** for **PV** (the *present value*), **0** for **PMT** (there are no *payments* to be made in this question), **10000** for **FV** (*future value*), and set both **P/Y** (*payments per year*) and **C/Y** (*compounds per year*) to **1**. Arrow back up to **N**. (**NOTE** that **PV** and **FV** need to be of *opposite sign* since this is a *loan* calculator. Use the white **(-)** key for the minus sign – NOT the blue "take-away" key!)

In green above the **ENTER** key you will see the **SOLVE** command. Press **ALPHA-ENTER** to solve for **N** and read the answer. If you wish for a more accurate answer, simply go back to the **MODE** menu and change the decimal places setting to **FLOAT**. This gives a result of 14.20669908.

Four different approaches to the same problem. Each demonstrates a different aspect of the functioning of the calculator; each suggests different learning experiences and outcomes. Each will be explored in greater detail in Learning Activities to come.

LEARNING ACTIVITY 2: COMPARING INTEREST

AIM   *To build student understanding of the differences between simple and compound interest.*

METHOD   *Use both tables of values and graphs to allow students to compare the effects of different interest types upon the growth of an investment.*

PROCESS   The simple interest formula is normally presented as **PRN** (where **P** is the *principal* or *present value* of the investment, **R** is the interest *rate*, and **N** the *term* or *number of interest calculations*). This formula, however, produces only the amount of actual *interest* which results. To find the total *amount* of the investment, use the formula:

$$A = P + PRN$$

Here the total is given by adding the *interest* to the original *principal*. Note that this may be expressed differently by factoring *(think and talk about this!):*

$$A = P(1 + RN)$$

Students may note the similarity in structure with the compound interest formula:

$$A = P(1 + R)^N$$

This feature may be a useful way for students to remember the two formulas.

1. Press **Y=** and enter the following (**NOTE** we use **X** instead of **N**):

    Y1 = P(1+RX)
    Y2 = P(1+R)^X

2. Use **2nd-QUIT** to return to the home screen and store values for **P** and **R,** or use the values from the previous learning activity
    (**5 000 STO-> P, 5/100 STO-> R**).

3. View the table of values (using **2nd-GRAPH**).
It is now possible to directly compare the effects of simple and compound interest. Students should record their observations from the table of values and then to compare these observations with others.

4. After noting the growth under both forms over the first 20 years, students should set up their graph screen by pressing the blue **ZOOM** key and choose **0:ZOOMFIT** (this may be found by scrolling *down* through the Zoom options, or by scrolling *up* to come up from the bottom!). This automatically tries to create the optimum viewing screen which will suit the chosen functions based upon the *x*-values which were already present.

5. By pressing the blue **TRACE** key, it is possible to explore the numerical values on each graph (use the **UP/DOWN** arrows to switch between graphs, and the **LEFT/RIGHT** arrows to move along the values of each. Note that the equation for the active graph appears in the top left corner of the screen.
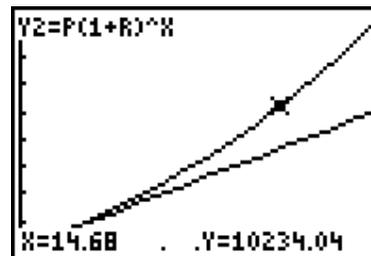
Now you are free to explore different values for the principal and the interest rate. Simply return to the home screen and store new values as desired.

LEARNING ACTIVITY 3: BUDGETS BY SPREADSHEET
CELLSHEET™ APP

AIM　　　*To introduce the basic functioning of a spreadsheet in setting up a simple budget.*

METHOD　*Set up a personal weekly budget, which will automatically update for changes.*

PROCESS　The spreadsheet is the centrepiece of software for all financial calculations. The *CellSheet™* App is a fully-functioning spreadsheet which is compatible with Microsoft® Excel®. Students may use a link cable and *TI-Connect* software to transfer spreadsheets between their calculator and a PC. Using the free *CellSheet™ Converter* software, this process is made even easier through the addition of a *TI-Tools* menu to their PC-version of *Excel®:* it is easy to create a spreadsheet on the computer and use this menu to send it straight to the calculator to be opened with *CellSheet™*.

For this Learning Activity, students will create a simple spreadsheet directly on their calculator.

1. Press the blue **APPS** key and choose the *CellSheet™* App.
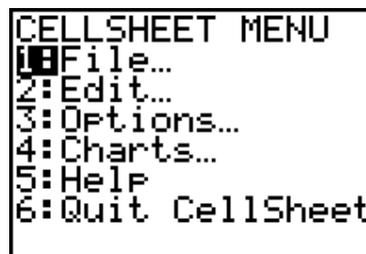
2. Press **ENTER** to read the Help Screen (shown above) and then again to open a spreadsheet. NOTE the **MENU** tab in the bottom right corner. Press the blue **GRAPH** key directly underneath to activate this and other "softkeys" that may appear within Apps. The five blue "graphing" keys directly underneath the screen often take on new roles in an App.

3. Choose the first option, **1:FILE…** and then option **3:New…** to create a new blank spreadsheet. You will be asked to name it. For this exercise we might call it "Budget". The first part of this name appears as shown in the top left corner of the new sheet.

4. As with any spreadsheet, the "cells" are named by referring to the letter of the column and the number of the row. Hence the highlighted first cell is **A1**. Use the blue arrows to move down to cell **A2.** The name of the cell appears at the bottom left of the screen, ready to receive values (text or numbers) or a formula for that cell.

5. To enter text (such as the days of the week), use the green **ALPHA** key to first choose the inverted commas (**"**) above the **+** key. Then type the letters you wish. In this case, enter the days of the week, beginning with Monday, as shown. To save time, press **2ⁿᵈ-ALPHA** at each cell, which locks the calculator into "ALPHA" mode until you leave that cell (either by pressing **ENTER** or using an arrow).

6. After entering "Sun", move down, leaving one cell blank and then enter **TOTAL** into cell **A10**. Move back to the top (To move quickly back to the top of the column, press **ALPHA** before you press the **UP** arrow).

7. Having entered our weekdays, move across the top row and enter our budget components: we will use headings such as Board, Lunch, Bus, Treats, Music (lessons), Movies and Other (just in case!). Decide what might be reasonable headings, and then enter appropriate amounts and when they might occur (for example, lunch might be $5 every day, bus fare might be $3 each school day, and perhaps $6 to go out on Saturday, music lessons could be $15 every Tuesday, and so on).

8. Assume that you pay your parents board each week on a Friday. Enter 0 into the first cell of each empty column (the first cell may not be blank), and 25 into **B6**. When entries have been made, move down to the **TOTAL** cell and move to the cell beside that (it should be cell **B10**). This will be the total for the "Board" or "Rent" column. To indicate that we are entering a formula, the entry must begin with an equals sign. For *CellSheet*™, we press the **STO->** key to produce this. Next we add the command to sum these values. To access this, choose **CATALOG** (**2ⁿᵈ-0**), press **S** (**LN**) and scroll down until you see the "sum(…" command. Press **ENTER** and this will be dropped into our cell. Finally, enter the range:

B2:B8. Note the use of the colon (:) to indicate the range from **B2** to **B8**. Close the bracket, press **ENTER** and *CellSheet*™ will add all the values from **B2** to **B8**.

9. Now, instead of having to do all that for each column, we will **COPY** that formula across the bottom of our sheet. Move to the cell **B10**, so that the total is highlighted as shown. To **COPY** we use **F3** (**ALPHA-ZOOM**). This causes the menu keys to change as shown.

10. Press **F1** (**ALPHA-Y=**) to select the range to be copied into, and then use the right-arrow to move across to the last column (in our case, that is row **H**). Press **F4** (**ALPHA-TRACE**) to **PASTE** the formula. The totals for each expense are given along row 10.

11. Move up to cell **I2** in order to total across for each day as well. Repeat step 8, summing from **B2** to **H2** for the first row. Then again use **F3, F1** and **F4** to copy and paste this formula into all the cells in column **I**. Notice the **ERROR** in **I9**, since this row has no entries (and, in particular, is blank in its first entry). Move up to this cell and press **DEL**.

BUDG	G	H	I
7	15		36
8		15	25
9			Error
10	15	15	**146**
11			
12			
I10: =sum(B10:H10)			[Menu]

12. We now have a functional weekly budget sheet, and can see that we are spending a grand total of $146 every week. We may use the day-by-day totals to isolate those days with the greatest expenses, and the column totals to see those activities which are more and less affordable. Changing any entry will automatically update the entire sheet. So now you are in a position to try to cut down those expenses and manage your budget effectively!

LEARNING ACTIVITY 4: STAYING INTERESTED
CELLSHEET™ APP

AIM   *To further extend students' skills in the use of a spreadsheet and their knowledge and understanding of simple and compound interest.*

METHOD   *Set up an automatic interest calculator using CellSheet™.*

PROCESS   Create a new spreadsheet, called **INTEREST**. We will use this, once again, to compare the differing effects of simple and compound interest, but in a more detailed way than previously. We will also create a spreadsheet which is completely controllable for all its values.

1. Set up the new sheet as shown. The only formula is the one displayed in **A6**. Notice that this formula is **=A5 + $B$3**. The **$** sign is accessed using **2ⁿᵈ-STO->** (or **RCL**) and indicates that this formula will always look precisely at the cell **B3**, no matter how or where it is copied to. This will become clearer after copying the formula in **A6** into the cells from **A6** down to **A15**. (Remember: **F3 -> F1 (Scroll down) -> F4**). As you arrow back up from **A15**, you will notice that the first cell reference changes at each cell (thus, the original **A5** becomes **A6**, **A7** and so on. This is called a **relative cell reference**. As the formula is copied down the column, the instruction of such a reference is to always look in the same *relative* position – in this case, look in the cell directly above for the value). The dollar signs indicate an **absolute** cell reference: this instructs the cell to always look for its value in the same place: in this case, cell **B3**.

2. Return to the top of the sheet. We will now create columns for *Simple Interest* (**S.INT**), *Simple Interest Total* (**S.TOT**, the sum of the principal and the interest earned), *Compound Interest* (**C.INT**) and *Compound Interest Total* (**C.TOT**).

3. For *Simple Interest*, we enter into cell **B5** (as shown), the formula **=$B$1*$B$2*A5**. Can you see why? The simple interest formula is **PRN**, the product of the **Principal** (or **Present Value, PV**), the interest **RATE** and the **TERM** (or **N**, number of interest periods). In this case, both **PV** and **RATE** are *absolute references* while **N** is a *relative cell reference*. **COPY** and **PASTE** this formula into the cells from **B5** to **B14**.

4. For *Simple Interest Total* we need only to add the interest found in column **B** to the original investment amount in **B1**. Hence, the formula will be **=$B$1 + B5**.

5. Next move to column **E**: we will complete the *Compound Interest Total* first, since the compound interest formula produces this amount. This formula is given by $P(1 + R)^N$, and so we enter: **=\$B\$1*(1 + \$B\$2)^A5**.

6. Finally, go back to cell **D5** and enter **=E5 - \$B\$1**, since the actual interest earned by the compound interest method will be the total less the original amount.

INTE	C	D	E
1			
2			
3			
4	S.TOT	C.INT	C.TOT
5	1000	0	1000
6	1050	50	1050

D5: =E5-$B$1   [Menu]

7. You have now created a fully automatic interest calculator. Changing any of the control values (**PV, RATE, STEP** or the first **N** value) and all the values in the sheet will update. Use the sheet to find the difference in interests for an investment of \$20 000 at 4.5%pa after 8 years. (**NOTE**: If you wished, you could now easily create a new column for this difference. **Note** also, to enter 4.5% simply enter it as 4.5/100 and let the spreadsheet do the calculation!).

INTE	A	B	C
4	n	S.INT	S.TOT
5	36	3000	23000
6	37	3083.3	23083
7	38	3166.7	23167
8	39	3250	23250
9	40	3333.3	23333

A9: =A8+$B$3   [Menu]

8. What about *monthly* interest calculations, *weekly* or even *daily* calculations? First, enter the interest rate over **1200** instead of **100**. Thus, for **5%pa monthly**, enter **5/1200**. Next, adjust the *starting value* of the term to whatever year you desire multiplied by 12. Thus, to find the monthly interest after 40 months (3 years and 4 months) enter **3 x 12** instead of **0** in **A5**, and then read off the required values!

```
CHARTS
1:Scatter…
2:Scatter Window
3:Line…
4:Line Window…
5:Bar…
6:Bar Window…
7:Pie…
```

9. Finally, the spreadsheet also allows graphical comparisons. Press **F5** (**MENU, GRAPH**) and choose option **4: Charts…** We will choose **3: Line Chart** for this information, completing the entries as shown below. We use the number of years (or interest terms) as our **Xrange** (**A5:A14**) and the actual interest amounts (rather than the total amounts) for our comparison (columns **B** and **D**). Choosing **DrawFit** allows the calculator to automatically choose a scale and axes which will display the data. This may be manually controlled from the **Charts** screen, option **4: Line Window**.

```
LINE CHART
XRange:A5:A14
YRange1:B5:B14
YRange2:D5:D14
YRange3:
Title:…P V COMP
AxesOn AxesOff
DrawFit Draw
```
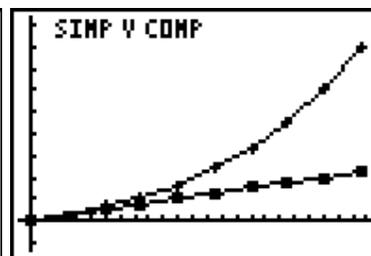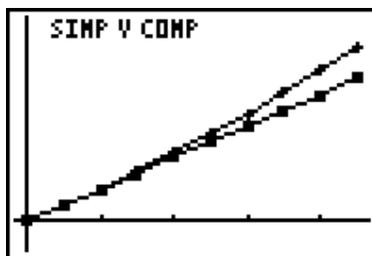
10. Press **CLEAR** to return to the spreadsheet, and change the step from **1** to **5**. In this way, we compare the differences, not over 10 years but over 50! All the previous settings are preserved, so simply return to **Charts**, choose **Line** and go down to **DrawFit** to observe a more dramatic comparison!

LEARNING ACTIVITY 5: SHARING THE WEALTH
CELLSHEET™ APP

AIM *To support students in their study of shares, tracking their values over a period of time.*

METHOD *Students choose 3 different companies and track their share prices over 6 weeks. They are given a virtual $1 000 to spend, and should buy enough shares of each to spend all of that, if possible.*

PROCESS Create a new spreadsheet, called **SHARES**. Enter the company names (I chose AMP, Patrick Holdings and Billabong), a starting number, such as 20 of each, then the first week's values. Across the top row, enter the week numbers from 1 to 6.

SHAR	A	B	C
1		nO.	1.00
2	AMP	20	13.49
3	PAT	20	15.75
4	BILL	20	7.83
5	TOTAL		741.40
6			
C5: =$B2*C2+$B3:→[Menu]			

1. In **C5**, enter the formula **=$B2*C2+$B3*C3+$B4*C4**. (**NOTE** the single **$**: this tells the spreadsheet to only look in column **B**, but to allow the other values to vary as the formula is copied.)

2. Students may now try different numbers of shares in the top set until they find a combination that uses most or all of their $1 000 budget.

3. Each week for 6 weeks, students may enter the share prices for their companies, and observe the effect on the total value of their shares.

4. They may find it useful to copy the company names again in rows 7-10. In cell **B7**, enter the simple formula **=B2** (which copies the value in **B2** directly

SHAR	A	B	C
1		nO.	1.00
2	AMP	20	13.49
3	PAT	22	15.75
4	BILL	49	7.83
5	TOTAL		999.97
6			
A1:			[Menu]

SHAR	A	B	C
4	BILL	49	7.83
5	TOTAL		999.97
6			
7	AMP	20	269.80
8	PAT	22	346.50
9	BILL	49	383.67
A9: "BILL			[Menu]

into this cell). Copy this formula into cells **B8** and **B9**. Finally, in cell **C7** enter **=$B2*C2**, which gives the total value for that company in that week. Copy this formula also into **C8** and **C9.**

5. As they add the new share prices week by week, students may copy these formulas across the sheet, or they may do so now when setting up and the values will be automatically updated.

6. Graphs of the individual share price fluctuation over the period should be compared with those of the total values of these stocks. Students may need to discuss the different information each graph gives. They should note, for instance, that the values of the Billabong shares are the lowest in the first graph, since their individual prices are much lower than the others, but their total value is the greatest (second graph) since we bought so many of them.
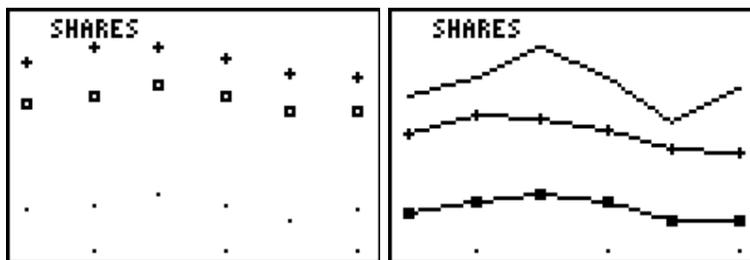
This is an interesting and very worthwhile exercise for students to undertake. It increases their understanding of the share market, and the influences which serve to control changes over time. Connections to gambling and probability should also be a feature of discussions.

LEARNING ACTIVITY 6: LOANS AND OTHER EXPENSES
FINANCE APP

AIM   *To build understanding of the relationship between term, interest rate and principal in relation to loan repayments and annuities.*

METHOD   *Students use the Finance App to investigate the effects of changes to each of the key variables on repayment of loans.*

PROCESS   Calculate the repayments on a loan of $100 000 at an interest rate of 6.75%pa over 25 years if interest is compounded (a) monthly; (b) Weekly.

1. Choose the **TVM Solver…** from the *Finance* App (or, if you are using a TI-83, from the FINANCE menu). Enter the values as shown (NOTE that the term, **N**, will be 25 x 12 = 300 for monthly payments, and 25 x 52 = 1300 for weekly payments over 25 years. NOTE, too, that the Payment/Year and Compounds/Year (**P/Y, C/Y**) will be 12 in the first example and 52 in the second).
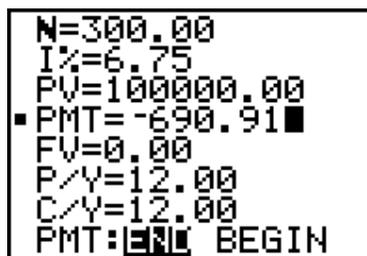
2. Move up to the **PMT** (Payment) entry and **SOLVE** (**ALPHA-ENTER**) to find that the monthly repayments will be $690.91, while weekly payments will be $159.31 (equivalent to $690.34 per month – (159.31 x 52)/12). This does not seem a lot, although 60 cents per week over 25 years is $780.
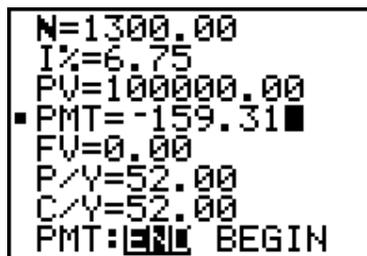
3. Students should now seek answers to other questions:
    a. What if I can only afford payments of $140 per week? How much can I then borrow?
    b. What if I can afford to repay more, say $800 per month? How much might this reduce the term of my loan?
    c. If I wished to repay my loan in 20 years instead of 25, what repayments would be required?
    d. If interest rates go up to 7 or 7.5%, how will this impact on my repayments?

The true power of technology lies in its ability to support such "what-if" games. Students are able to ask realistic questions and to receive immediate and meaningful responses to these.

LEARNING ACTIVITY 7: A BALANCING ACT

The previous investigation may be taken further using the graphing capabilities of the calculator. First, we will need to change the graphing **MODE** to **PAR** (parametric) as shown.
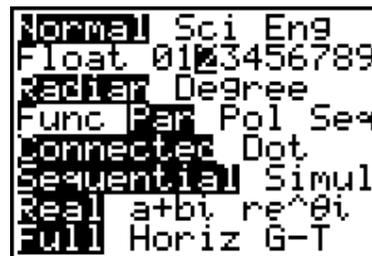
Press **Y=** and press $\boxed{\textbf{X,T,θ,}n}$ to enter **T** in **X₁T** as shown. Arrow down to **Y1T**, then press **APPS**, choose **1: Finance**, and then **9: bal(**. Again enter **T** as shown.

This will graph the balance remaining on the loan over the 25 year period.

Before viewing the graph, we need to set up the viewing window – this is most easily done by pressing **ZOOM** and choosing the last option, **ZoomFit**. Students may then press **TRACE** and follow the decline of the balance over time, slowly at first and much more rapidly towards the end of the loan period.
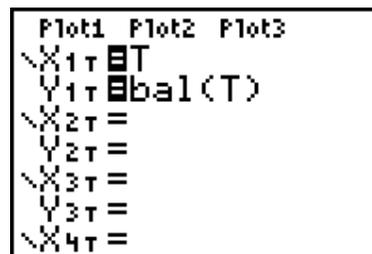
Return to the **MODE** screen and arrow down to the last option, then move across and choose **G-T** by pressing **ENTER**. Now press **GRAPH** and students may observe the graph and table of values at the same time.
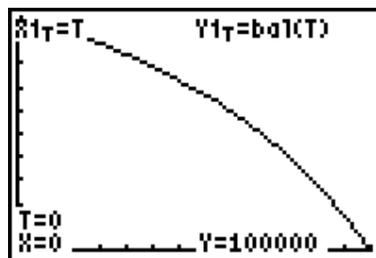
LEARNING ACTIVITY 8: SOME NICE THINGS TO KNOW

Two further features of the **Finance** capabilities of the calculator can prove extremely useful to students, both in class and examination.

1. Days Between Dates

The dbd(…) command may be found under **APPS -> 1. Finance…** as option **D: dbd(**
As the name suggests, it calculates the number of days between two given dates. This is a very useful capability when students are calculating credit card interest, since this always involves a simple interest computation for the number of days outstanding. Simple arithmetic errors by students can cost valuable time and marks, and this little utility can help to avoid such errors.

For Australian dates, students should use the format
DDMM.YY (US dates use a different format).
For example, to calculate the number of days between
August 21, 2003 and December 31, 2003, the format
would be as shown.

```
dbd(2108.03,3112
.03)
              132
dbd(2402.56,2108
.03)
            17345
■
```

Of course, this is also a feature that can prove of great
interest to students, and a source of interesting statistical data. The second example shown, for instance, calculates the number of days that I have been alive!

2. Effective and Nominal Interest

Another great little feature of the **Finance** App may be found under options **C:Eff(** and **B:Nom(**.

The first give the *effective rate of interest*, the second the *nominal rate*.

To illustrate, consider an interest rate of 8% pa compounded
monthly. An investment at this rate (or a loan) will clearly
increase more rapidly than one at 8% pa, with only one
compounding period each year. So what would be the
equivalent annual rate that would return the same amount?
This is called the *effective* rate, while the original rate is
called the *nominal* rate.

```
▸Eff(8,12)
       8.299950681
▸Nom(8.3,12)
       8.000045843
```

From the example shown, the *effective* rate for 8% pa compounding monthly is approximately 8.3% pa.

INVESTIGATION

What you expect the graph of the **Eff()** function to look like for a given interest rate (say 8%) if the number of compounding periods per year increases?
Into **L1**, enter the values 1, 2 (six-monthly), 4 (quarterly), 12 (monthly), 26 (fortnightly), 52 (weekly) and 365.25 (daily). At the home screen, enter **Eff(8, L1) -> L2** and plot!

2. DATA ANALYSIS

The graphic calculator is a surprisingly powerful tool for statistical representation and analysis, going well beyond the scope of most school courses in its capabilities. At the same time, it is simple to use, supporting students in ease of entry of data, and the variety of plot-types and statistical computations that are readily available.

Learning Activities	Content Reference	App
9. Getting started with data	Displaying data	
10. People graphs	Displaying data	
11. Can you feel the warmth	Interpreting data	Datamate
12. What is normal?	Normal Distribution	
13. Data, data everywhere	Correlation	

LEARNING ACTIVITY 9: GETTING STARTED WITH DATA

AIM   To introduce the main statistical features of the graphic calculator.

METHOD   Students will enter and view a set of scores, using the histogram to produce a frequency distribution table.

PROCESS   Enter the following scores into **L1**:

12	15	11	12	8	17	15	13	6	12
15	10	9	16	14	15	17	19	15	12

1. The **STAT** key is located on the second row of black keys, beside the blue left arrow. This is where you will start when doing "stats". To enter data, simply choose the first menu item, **Edit…** This takes you to the **List Editor** where you may enter your scores directly into the first of 6 lists, called **L1**. Be careful when entering data – most mistakes are made at this stage! Use your finger to move along the data set, or have a friend read out the scores. Notice that when finished, the list entry on the calculator reads **L1(21)**? This is quite correct, since the last entry was the previous one, the **20th** score, as required.
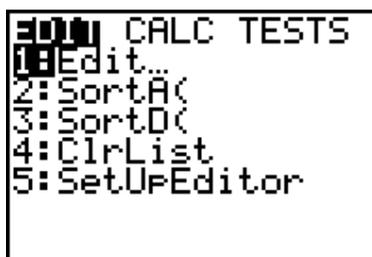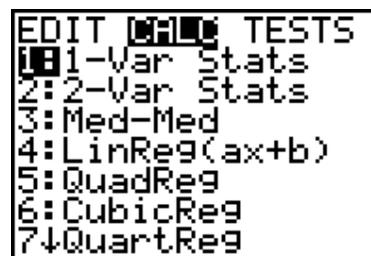
2. Press the **STAT** key again, but this time, move across using the right arrow to the **CALC** menu to do some statistical CALCulations. Choose the first menu item, **1: 1-Var Stats** (you will rarely do anything else in this course). The command **1-Var Stats** is dropped into the Home Screen with a space and a flashing cursor. Since all your data is in **L1**, there is no need to add anything else (this command automatically looks in **L1** unless told to look elsewhere), so just press **ENTER**.

3. Two screens of statistical information appear (NOTE the down arrow at the bottom left corner of the screen, beside the **n**? this indicates that there is more to be seen by scrolling down. The first item on the list is the **mean** (x with a bar above it). The average of this set of scores was 13.15. The next two items give the sum of the scores and the sum of the squares of the scores. Next we find two measures of spread: the standard deviation for a sample (with an **S** for sample) and the population standard deviation (with a Greek sigma). The number of scores follows (here, of course, **n**
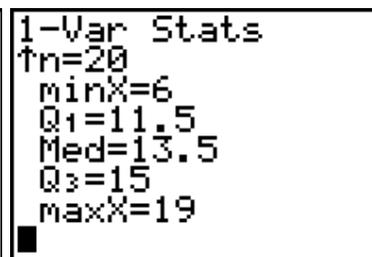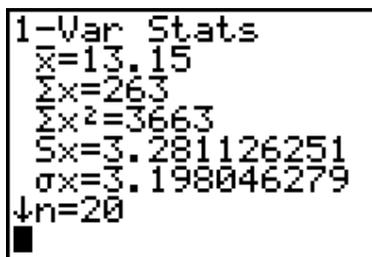
should equal 20), then the **minimum** score (6), the first quartile, the **median** or middle score (13.5), upper quartile and the **maximum** (19). Most often students will use the mean, median, range (maximum – minimum) and, later, the interquartile range (Q3 – Q1) and the standard deviations. Pres **CLEAR** when the required values have been noted.

4. We will now set up the graph screen to view a histogram of these scores. In this case, since the scores potentially range from 0 to 20 (it was a test out of 20) and the most times a score was repeated was likely to be less than 5, we may set up the **WINDOW** menu as shown.

5. To view a function graph, we would go to the **Y=** menu. To view a *StatPlot*, use **2nd-Y=**. Choose the first menu option, **1: Plot1** and press **ENTER**. Set the screen up as shown and then press **GRAPH**. Press **TRACE** to move across and note the number of times each score occurred. It is easy to see the *mode*, the most popular score (15 occurred 5 times).
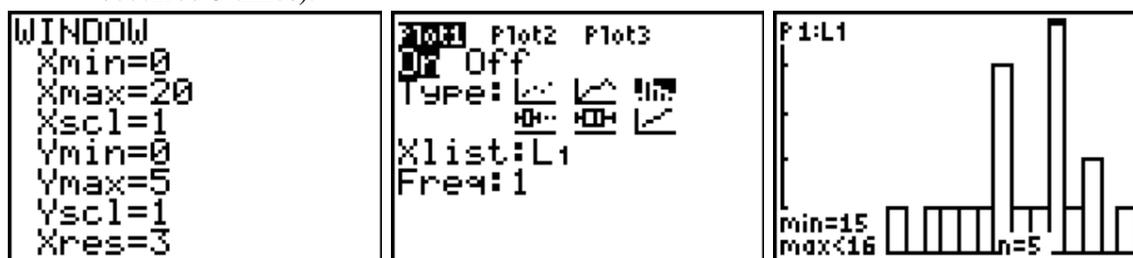
6. By noting the frequency of each score, students may use these to construct a frequency distribution table. This is a great time saver when there are a large number of scores and there is a greater risk of tally errors using traditional methods.

*This method of creating graphical frequency distribution tables is quite versatile, extending even to grouped data. If our scores were grouped in, say, groups of 5, then simply changing the **Xscl** value from 1 to 5 in the **WINDOW** menu will collect automatically into groups of 5.*

LEARNING ACTIVITY 10: PEOPLE GRAPHS

The Box-and-Whisker plot is becoming increasingly popular as a tool for representing statistical information, particularly when there is more than one set of data to be viewed. NOTE from the plot screen above that there are two box-plot options. Students should generally use the first, since it displays any outliers that may exist above or below the scores. In this case, the score 6 constitutes an outlier, lying more than 1.5 times the interquartile range away from the median.
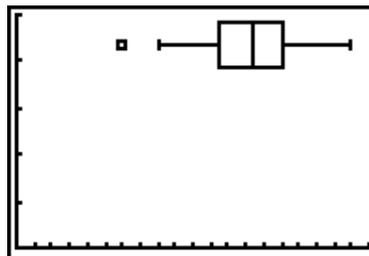
This type of graph is excellent when comparing different data sets visually, as shown. It also provides quite specific information, giving the median, upper and lower quartiles, minimum and maximum scores.
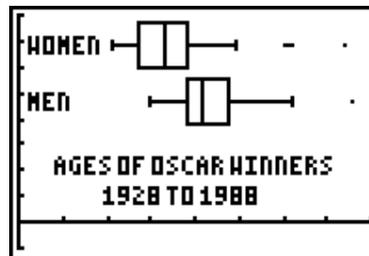
This activity leads to the class creating a "human" box and whisker plot, among others.

AIM   To introduce to students the main features of a box and whisker plot.

METHOD   Students will create their own graph from information about their birthdays.

PROCESS   Birthdays are interesting. We all have one, and it is always fun to find out if we have a "birthday buddy" – someone who shares the same birthday. In this activity, we use birthday information to create the main statistical graphs for this course: pie chart, histogram, stem and leaf and box and whisker plots. Each uses a different aspect of their birthday and demonstrates the different uses of the graph types.

1.   **Pie Chart**

For this activity, we might use the *season* of their birth. Explain to the class that we are going to line up in order of the season of our birth. Ask them to think about and discuss what they expect to happen here: is there likely to be any particular pattern in this, or would we expect people to be evenly spread among the four seasons?

Call out all the Spring babies – those born in September, October or November. Have them form a line (for a class of 30, this may need to be done outside, since it begins with one long line). Note that order is not important here.

Next call out the Summer babies – December, January, February. Follow with Autumn (March, April, May) and, finally, Winter (June, July, August).

Have students look along the line and comment if any pattern is visible. You may want each season to step forward.

Finally, while remaining in their order, have the students form a circle and sit down, equally spaced. Now they have formed a human pie chart. In turn, each season should stand, or hold up their hands for everyone to see. Is it easier now to see how the children are distributed through the seasons? Is the result that was expected?
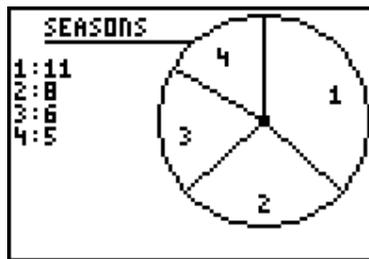
Discuss the 360 degrees of the circle and how each student is related to this: if there are 30 students in the class, this is not hard. If not, they may use their calculators to calculate how many degrees each student represents.

On returning to class, students should draw up the pie chart they have just created. If they have the *CellSheet*™ App, they should enter their numbers and compare their pie chart with that generated by the calculator. Find the percentage of the class born in each month, and ask them to comment on this result in their workbook.

2. Histogram

For this graph, you may use their seasons, or it may be more interesting to use their month of birth. Describe the activity, and then have students record any predictions they may expect in their journal or workbook. Talk in class for a few minutes about these expectations: do they expect that they will be equally spread among the months? Why, or why not?
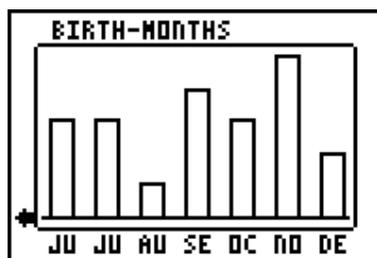
Once again, call students out by month of birth, and have them line up in parallel rows, equally spaced (they may space themselves by putting one hand on the shoulder of the student in front, and one hand to touch the student at their left or right. Once again, this is probably best done outside with plenty of room.

Have students sit down, and then have each month stand up and observe the human histogram they have formed. Record the data in the calculator's list editor (Press **STAT**). Enter the numbers from 1 to 12 in **L1**, then the numbers in each month in **L2**.

Before returning to class to write up this activity and to draw their histogram, have the last student in each column reach across to the last person in the next column, forming a line graph with their arms. Have students talk about this, and why the line graph would NOT be appropriate in this case. Ask them to think about situations they could model where a line graph would be better than a histogram.

Why was a histogram a better choice for this data than a pie chart? Why was a pie chart the better choice for the first activity?

3.   Stem and Leaf Plots

For this activity, we will use the *day* of their birthday: i.e. if they were born on the first or the 21st of a month. The class will be divided into numbers between 1 and 31, ignoring the actual month of their birth: we are only interested in the day of the month. Again, have the class discuss what they expect to see in this graph: are they likely to be equally spread through the 31 days available? Why, or why not?

Call out all those with single digit days: line them up in increasing order from a starting point: call them 01, 02, 03, etc. Ask them to sit down. Now call out those born on a day between 10 and 19: line up in order parallel to the first group, and sit down. Line up the 20s next, sit them down, and then the 30s.

```
bIRTH DATE
0 146789
1 233555689
2 112246777899
3 011
_____
UNTITLED        [MENU]
```
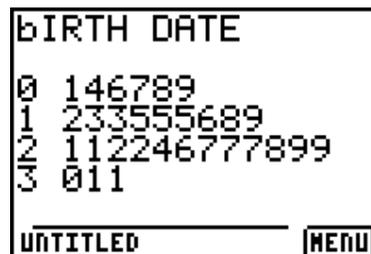
They have formed a stem and leaf plot. Each line should stand up in turn and study the graph, noting any features of interest.

Finally, ask the boys to separate from the girls: girls on one side of the middle line, boys in order on the other to form a back-to-back stem and leaf plot. Look, think, talk.

Go back to class and write this up: draw both stem and leaf plots. Enter the data into lists as before and look at the graphs formed. What is the advantage of a stem-and-leaf plot over a histogram?

4.   Box and Whisker Plots

For this activity, students will work out the day of the year that their birthday falls on: someone born on January 1 would be 1, someone born on December 31 would be either 365 (or 366 in a leap year). Students may calculate their birth number in a number of ways.

By writing the list of months and their days on the board, students may easily calculate their number by adding the days in the months up to their birth month, and then the date within that month.

Alternatively, they may use a calculator function, **dbd(***date1, date2***)**, which stands for "days between dates" (introduced in the previous chapter). This function may be found in the **CATALOG** or under **APPS -> Finance -> D:dbd(**. The format for Australian dates is **DDMM.YY**). This means that for a student to enter March 15, 1974, they should enter 1503.74. This to find the number of days between January 1 and March 15 in 2003, enter **dbd(0101.03, 1503.03)**. The result is 73.

```
L1      L2      L3      1
103
104
134
136
▮▮▮
194
201
_____
L1(16) =194
```

Once everyone has calculated their birth number, all students may come to the front and enter their number on the teacher's ViewScreen™ calculator in **L1**. Return to the **STAT** menu and choose **2: SortA(** to sort **L1** in ascending order, and students can easily see if they have a "birthday buddy" in their class.
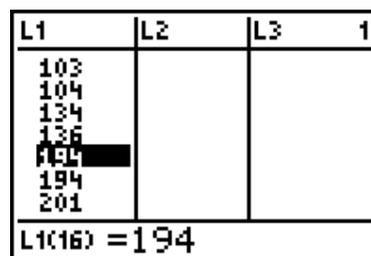
Discuss what the likelihood is of sharing the same birthday: surprisingly, in a group of 30, it is almost 50%!

Return to the **STAT** menu and choose **CALC:1:1-Var Stats** to compute the summary statistics on this birthday data in **L1**. Scroll down to observe the quartiles, median, minimum and maximum scores.

```
1-Var Stats
↑n=31
 minX=4
 Q₁=70
 Med=194
 Q₃=277
 maxX=346
```

The class will be asked to form a straight line (probably outside the classroom for a class of 30!). First ask all those with birthdays between 4 and 70 to line up at one end (the order within this group is not important). You may like to make up signs for **minX, Q1, Med, Q3** and **maxX.** Give the **minX** and **Q1** signs to those students with these scores.

Next send out those between 70 and 194, with the last person taking the **Med** sign.

Follow with those between 194 and 277 (last person takes **Q3**), and then the final group to line up last, with **maxX** on the end.

Once the class is lined up outside, have them cluster into their four groups and sit down. Those holding the signs should be at the end points of their groups and they may stand. Ask for observations regarding the way the class has been divided: there should be equal numbers above and below the **median**; their should be approximately equal numbers in each of the four groups (the **quartiles**). Ensure that students understand this aspect of the box and whisker plot.

```
P 1:L1

Q1=70    .    .    .
```
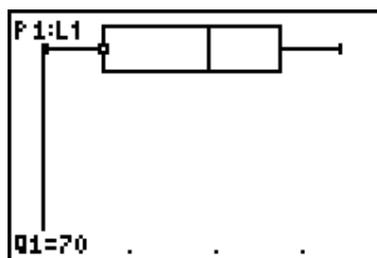
Finally, it may be possible to have students line up (at least approximately) in their relative positions out of 365. They may observe some stretching of the quartiles.

At this point, students may return to class and plot the data in a box and whisker plot on their calculator, observing the shape and skewness of the graph (if any) and discussing their expectations regarding this set of data.

```
P 1:L1

Q1=174 .    .    .
```
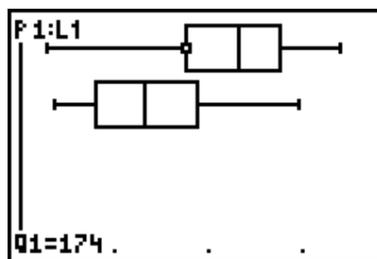
It may be well worthwhile repeating this activity after separating the boys from the girls, since box and whisker plots are fundamentally tools for comparison. Again, talk about the results: are there any surprises? Have students report on this activity, drawing particular attention to the choice of different graph types for different types of data.

LEARNING ACTIVITY 11: CAN YOU FEEL THE WARMTH?

AIM   To use real world data to introduce lines of best fit.

METHOD   Students use a data logger to gather temperature data in both Centigrade and Fahrenheit units, then derive a line of best fit for the resulting plot.

PROCESS   This simple experiment may be done in a normal classroom by simply bringing in some hot and cold water.
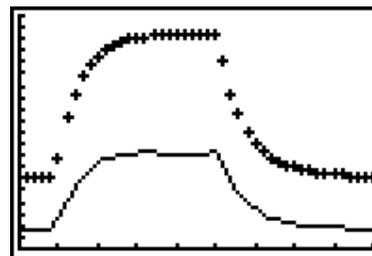
Set up the **DataMate** App and **CBL2™** data logger with two stainless steel temperature probes in Channels 1 and 2. One should be set to read degrees Fahrenheit and the other degrees Centigrade. Set the time graph to read data at one-second intervals for 45 seconds.

Set up a cup of cold water and another of hot water. Start data collection, holding the probes in the air for 5 seconds, then 20 seconds in the hot water and the remaining time in the cold water. Quit the DataMate App, and set the **StatPlots** to show **L1** (time) against **L2** (degrees Centigrade), and then **L1** against **L3** (degrees Fahrenheit). Use **ZoomStat** if desired to display the screen above.

Have students discuss what they observe from these graphs: the two curves are exponential, as temperature rises rapidly, and then falls away.

The power of this experiment for our purposes, however, arises when the calculator plots degrees Centigrade (**L2**) against degrees Fahrenheit (**L3**). Set up **Plot3** as shown, then **ZoomStat**.
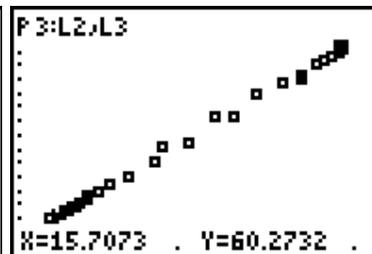
Students should discuss what they see: why is the line very close to being straight. Press **TRACE** and have students observe the motion of the cursor as it traces through the 45 seconds, but jumps around the graph screen. Explain what is happening.
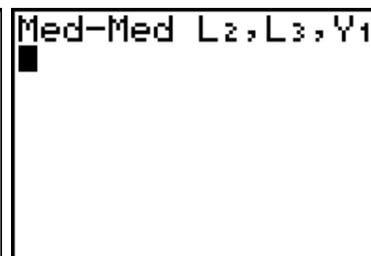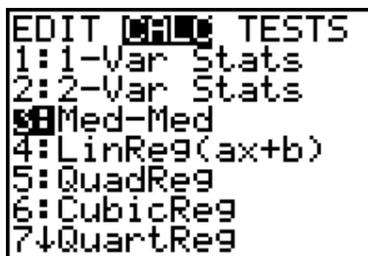
This activity presents an opportunity for algebraic modelling, as the students attempt to fit a linear function to the plot.

Begin with **Y1 = X**, view the graph and vary the gradient and y-intercept until satisfied with the fit. This activity is always valuable, reinforcing understanding of gradient and intercepts and engaging students actively in the process.

If desired, students may be exposed to the regression lines available in the **STAT -> CALC** menu. Some discussion may occur when appropriate regarding the difference between *median-median* and *linear* regression lines. In this example, we would expect little difference between the two. *Why?*
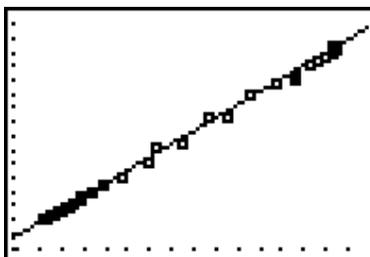
Finally, it may be worthwhile inviting students to simply enter the freezing and boiling points of water into lists as shown. Once again, set up a scatter-plot for these two points, and then compute the line of best fit using the *linear regression* available.

Compare the values for this line with those arising from the data. Discuss the differences.

LEARNING ACTIVITY 12: WHAT IS NORMAL?

AIM   To introduce the normal probability distribution and its key properties.

METHOD   Students use the calculator tools available to analyse the normal distribution..

PROCESS   The concept and properties of the normal probability distribution are of particular importance to candidates external examinations, and it is of some benefit for them to understand the implications.

This activity begins with a visual display and subsequent discussion of a normal distribution. We begin with one which has a mean of 60 and a standard deviation of 12.5, since these are the properties generally used by the New South Wales Board of Studies for HSC examination results.

The **normalpdf(** (normal probability distribution function) may be found as the first item in the **DISTR** menu (**Shift-VARS**, just below the blue arrow keys). For this activity, students should begin in the **Y=** menu, in **Y1** (all other functions and stat-plots should be off). Go to the **DISTR** menu and select item 1, depositing **normalpdf(** into **Y1**, as shown.

Students should next go to the **WINDOW** menu and set the Window Settings as shown. Finally, press **GRAPH**.
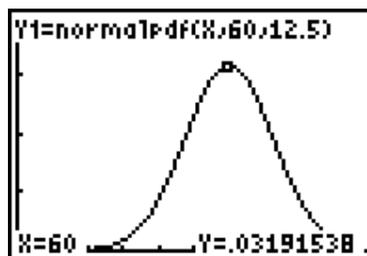
The familiar shape of the bell-shaped curve appears, and students (using the **TRACE** facility) should notice at very least that the mean coincides with the modal value, as the "high point". The further above and below the mean, the fewer and fewer scores may be found. In the context of an examination, students should explore the extremities, observing that very few students will score below around 25 or above 100. While in **TRACE** mode, simply entering a value (such as **60**) will take the cursor directly to that point. Observe that the y-values (heights) are percentages, and should be multiplied by 100 to be meaningful. *Discuss the implications*.

It may be timely to refer at this point to other examples of the occurrence of the normal distribution: everywhere from heights and weights to the numbers of eggs laid by 1000 chickens each day in a hatchery; even to the differing sizes of bolts produced in a factory.
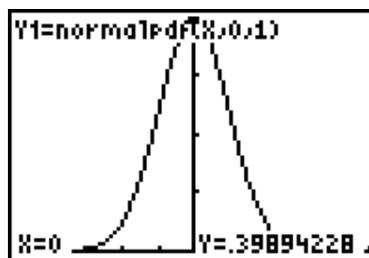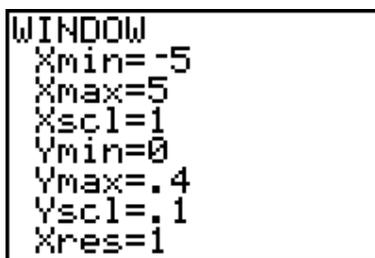
Have the students return to the **Y=** menu and change the mean from 60 to 0, the standard deviation from 12.5 to 1 (thus, **Y1 = normalpdf(X, 0, 1)**). Adjust Window Settings as shown, and then view the graph. Discuss the features of this general normal distribution curve.

It is now useful to consider the areas under this curve between particular values, since these provide *probabilities* of events occurring within a specified range. For this purpose, we return to the **DISTR** menu, but right-arrow across to **DRAW**, then choose **1: ShadeNorm(**. Entering –1 and 1 on the home screen as shown produces, not only the shaded normal curve between these values, but the area, indicating approximately 68% probability of a score falling within 1 standard deviation from the mean in a normally distributed population of scores.
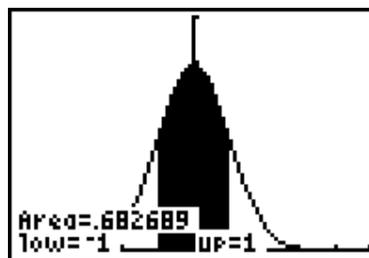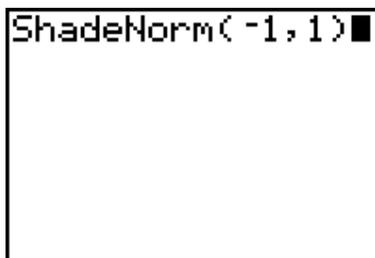
Clearly this will need to be discussed with students: give examples from different tests: for a test with a mean of 65 and a standard deviation of, say 15, then we expect over two thirds of students to score marks between 50 and 80. Use the term *probably* to describe this situation.

Students should now investigate the probabilities associated with scores falling within 2 and 3 standard deviations either side of the mean (arriving at probabilities close to 95% - *very probably* - and 99.7% - *almost certainly* - respectively).

After further discussion and exploration, it is helpful to draw students back to the Higher School Certificate scenario. Ask them to make predictions related to different scores in the HSC: between what scores will students *almost certainly* fall? What does this mean for them? Between what scores will candidates *very probably* fall? Plot the normal curve again for a mean of 60 and a standard deviation of 12.5, and ask students to verify that the probabilities for the different intervals (in this case, between 47.5 and 72.5, between 35 and 85, and between 22.5 and 97.5) correspond to those already encountered.

The introduction to the concept and formula for **z-scores** can occur prior to this activity, or following it. After visualizing the two normal distribution curves (for mean of 60 and standard deviation of 12.5, and that for 0 and 1), it is readily seen that the curves are identical

in shape. Hence it is possible to move easily from one representation to the other, and the formula which performs this transformation may be now appreciated.

A simpler approach (less visual but certainly more efficient) involves another option from the **DISTR** menu, **2:normalcdf(**, which is the normal cumulative distribution function. As the name suggests, this provides the accumulated value between two given points, or the area under the curve. The probabilities between 1, 2 and 3 standard deviations from the mean are easily verified as 68%, 95% and 99.7%.

```
DISTR DRAW
1:normalpdf(
2:normalcdf(
3:invNorm(
4:tpdf(
5:tcdf(
6:X²pdf(
7↓X²cdf(
```

```
normalcdf(-1,1)
         .6826894809
normalcdf(-2,2)
          .954499876
normalcdf(-3,3)
         .9973000656
■
```

It is similarly easy to verify that, for a mean of 60 and a standard deviation of 12.5, scores of 47.5 and 72.5 correspond to –1 and 1 standard deviations away from the mean, while 35 and 85 lie exactly 2 standard deviations away.

```
normalcdf(47.5,7
2.5,60,12.5)
         .6826894809
normalcdf(35,85,
60,12.5)
          .954499876
■
```

```
invNorm(.90,60,1
2.5)
          76.01939458
invNorm(.90)
          1.281551567
```

Finally, if we wish to know what score would be needed in order to lie ahead of 90% of students, then the **invNorm** command readily offers such information: for a mean of 60 and SD of 12.5, this would be a score of 76 (approximately 1.28 standard deviations above the mean).

The normal distribution describes data which is *continuous*, usually that which involves some form of measurement in order to produce the scores. While many real-world activities from a surprising range of sources may be described in this way, it is also useful to be able to describe discrete data, and for this we may use, among others, the binomial distribution.

LEARNING ACTIVITY 13: DATA, DATA EVERYWHERE

AIM   To draw on student generated data in order to introduce correlation.

METHOD   Students record birthday data (days and months) and explore relationships.

PROCESS   Students carry data around with them all the time. They especially enjoy birthday data, since it is personal and fun. It is also surprisingly revealing statistically.

This activity can involve just one teacher ViewScreen™ calculator into which class members enter their birthdays, or students can enter the data onto their own calculators as it is called out. The activity works best with a good-sized class – around 30. If the class is small, then members can enter their own birthdays and those of one or two loved ones to increase the sample size.

Into **L1**, enter the day of the month of each class member (numbers from 1 to 31). Into **L2** will be entered the month of their birth (numbers from 1 to 12). By the way, students will naturally look out for "birthday buddies". Talk about the likelihood of these occurring and perhaps discuss the somewhat surprising result that, in a class of 30, the probability is close to 50% of two people sharing the same birthday!

Discuss the scatter-plot that will result from plotting **L1** against **L2**. *What do students expect to see?* The correct answer here, of course, is *nothing*! The scatter-plot should be random, with points all over the screen.

It is now appropriate to introduce the concept of zero correlation – students will immediately appreciate this concept.

Now go back to the **STAT** menu and choose option **2:SortA(** (*Sort Ascending*). Sort both **L1** and **L2**.

Importantly, discuss what students might expect to see from a scatter-plot of the data now! **View** the data by going back to the list editor (as shown). Students should realise that the data now bears no relation to themselves: the link between days and months has been broken. These are just two sets of random numbers, in ascending order.
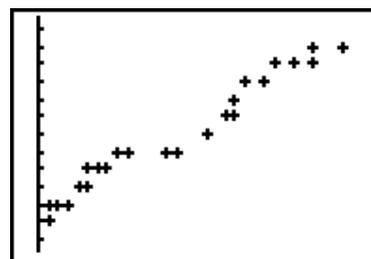
As should be expected, however, the plot appears surprisingly linear! Once again, discussion follows. This should lead into further work on correlation – this should be much closer to 1.

The opportunity (as in the last learning activity) to attempt a linear model here is to good to pass up. Begin again with **Y1 = X**, and students (individually or in small groups) should attempt to find the line of best fit for this data set. For example, the line **Y1 = 0.3X + 3** is shown.

This activity will generate some discussion and, ideally, some arguments and disagreement about what constitutes a "line of best fit". Different groups will arrive at different results and will claim that their result is superior to those of others. *Grasp this teaching moment* and ask groups to justify their point of view.

This leads naturally into the variety of methods used to compute such lines of best fit, and again attention may be drawn to the two methods available on the calculator. The **Med-Med** regression, like the **median** itself, is largely unaffected by outlying results, interested only in the *numbers* of scores, not the *value* of those scores.

```
Med-Med
 y=ax+b
 b=1.949612403



LinReg(ax+b) L₁,
L₂,Y₂
```

The **linear** regression model behaves more like the mean, and is affected by outlying scores. In the example shown, we might expect little difference between the two models, since the scores are fairly tightly packed, with no outliers. Note that this equation fits in well with our estimated line of best fit, with a gradient close to 0.3 and a y-intercept close to 3.

```
LinReg
 y=ax+b
 a=.3128976566
 b=2.604703667
 r²=.962993536
 r=.9813223405
■
```

To reveal the correlation coefficient (**r**) it is necessary to use the **CATALOG** (Shift-0) and choose the command **DiagnosticOn**.

The resulting graph, as might be expected, displays two very similar "lines of best fit".
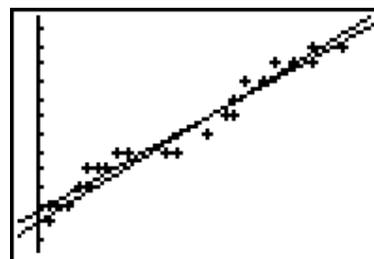
3. MEASUREMENT

The study of measurement at this level should be a very practical encounter with the important skills and ideas involved. From consideration of the importance and implications of accuracy and precision, to getting outside and performing real tasks, which put their studies into a practical context, students should be given opportunities to engage actively in all aspects of this topic. The available technology offers wonderful opportunities in this respect, from unit conversions and interactive geometry to effective teaching and learning sequences, which offer thorough coverage of several key aspects of this module.

Learning Activities

	Content Reference	**App**
14. Precision and Accuracy	Units of Measurement	*FunSci, SciTools*
15. Scientific Notation	Units of Measurement	*FunSci, SciTools*
16. Units and Conversions	Units of Measurement	*FunSci, SciTools*
17. Covering the Area	Area and Volume	*CabriJr™*
18. Almost the same	Similarity	*CabriJr™*
19. Am I right or not?	Right-angled triangles	*CabriJr™*
20. Where in the world?	Spherical geometry	*World Geography*

TAKING GOOD MEASUREMENTS
FUNDAMENTAL TOPICS IN SCIENCE AND SCITOOLS APPS

AIM   To review with students key aspects of the process of measurement, in terms of accuracy and precision, and to consolidate previous work on scientific notation.

METHOD   Using the extensive teaching sequences of the *Fundamental Topics in Science* and the complementary *SciTools* Apps, students may explore and consolidate knowledge and skills required for this unit.

PROCESS   The teaching sequences and associated tools delivered by these two Apps offer a very effective means by which students may be exposed to the key ideas of this part of the Measurement unit. Working individually, in small groups, or as a class with the teacher using the ViewScreen™, students may be led or work independently through the relevant topics.

```
SCIENCE SECTIONS
1:FUNDAMENTAL TOPICS
  ┏━┓SCIENTIFIC METHOD
  2:PRECISION & ACCURACY
  3:SCIENTIFIC NOTATION
  4:UNITS & CONVERSIONS
↓5:DATA & GRAPHS
[EXIT]    [ UP ]    [HELP]
```
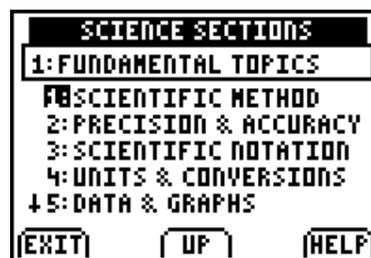
Although this App was designed by and for science teachers and their students, most of the topics are very relevant to non-calculus senior mathematics.

LEARNING ACTIVITY 14: PRECISION AND ACCURACY

Each chapter in this interactive textbook consists of three parts: an Introduction, Concepts and Activities. Using a mixture of information, animated graphics and interactive learning tools, students are carefully and thoroughly introduced to the key concepts and skills of the topic. They are then provided with opportunities to practise and consolidate this knowledge and skill base.

```
SCIENCE SUBSECTIONS
1:FUNDAMENTAL TOPICS
2:PRECISION & ACCURACY
  ┏━┓INTRODUCTION
  2:CONCEPTS
  3:ACTIVITIES
[EXIT]    [ UP ]    [HELP]
```

```
PRECISION & ACCURACY


      ⎡ ˙ ₊˙₊ ⎤


/PRECISE & ACCURATE:/
     AWESOME!        ✛
```

```
PRECISION
PRECISION IS THE EXTENT
TO WHICH A SERIES OF
MEASUREMENTS OF THE SAME
QUANTITY MADE IN THE SAME
WAY AGREE WITH ONE
ANOTHER.
                      ✛
```

```
WHEN MEASURING ...
    ⌐30   RECORD EACH DIGIT
    ⌐          KNOWN WITH
    ⌐20        CERTAINTY...
    ⌐
    ⌐10    ┌──────────┐
    ⌐      │    _     │
    ⌐      └──────────┘
                      ↕
```
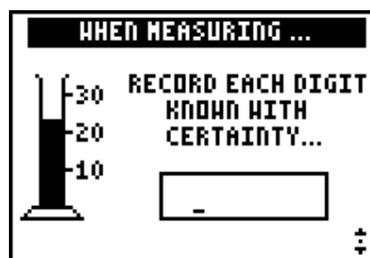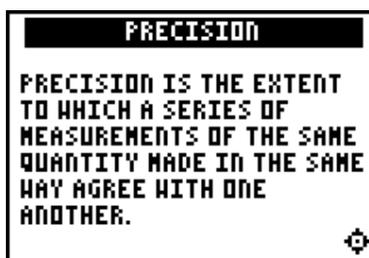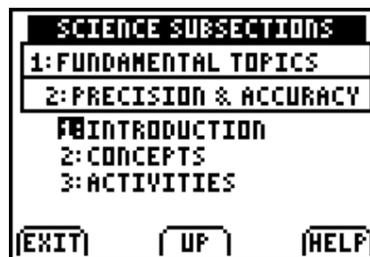
From introducing the ideas of precision and accuracy (and distinguishing between them) this unit moves on to significant figures, explaining the rules of significance and then providing games for students which will reinforce these skills. Further practice is given in reading from a variety of gauges and measurements, and in choosing different measurements and different scales and rules in a variety of contexts. Many teachers find the rules for significance confusing at times: this unit provides a welcome clarity.

```
┌─────────────────────────┐  ┌─────────────────────────┐  ┌─────────────────────────┐
│ ▓THE BIG SIG-FIG DIG!▓  │  │ ▓WATCH YOUR SPEED!▓     │  │ ▓DO YOU MEASURE UP?▓    │
│                         │  │          B              │  │      ┌200               │
│ MARK EACH UNDERLINED    │  │        ╱──┼──╲           │  │      │      CHOOSE YOUR │
│ DIGIT 'SIG' OR 'NOT'.   │  │       │   │   │          │  │      │     MEASUREMENT, │
│                         │  │       │   +   │          │  │      ├100  THEN THAT OF 3│
│         ┌──────┐        │  │        ╲     ╱           │  │      │     OTHER STUDENTS.│
│         │ 7030 │        │  │         0  100           │  │      │                  │
│ SCORE: 80└──────┘       │  │                         │  │ MEASURE1?               │
│      ┌SIG┐┌NOT┐┌MENU┐  │  │ ┌60┐┌62┐┌64┐┌66┐┌MENU┐ │  │ ┌150┐┌160┐┌170┐┌180┐┌MENU┐│
└─────────────────────────┘  └─────────────────────────┘  └─────────────────────────┘
```
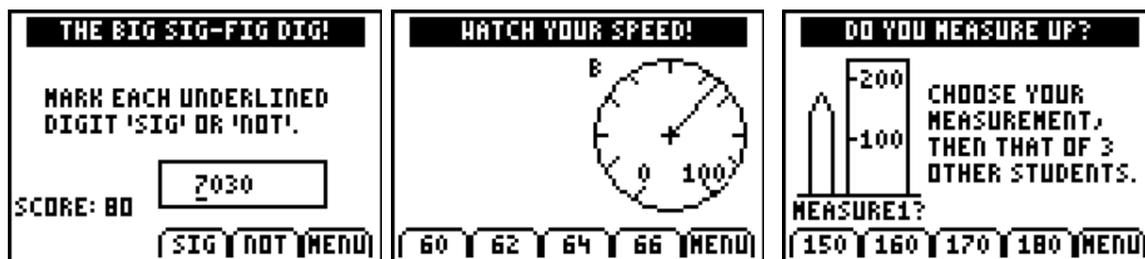
This is a careful and well-prepared learning sequence which will be of benefit to all students, especially if accompanied by discussion and teacher-reinforced guidance.

```
┌─────────────────────────┐        ┌─────────────────────────┐
│ ▓   SELECT A TOOL   ▓   │        │ ▓ SIG-FIG CALCULATOR ▓  │
│ ▓1:SIG-FIG CALCULATOR   │        │   2.56            [3]   │
│ 2:UNIT CONVERTER        │        │ *5E⁻3             [1]   │
│ 3:DATA/GRAPHS WIZARD    │        │ ──────────────────────  │
│ 4:VECTOR CALCULATOR     │        │   1.28E⁻2               │
│                         │        │ →1E⁻2             [1]   │
│                         │        │                         │
│ ┌EXIT┐                  │        │ ┌EXACT┐ EE ┌DEC┐SCI ┌EDIT┐│
└─────────────────────────┘        └─────────────────────────┘
```
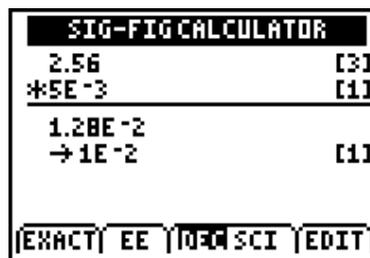
When students have worked through the sequence and activities for this chapter, they may then make use of the *SciTools* App which serves as a complement to *Fundamental Topics In Science*. Where *FunSci* provides interactive learning sequences, *SciTools* provides exactly that: a toolkit of useful support when students are dealing with measurement. As shown, these consist of a Significant Figure calculator, a Unit Converter, a simplified menu-driven means for dealing with data and graphs, and a vector calculator.

The Sig-Fig calculator allows students to enter actual calculations and have the significant figures of the component numbers displayed, as well as those of the resulting answer. A very useful tool – once students have completed the section on scientific notation, since this will be needed to interpret the result!

LEARNING ACTIVITY 15: SCIENTIFIC NOTATION

SCIENTIFIC NOTATION NUMBERS ENCOUNTERED IN SCIENCE AND TECHNOLOGY ARE OFTEN VERY LARGE... 598000000000000000000000 kg **MASS OF THE EARTH**	**SCI. NOTATION FORM** $P * 10^K$ 1) $1 \le P < 10$ 2) ALL DIGITS IN P ARE SIGNIFICANT 3) K IS AN INTEGER	**SELECT AN ACTIVITY** D->S **S->D** CONVERT SCIENTIFIC NOTATION TO DECIMAL. +-*/ SELECT & PRESS [ENTER]
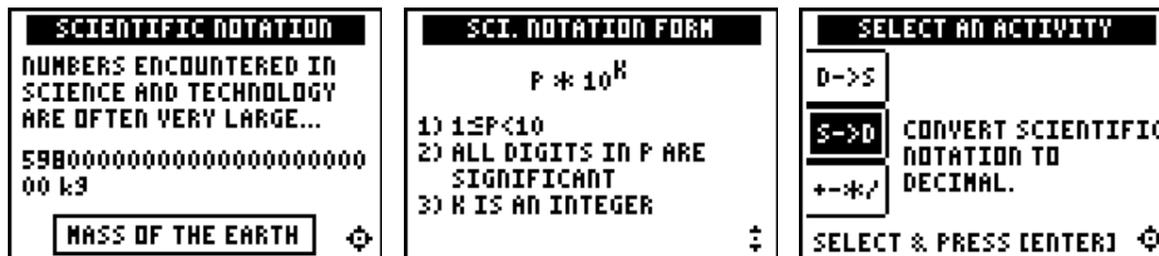
The skills involved in using scientific notation are very important in senior studies. Students need to be able to move easily between the (normal) decimal form and the scientific notation form for very large and very small numbers. They also need to be able to interpret the calculator format (5.2E3, for example, representing 5×10^3 or, simply, 5 200).

All of these skills are explained and consolidated in this chapter of the *Fundamental Topics in Science* App. The multiple choice quiz questions which provide the activities for this unit are well-sequenced and serve to effectively build skills that are needed for this topic.

As noted already, the ability to work with scientific notation is a fundamental requirement for this course, and for using the various tools provided by the graphic calculator, including working with the conversion of different units of measurement, which follows.

LEARNING ACTIVITY 16: UNITS AND CONVERSIONS

This interesting, informative and varied teaching sequence explores key understandings and features associated with units of measurement, focusing in particular, of course, upon the metric system.

WHY CARE ABOUT UNITS? IMAGINE AN ANIMAL HAVING THESE DIMENSIONS HEIGHT = 5 LENGTH = 14 WEIGHT = 4	**WHY CARE ABOUT UNITS?** MAYBE YOU ASSUMED HEIGHT = 5 METERS LENGTH = 14 METERS WEIGHT = 4 METRIC TONS	**WHY CARE ABOUT UNITS?**

The various prefixes are introduced, explained, and then conversions between different units are reinforced.

ANOTHER EXAMPLE SEVERAL CONVERSIONS CAN BE CHAINED. CONVERT 75km/hr TO m/s $\frac{75km}{hr} \cdot \frac{1hr}{60min} \cdot \frac{1min}{60s} \cdot \frac{1000m}{1km}$ = 21m/s	**SI PREFIXES** EXPRESS THE DIAMETER OF THE EARTH(1.2756E4 km) in cm. A) 1.2756E9 B) 1.2756E-1 C) 1.2756E6 D) 1.2756E3 YOUR SCORE:20 [MORE][MENU]	**DIMENSIONS** WRITE A FORMULA RELATING MASS m, VOLUME V, AND DENSITY d (g/cm^3). A) d=m/V B) d=mV C) d=V/m D) m=V/d YOUR SCORE:30 [MORE][MENU]

Once students have worked through the unit on Units and Conversions, they are free to use the wonderful *SciTools* **Unit Converter**.

This package allows students to convert between all the common units of measurement – and quite a few uncommon ones as well. It also provides the standard constants, with the facility to convert these to other units as well. All calculations and results are presented using scientific notation, so it provides further practice for students in this area, as well as a wonderful collection of tools for exploration!

```
┌───────────────────────┐   ┌───────────────────────┐
│     UNIT CONVERTER     │   │        LENGTH          │
│ 1:LENGTH    7:MASS     │   │ fm   Å   mm   cm   m   │
│ 2:AREA      8:FORCE/WT │   │ km   Mil  in   ft   yd │
│ 3:VOLUME    9:PRESSURE │   │ fath rd   mi   Nmi  ltyr│
│ 4:TIME      A:ENERGY/WORK│ │ 2.6E0 km▶              │
│ 5:TEMP      B:POWER    │   │        2.6E6 mm        │
│ 6:VELOCITY  C:SI PREFIXES│ │ [CONSTANT][EXPT][COPY][EDIT]│
│ [CONSTANT]             │   │                        │
└───────────────────────┘   └───────────────────────┘

┌───────────────────────┐   ┌───────────────────────┐
│       CONSTANTS        │   │      SI PREFIXES       │
│ Na   kB   kc   e⁻   R  │   │ f    p    n    µ    m  │
│ G    g    me   mp   mn │   │ c    d   base   k    M │
│ µ0   ε0   h    c    u  │   │ G    T    P    E       │
│ 2.9979245BE8 m/s       │   │ 1E0 µ▶                 │
│ SPEED OF LIGHT IN VACUUM│  │        1E-3 m          │
│ [CONVERT] [EXPT][COPY] │   │ [CONSTANT][EXPT][COPY][EDIT]│
└───────────────────────┘   └───────────────────────┘
```

The SI prefixes module is also a boon, allowing students to investigate the value of the various metric prefixes. This handy App should never be far from students who are working with measurements in mathematics, science or other curriculum areas.

Together with *Fundamental Topics in Science*, *SciTools* provides a powerful collection of tools and learning experiences associated with measurement, and provides a superb complement to any senior mathematics course.

LEARNING ACTIVITY 17: COVERING THE AREA
AREAFORMULAS APP

AIM   To review and consolidate the main area formulas required for work on areas and volumes.

METHOD   The *AreaForm* App provides a clear and thorough coverage of areas of six plane figures, which form the basis for this course.

PROCESS   The App provides definitions, formulas (and animated explanations for these formulas), along with examples, and practice quizzes involving plane and composite figures.
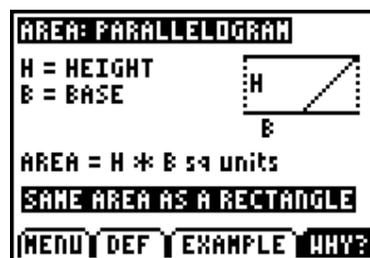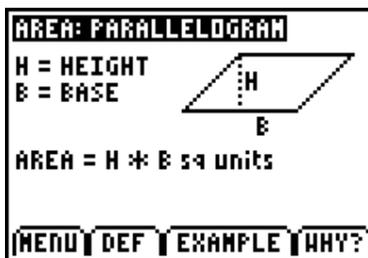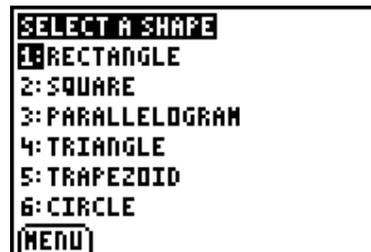
This App provides an invaluable teaching and learning resource for most levels of high school. In the case of senior non-calculus mathematics, it offers a necessary review of work, which should have been well-established in earlier years. This work can then form the building blocks for further work on volumes of simple and composite shapes.
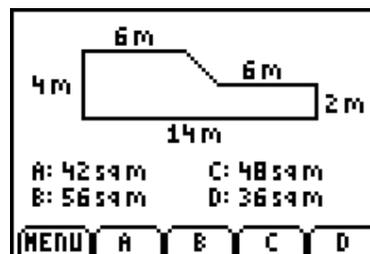
Formulas, of all sorts, of course, are easily dealt with using in-built features of the graphic calculator. Consider, for example, the volume of a cone with base radius 8 cm and height 12 cm. As shown, store the values to these variables using the **STO->** key (just above the **ON** key!). Then enter the formula exactly as it appears on the formula sheet and the calculator will substitute and evaluate the result.
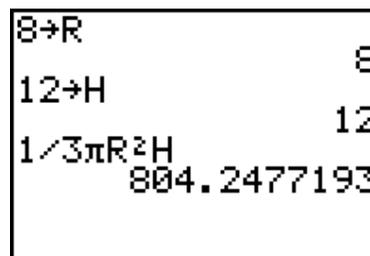
If the value that is required is not the subject of the formula, then the in-built equation solver will allow all other values to be found (Look under the **MATH** menu, the very last entry). Enter the formula as an equation equal to zero as shown. If we wished to find the height required for a volume of 400 cm^3 with a base radius as given, then simply enter these values as shown and **solve** for the height (move to **H** and press the green **ALPHA-ENTER**).
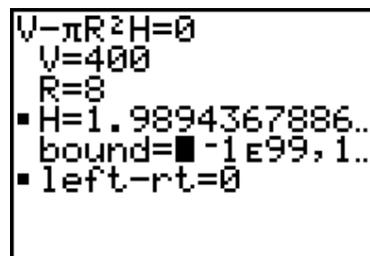
## LEARNING ACTIVITY 18: ALMOST THE SAME!
CABRIJR™ APP

AIM   To build understanding by students, and the ability to solve problems involving similar figures.

METHOD   Using interactive geometry software, students can explore, discuss and consolidate their understanding of similar figures.

PROCESS   Using the *CabriJr™* App (or other interactive geometry software), students begin by creating plane shapes similar to those given.

Begin with a triangle (choose **Triangle** from **F2** and click three times. Choose any of the sides, select **Parallel** from **F3** and construct a line parallel to that side, as shown.
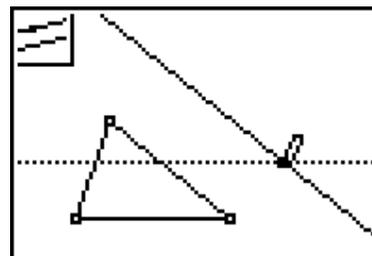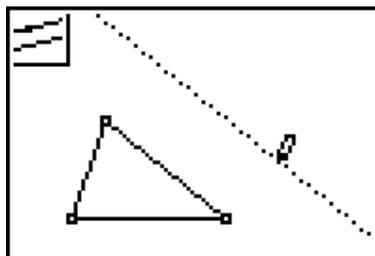
Using the point created on the parallel line, construct a new line parallel to another side of the triangle, and pass it through the chosen point. Do this a third time with the third side of the triangle and pass this parallel through any point anywhere on either of the two existing parallel lines.
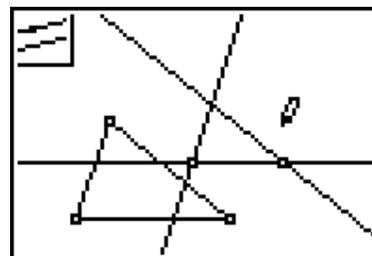
Investigate the figure formed. Make conjectures about its properties and use the interactive geometry software to test their conjectures. Note that the equality of angles may be verified, not only by actually measuring, but by moving the new triangle around to overlay the different angles, as shown.
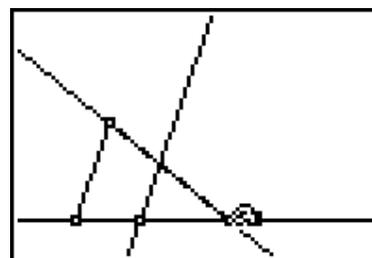
Have students record both their conjectures and their observations. This activity is best done in pairs or small groups, ensuring discussion and verbalization: it is important that students attempt to put into words what they are thinking.

Invite conjecture concerning the sides. How might these theories be tested? They may need an introduction to the **Calculate** command: measure two corresponding sides (discuss this idea!) and then select **Calculate** from **F5**: choose one length, press an operation (in this case, the division operation) and select the second side length. The result is the ratio of the two sides.
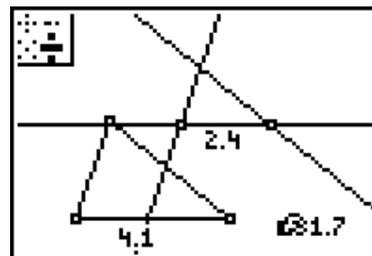
Why does changing side lengths on the original triangle **not** result in any change to this ratio, while changing the second triangle does? Investigate the ratio of other sides.

Students should prepare a brief report on their investigation, outlining the steps taken in the constructions, their conjectures and observations, and their conclusions regarding the properties of similar figures. Follow on with other plane shapes: quadrilateral, pentagon…

Invite students to create similar figures which are **not parallel** to the sides of the original figure: can this be done? What properties of similarity may be used?

It is useful for students to actually use the properties of similar figures in practical situations, such as using a shadow stick to find the height of a tree or building. This is best done outside the classroom.

There may well be benefit, then, after completing this activity, in inviting students to create a model using *CabriJr*™, as shown. Begin with two parallel segments, and then create two parallel lines through the high point of each. Measure the corresponding lengths and then the corresponding angles. This activity further consolidates previous work on similarity, and then provides a useful introduction to the ideas of trigonometry.
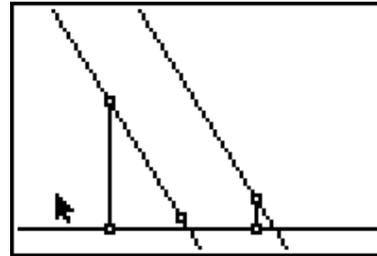
The similar figures method involves measuring the lengths of sides and shadows (if we know 2 sides of a right-angled triangle, we know the third!). A trigonometric method would require only the knowledge of one length and one angle, hence is more efficient.

LEARNING ACTIVITY 19: AM I RIGHT OR NOT?
CABRIJR™ APP

AIM   To build understanding by students, and the ability to solve problems involving right-angled triangle trigonometry.

METHOD   Using interactive geometry software, students can explore, discuss and consolidate their understanding of Pythagoras' theorem and trigonometry.

PROCESS   Using the *CabriJr™* App (or other interactive geometry software), students begin by creating right-angled triangles.

Begin with a segment (choose **Segment** from **F2** and click twice). Choose **Perp** from **F3** and construct a line perpendicular to the segment. Now select **Triangle** from **F2**, and choose the original two points of the segment and a point anywhere on the perpendicular line. If desired, use the **Hide/Show** option from **F5** to hide all but the triangle. Measure the three sides and one of the angles as shown.
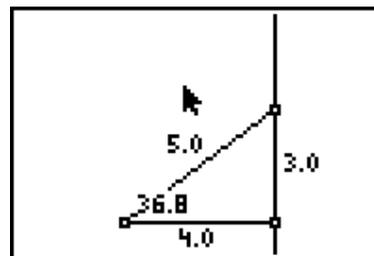
Invite students to investigate the relationship between the sides and the angle. They may begin by setting up a table as follows:

Opposite	Adjacent	Hypotenuse	Angle
3.0	4.0	5.0	36.8
2.9	4.0	5.0	36.1
2.8	4.1	5.0	34
2.6	4.3	5.0	31.1
2.5	4.4	5.0	29.2

Note that in this investigation, we are keeping the hypotenuse constant. This can be easily achieved by adjusting the vertical side (the opposite) and then moving across and adjusting the adjacent side to set the hypotenuse back to 5.
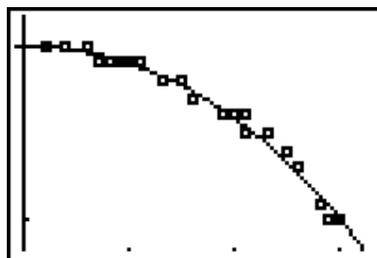
Students are encouraged to produce a table of, perhaps, 30 values, making minor adjustments to the opposite side from 3.0 down to very small (say 0.2).

These values should then be entered into **L1** and **L2** on their calculators, and a stat-plot produced of **L1** against **L2**. The resulting scatter-plot should be recognizable as being roughly one quarter of a circle, which may be verified if desired by entering into **Y1** the formula for a circle of radius 5. Although not required for this course, this provides an interesting and somewhat surprising modelling exercise for students and certainly suggests further questions. Discussion should certainly be drawn to the error involved: what is the source of this error? Could it have been avoided? How might it be minimized?

The table created, of course, is ideal as an introduction to trigonometry, allowing students a plentiful source of data by which they may study the sine, cosine and tangent ratios for these angles. Once again, using the lists on the graphic calculator, students may readily store **L1/5 -> L3** and **L2/5 -> L4**. These values should then be compared to those of the sine and cosine for each of these angles. Finally, **L1/L2 -> L5** should produce the tangent values.

If desired, a command such as **sin^{-1}(L3) -> L6** (or **cos^{-1} (L4)** or **tan^{-1} (L5)**) should each return approximately the same angles that we measured from our many triangles. Once again, useful discussion may pursue the errors here.

In this way, students are given tools to explore real values of trigonometric functions for many different angles – of course, this activity does not rely on the interactive geometry software. It could have been carried out by students actually creating for themselves many triangles which have 5 (or 10, which is a more realistic size for real triangles) as their hypotenuse.

The interactive geometry feature, however, makes generating a large variety of right-angled triangles quite easy. Students may readily practice finding missing sides and missing angles by generating different triangles each time, doing the calculation – and then checking their result. Another alternative, of course, involves the teacher with a ViewScreen™ calculator presenting questions for the class to work with. The example shown, for instance, could involve students using two different methods to find the length of the missing side.
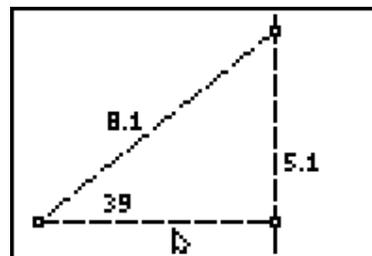
The possibilities are great!

LEARNING ACTIVITY 20: WHERE IN THE WORLD?
WORLD GEOGRAPHY APP

AIM   To support students in their learning of spherical geometry in the context of world time zones.

METHOD   Using the *World Geography* App database as a source of graphs and information related to time zones.

PROCESS The *World Geography* App provides maps and information for all of the 192 countries in the world today.

Using both the maps and the longitude information, students are provided with the means to investigate time differences between any locations they choose.

Further, latitude information can form the basis for investigation of spherical geometry properties of points on the globe.

4. PROBABILITY

The ability of the graphic calculator to generate random numbers is not unique: most calculators can carry out this function. The capacity to record, manipulate and even to view these results graphically, however, is a powerful and important learning feature, which should be used. Simulations, however, MUST be preceded by hands-on experiment if students are to appreciate and understand the true nature of probability. It is only when classroom experiments have established this understanding that the power of the technology may be utilized to generate larger amounts of data than could realistically be created by manual means. And then the fun really starts…

Learning Activities

Learning Activities	Content Reference	App
21. Introducing simulations	Language of chance	
22. Finding Pi experimentally	Applications	
23. All at Sea	Applications	
24. What is the chance of that?	Probability	ProbSim

LEARNING ACTIVITY 21: INTRODUCING SIMULATIONS

AIM   To introduce to students both the ideas of simulation and the simple calculator
tools which may be used to support this understanding.

METHOD   Students use the probability features of the graphic calculator to create their
own simple simulations.

PROCESS   Begin by inviting students to explore the **PRB** option of the **MATH** menu.

The first option is **rand**, which
students will quickly recognize
as the same feature available
on most calculators. It
produces a random number
between 0 and 1.

More powerful and useful for
our purposes is option **5:
randInt(**. This function
produces a random integer
between the two given values.
In the first example shown,
then, students may simulate the
rolling of a normal unbiased
die. In the second, the calculator simulates the rolling of two dice (note the use of braces **{ }**
in order to link the pair of numbers).

Moving back to a simpler concept is an experiment, which
produces only two possible results: heads or tails, left or right,
up or down, as shown.

This last experiment provides a suitable starting point for the
linking of experimental and theoretical probability. Invite
discussion regarding the expectations of students regarding the
outcomes of such an experiment. If they work in pairs, one
partner may generate tosses of the coin while the other records the result. A relevant question
here concerns the true nature of this experiment: in reality, we may be checking the true
ability of the calculator to produce random results!

Some special features of the calculator may be used to advantage
once students have used manual methods for generating and
gathering their data. The **seq** command (**LIST – 2nd-STAT –
OPS -> 5:seq(**) generates a sequence of numbers, which can be
assigned to a list. In the example shown, a sequence of 10
random numbers between 0 and 1 are stored into **L1.**

Also from the **LIST** menu, students may choose the commands **sum** (under **MATH**) and **dim**
(under **OPS**, which gives the dimension or length of the list). In the example shown, the sum

of **L1** (the number of occurrences of **1**) is divided by the number of elements in the list (10) to calculate the proportion of 1's in the first sample of 10 (in this case 0.7 or 70%).

Use **2nd-ENTER** to recall the previous entries, and the experiment may be quickly and easily repeated – note that the second time it runs, a result of 0.3 occurs!

```
seq(randInt(0,1)
,X,1,10)→L1
{0 1 0 1 0 1 1 …
sum(L1)/dim(L1)
            .7
```
```
sum(L1)/dim(L1)
            .7
seq(randInt(0,1)
,X,1,10)→L1
{0 0 1 0 1 0 1 …
sum(L1)/dim(L1)
            .3
```

Students may run this experiment 10 times initially, pooling their results across the class.

Of course, by changing the 10 to 100, or 500, much larger samples may be easily generated, and observations made concerning these results.

```
sum(L1)/dim(L1)
            .3
seq(randInt(0,1)
,X,1,100)→L1
{1 0 0 0 1 0 1 …
sum(L1)/dim(L1)
          .46
```
```
seq(randInt(0,1)
,X,1,500)→L1
{0 0 0 0 0 0 1 …
sum(L1)/dim(L1)
         .534
```

It is very important, of course, that students reflect and report on their results. What were they expecting? *Why* did they expect this?

Invite the class to create their own simulations of other practical situations: traffic lights, lotto draws, horse races, and more. They should then comment on the strengths and the limitations of their models.

LEARNING ACTIVITY 22: FINDING PI EXPERIMENTALLY

With thanks to *Michael McNally, Lower Canada College, Montreal, Canada,*
mmcnally@lcc.ca

AIM   To build student understanding of the power of simulation as a means of
                  seeking answers to quite diverse mathematical questions.

METHOD   Using the random number generating facility of the graphic calculator,
                  students use a very visual means of approximating the value of Pi.

METHOD   Create a [-3, 3] x [-2,2] WINDOW on your
                  calculator.

```
seq(-3+6rand,X,1
,50,1)→L₁
(-.136795476 -1…
seq(-2+4rand,X,1
,50,1)→L₂
(.0871558764 -.…
Circle(0,0,1)
```

Scatter 50 random points in this window.
Seq(-3+6*rand*, X, 0, 50, 1) -> L_1
Seq(-2+4*rand*, X, 0, 50, 1) -> L_2

Create a StatPlot using the plus sign as your Mark.
Superimpose a circle with centre at the origin and radius of 1. This can be done from the
**HOME** screen by accessing the **DRAW** menu.
2nd/**PRGM**(**DRAW**)/#9 Circle. Enter *Circle*(0 , 0 , 1).

How many points landed inside the circle (if a point lands
on the circle, flip a coin to decide if it is in or out)?

Some questions:
- *What is the area of the WINDOW?*
- *What is the area of the circle?*
- *What is the theoretical probability of a point landing in the circle?*
- *What is the experimental probability?*
- *Assuming these probabilities are equal, solve for an approximate value of PI.*
- *Pool a series of results from different students and try to get a better estimate of PI.*

LEARNING ACTIVITY 23: ALL AT SEA
With thanks to Doug Clarke

AIM   To further extend student understanding of the role of experimental probability in answering mathematical questions of all types.

METHOD   By collecting readily available data from a class experiment, students may use experimental probability to find questions related even to world geography!

PROCESS   *What percentage of the world's surface is covered by land?*

There are many ways to seek an answer to this question (including perhaps using the *World Geography* App and adding the areas of every country) but most are tedious and time consuming, or involve simply finding a reference book. A nicer method uses sampling.

Use an inflatable globe of the world. Have students take special note of, say, the thumb of their left hand. Throw the globe to any student in the class and ask them to take note of whether the their left thumb landed on land or sea. If land, then enter 1 into **L1**; if sea, then enter 0.

Have the student with the globe throw it to someone else. Again, record the result with a 1 or a 0. Stop after 10 throws and observe how many "1"s are in the list – or go to the Home Screen and enter from the **List** menu **sum(L1)/dim(L1)**. (**sum(** may be found as item 5 under **MATH** on the **List** menu; **dim(** is item 3 under **OPS** and gives the *dimension* or number of items in the list). Record the percentage shown.

Continue the game for another ten throws, again recording the percentage total. As the total number of samples grows, students should observe this percentage approaching a value which gives an estimate of the percentage of the earth's surface covered by land.

Discussion should follow concerning this effect: the greater the sample size, the better the estimate. Discussion should also focus upon limitations of the method: what factors may reduce the accuracy of the result?

Ask the students to consider other sampling methods, which might be used to answer this question.

LEARNING ACTIVITY 24: WHAT IS THE CHANCE OF THAT?
PROBSIM APP
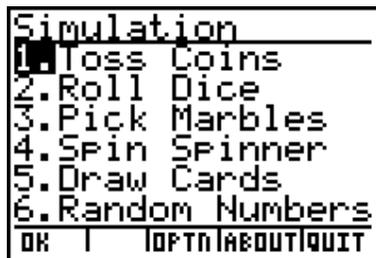
There is absolutely no argument that students need to engage physically with as wide a variety as possible of chance situations before turning to computer- or, in this case, calculator-based simulations. Tossing coins, rolling dice, picking marbles, numbers and other objects, drawing cards and even spinning spinners: there is much to be learned from the physical engagement in such activities. Tempting though it might be for ease and convenience or even for a quieter classroom, it is vital that students of every age engage in such activities to lay solid foundations for their future work in probability. Further, while such "classroom experiments" offer clear and easily recognizable links to simple number statements, it is also critically important that students engage with "real world" aspects of this important aspect of mathematics: *How often do you get caught at those traffic lights? Why do they always seem to be out of my size? How can I increase my chance of winning in the school Easter raffle?*

Once such foundations are established, then the world of simulation comes into its own. Students may quickly and easily generate vast amounts of experimental data, which can be readily displayed in a variety of ways. Using the *ProbSim* App, they can control and manipulate the experiments, too, bringing a richer and deeper understanding of the relationship between the number of ways in which an event can occur and the frequency of a "win".
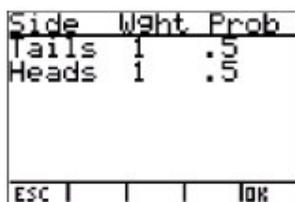
The example given here is the first menu choice, the Coin Toss. Selecting this takes the user to a blank screen. Choose **TOSS** to begin the simulation with a coin toss. Choose **SET** to go to the **SETTINGS** screen (below) where the user may control various aspects of the experiment. Here we decide if we want to toss 1, 2 or 3 coins, and other variables. Pressing **ADV** "advances" on to the next screen, where the

weightings of the coins may be varied. In this initial experiment, we will leave the coins equally weighted and so with an equal probability of head or tail.
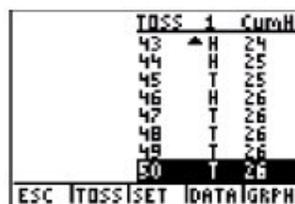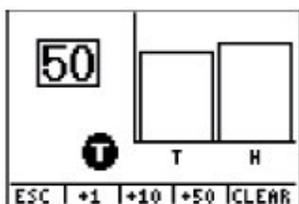
After pressing OK we begin with a single coin toss and then select **50** to see the result of 50 trials. If desired, students may select another 50, and so on to observe the long term results. Finally, pressing **DATA** allows students to inspect the actual results, along with a cumulative count of the number of heads obtained. These results may be stored in lists for further study.

This simple experiment demonstrates both the ease of use and the clarity of the display for this App. It is suitable for use with students from quite early years through to seniors and offers a wealth of investigations and experiments which will build student understanding of probability concepts.

5. ALGEBRAIC MODELLING

The use of technology in the modelling of linear and non-linear relationships can be considerably enhanced by reference to "real-world" occurrences, and particularly so if such models can be generated by the students themselves from a variety of sources. This unit examines a range of modelling tasks which are closely linked to the data strand through the use of data loggers as a means of generating results which can be modeled using the functions required for this course: linear, quadratic, cubic, hyperbolic and exponential. Earlier units have already introduced these ideas of algebraic modelling, in the contexts of Data (Activities 11 and 13) and Geometry (Activity 19). Such activities should also be introduced in the context of modelling in order for students to appreciate the links that exist across the modules of the syllabus.

Pythagoras claimed that "All is number". He may have been closer to the mark if he observed that "All is modelling".

Learning Activities

Learning Activities	Content Reference	App
25. Walking the line	Linear relationships	*CBL™/CBR™*
26. How long is a piece of string?	Linear relationships	
27. Guess my rule	Non-linear relationships	*GuessCoeff*
28. The Bounce Factor	Non-linear relationships	*CBL™/CBR™*
29. Move into the light	Non-linear relationships	*DataMate*
30. Turn up the heat	Non-linear relationships	*DataMate*
31. Models in fashion	Non-linear relationships	
32. Nuts and bolts of algebra	Basic skills & techniques	*PrettyPt/Symbolic*

LEARNING ACTIVITY 25: WALKING THE LINE
CBL™/CBR™ APP

AIM   To build student expertise in working with linear algebraic relationships, particularly with regard to gradient and y-intercept.

METHOD   The use of a motion-detector is an excellent way for students to physically represent the algebraic concepts of gradient (speed of movement) and y-intercept (starting position).

PROCESS   Begin with students informally moving in front of the motion detector, generating real-time graphs which the class should interpret. Attention should be drawn to the horizontal axis (time in seconds) and the vertical axis (distance in metres).

In the example shown, note that the motion began almost exactly 3 metres from the CBR™, and that the movement towards the CBR™ occurred for approximately 7 seconds.
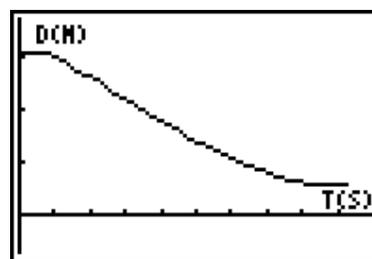
Suitable warm-up activities include asking students (with help from class-mates where necessary) to make mountains and valleys (*Do I start close or far away?*), move fast and slow, towards and away from the source – all building student understanding of the concepts required to interpret such graphs. Some use of the *Distance Match* Application (part of the *Ranger™* program) will also help to build skills and understanding.

When students are sufficiently comfortable with the working of the device and the interpretation of the results, it is time to have someone again walk a simple straight line (like that shown above) and then quit the *Ranger™* program, and turn on the graph grid (**FORMAT: GridOn**). Now the real work begins!
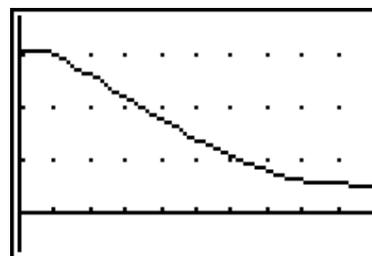
Explain that the task is to try and match a straight line (linear function) to the main part of the graph shown. Since this is a modelling task, it is usually a good idea to begin with the graph **Y1 = X** (as shown). *How do we adjust the function to match the graph?*
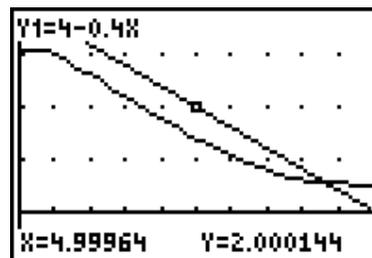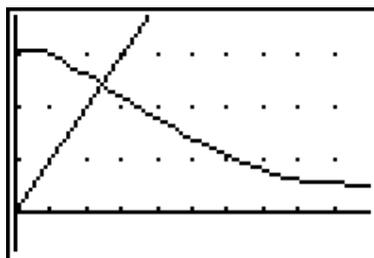
The trial-and-error nature of this activity is quite important: students learn more from their mistakes than from correct responses. Even with no background in this type of task, students will quickly discover the ideas of negative slope and y-intercept, and keep adjusting the values until they arrive at a result that most are happy with. While they should begin by counting the grid points to arrive at a "rise over run" value for the gradient, students will quickly move away from the fraction to the decimal form, since this is more easily adjusted.

LEARNING ACTIVITY 26: HOW LONG IS A PIECE OF STRING?

AIM   To build student expertise in working with linear algebraic relationships, particularly with regard to gradient and y-intercept.

METHOD   This is a simple activity in which students generate linear data using measurement of the length of a piece of string or rope.

**PROCESS**   Some discussion concerning the need for accuracy of measurement may need to precede this activity.

Pairs or small groups of students are issued with pieces of string or rope of different lengths. These are measured carefully and students record a zero in **L1** and the length in **L2.**

The instructions now are simple: tie a knot in your rope, and measure the new length! The knots do not have to be terribly tight, or very loose, as long as they are consistent. Use **L1** to record the number of knots, and **L2** the length of the rope.
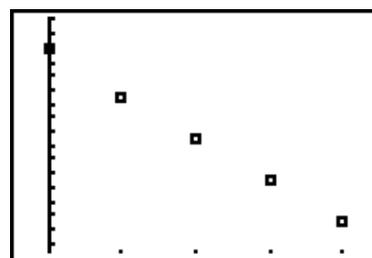
Observations should be recorded, and then an effort made to match a linear function to the values.
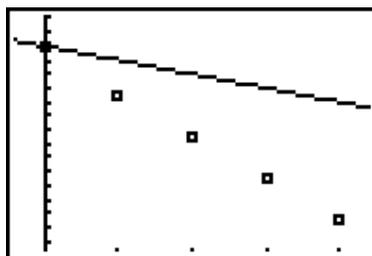
The negative gradient in this situation is worthy of comment: ask students to explain in their own words the implications of this feature. Similarly, the significance of the y-intercept in relation to the starting value.
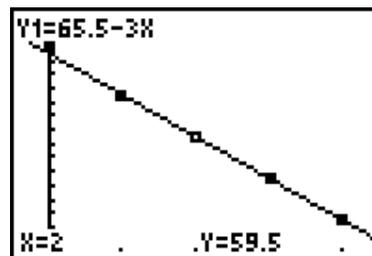
It is important that students attempt to take account of the error that is a natural part of such an experiment: it should not be "airbrushed" away, but sought out and discussed.

Finally, experiments such as this one provide excellent introduction to lines of best fit. Once the students are satisfied with their own curve-fitting efforts, they should compare this with that derived from the calculator. Again, there may be profit gained by comparing the two linear models available: Med-Med and LinReg. In this case, we would expect there to be little difference between them since the scores are very consistent.

Two other experiments which lead to linear functions have already been described in previous sections: the comparison of Centigrade with Fahrenheit temperatures in learning activity 11, and the birthday data from activity 13. Each provides valuable practice for students in generating their own linear functions which model these situations.

LEARNING ACTIVITY 27: GUESS MY RULE
GUESS MY COEFFICIENT APP/TABLE OF VALUES/TOPICS IN ALGEBRA PT 1

AIM To build student expertise in working with linear algebraic relationships, particularly with regard to gradient and y-intercept.

METHOD This activity is intended to build confidence in working with both graphs and tables of values, which represent linear functions.

PROCESS Simple Guess My Rule activities should have been a familiar practice since the beginning of high school. Indeed, no treatment of early algebra is complete without work on the numerical basis, which underlies all algebraic forms.

Many programs exist which offer Guess My Rule activities. The *Guess My Coefficients* App is an excellent tool by which students may build their skills in linking the algebraic and graphical forms, for a range of function types, linear, quadratic and (not required for this course) absolute value.

While this App provides a wonderful tool for learning about graphs, it does not offer a table of values option.
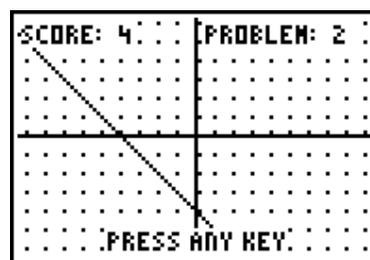
This is easily remedied, however. Using a ViewScreen™, simply enter a simple linear function in the **Y=** menu (without showing the students), show the **TABLE** and then turn on the overhead projector. Students are then instructed to make their calculator look the same as this.

If they do not know already, they will quickly learn to recognize the "magic value" (*What are the numbers going up or down by?*), corresponding to the gradient in the graphical form; this goes next to the **X** term in the rule. In the example shown, the rule is of the form **Y1 = 2X + ?** since the numbers are going up by 2. The missing number may be found by substituting any **X-**value into the formula you have and seeing what must be done to make it work: for example, if we use **X** = 3, **Y1 = 2 x 3 = 6**, but our table says that when **X** = 3, **Y1** = 3, so we should subtract 3. This gives the rule as **Y1 = 2X – 3**.

Later, after some practice at substituting, students are likely to notice that this constant term always corresponds to the value of **Y** when **X = 0**.

Students will quickly build confidence and competence in interpreting the rule for linear tables of values. It is a simple thing to put a variety of linear functions into **Y1** with the overhead projector turned off, and then display the table of values for students to match. The same technique can be used for the graphical representation, not just for linear functions but for quadratics and others as well.

Consider the example shown above. When students calculate the difference between terms, they find that the value is not constant, and so the function is not linear. However, when they take the difference of these difference values, they do arrive at a constant value, indicating that the function is quadratic (degree 2). If it takes 2 steps to reach a constant, the function has degree 2; if three steps are needed, then the function is cubic – degree 3.

Note, too, the zero values of this function: this indicates a quadratic of the form
$$Y1 = A(X + 3)(X – 2)$$
In this case, in fact, the value of **A** is 1.

Of course, quadratics and cubics are far more difficult to evaluate than linear functions. However, if students are at least taught to recognize the importance of zeros and y-intercepts, then much can be achieved.

The table of values has other applications as well. When applied to equation solving, they provide a very useful place to start, especially when used as an extension of the previous work on guessing the rules for linear functions.

Consider the example shown. Students should first be asked to find the rules for the two functions, **Y1** and **Y2**. (In this case, **Y1 = 4X – 1** and **Y2 = 2X – 3**.

Now they should find the value of **X** where both functions are the same (**X = -1**). Clearly this is the point where **4X – 1 = 2X – 3**.
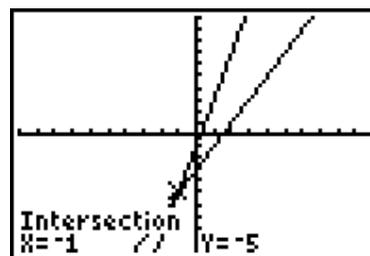
While students will certainly have encountered equation solving prior to this course, they may well have little understanding of what it represents. The table of values is a powerful way for this to be made clear. This concept is further reinforced by use of the graphical representation shown, where students appreciate visually the nature of this point of intersection. Some time should be spent reinforcing these concepts with simple linear and non-linear examples, and then it may readily be extended to the solving of simultaneous equations. The **Intersection** option of the **CALC** menu (shown) is a powerful tool for students to use in this regard.

Finally, Chapter 2 of the *Topics in Algebra* App, **Linear Equations**, gives a superb coverage of the solving of linear equations, first using tables and graphs, and then
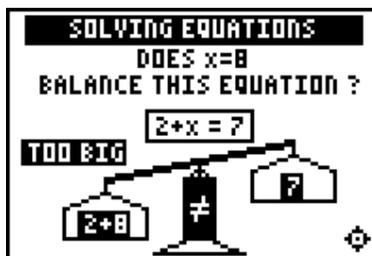
symbolically using algebra. As with other topics in this excellent interactive textbook, the coverage of this important unit is achieved carefully and thoroughly. Through the introduction, observations and activities, students are provided with a wealth of appropriate learning experiences to support them.

## LEARNING ACTIVITY 28: THE BOUNCE FACTOR

AIM   To build student expertise in working with quadratic algebraic relationships, based upon real-world data.

METHOD   This is a simple activity in which students generate quadratic data using the motion detector in two different ways.

PROCESS   This activity may begin with the throwing of a soft ball or other suitable object to and from students around the room. Draw their attention to the path of the motion, and have them sketch this motion on a pair of axes.

Next have students try to model that motion themselves by moving in front of the CBR™. Invite discussion and contributions from others in order to try to improve the graphs produced. This activity quickly reveals the nature of student understanding regarding parabolic motion. Particular attention should be drawn to the different units being used in both situations. Their first sketch graph will have displacement on both axes, horizontally and vertically. The motion detector captures time on the horizontal axis and displacement on the vertical. Although both graphs may look similar (and certainly both are parabolic), they are measuring quite different things.

Finally, conduct a ball bounce experiment, using the motion detector. Each bounce is an inverted parabola. Have students isolate one of the parabolas and attempt to match a quadratic function, taking note of such features as the coordinates of the maximum turning point. Suggest the use of the quadratic form $Y = A(X - B)^2 + C$ as the basis for their model, and have them record from their graph the coordinates of the turning point, as shown.
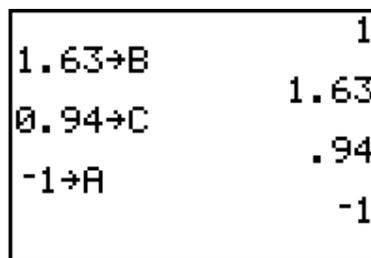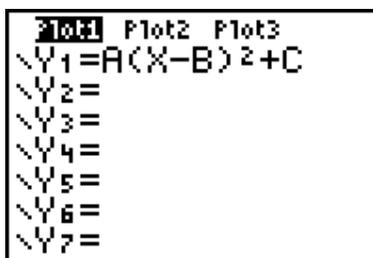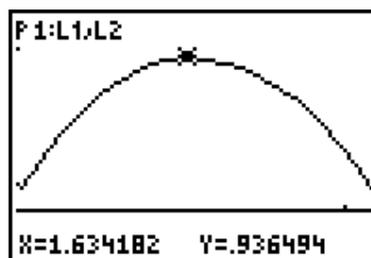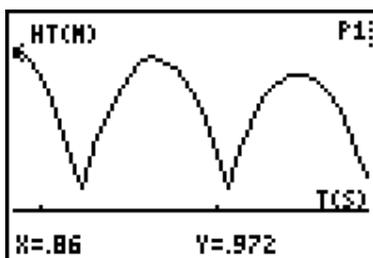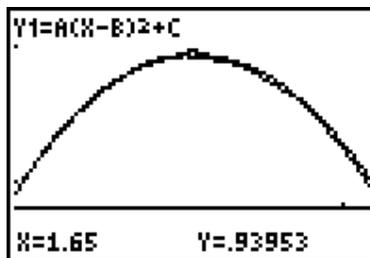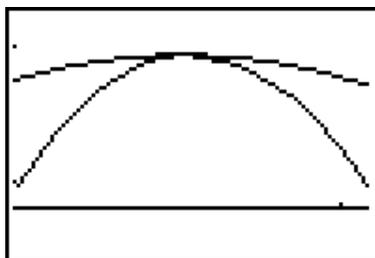
A little trial and error with the values for **A**, then, leads to a good model for the motion. Changing the viewing window allows students further practice as they repeat this exercise for the other parabolas available from the ball bounce.
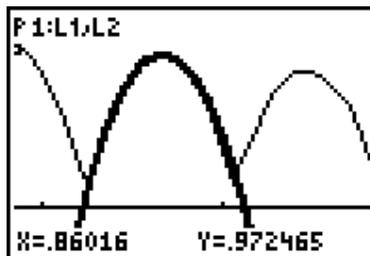
This activity brings with it another bonus. If the ball bounce is carefully recorded, the high points of each parabola actually decrease exponentially: students should verify this, both by modeling, but also by observing that the ratio of consecutive terms in such a sequence should be (fairly close to) equal! This "bounce factor" could be measured for a variety of different balls – in the example shown, the bounce factor was around 0.9, making it most suitable for this activity.
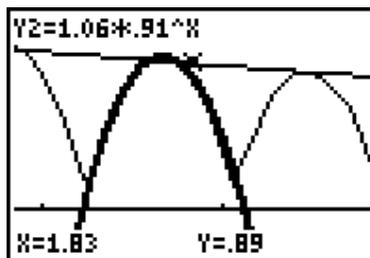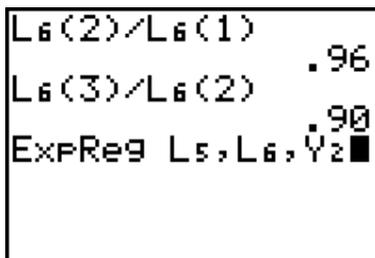
As always, the activity should conclude with students recording their actions, conjectures, observations and conclusions.
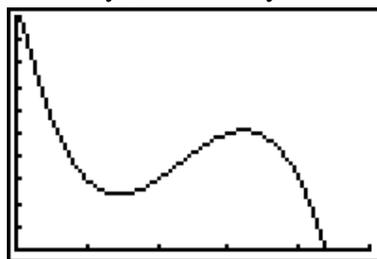
Cubic equations are not so readily modeled from real-world data, although a very nice exploration involves designing a roller-coaster ride which must fit into a space 5 metres long, and should be 10 metres in height. (Try 2 – (x-1)(x-2)(x-4)!)

LEARNING ACTIVITY 29: MOVE INTO THE LIGHT
## *DATAMATE* APP

AIM To model motion which is hyperbolic in shape from real-world data.

METHOD This is a simple activity in which students generate hyperbolic data using the motion detector in combination with the light probe.

PROCESS Using a CBL2™ data logger, *DataMate* App and both CBR™ and light probe, students may gather data on light intensity as a function of distance from the source. This experiment is best done in a darkened room with a strong light source, such as an overhead projector.

Once again, students should begin by estimating what they think the graph of the light intensity will look like. Will it be linear? Positive or negative gradient? If not linear, then what should we expect in the curve?

Discussion should lead to the recognition that the graph should be hyperbolic. This is a graph of distance against light intensity. Attention should be drawn to the natural behaviour at the extremes: what is happening when distance is zero? As distance from the light source becomes large?
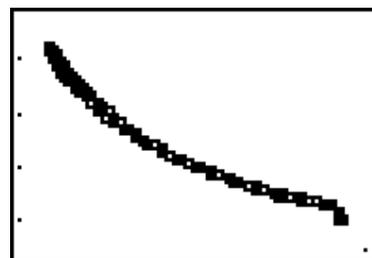
The graph shown was produced by simply moving the CBL2™ and probes back and forth in front of the light source over a 5 second interval. The hyperbolic nature of the curve may be clearly seen, but if the graph is **TRACE**d, then the actual nature of the motion will be revealed through the sequence of points jumping back and forth rather than following the line of the curve.

The fact that the function being modeled is actually the square of the hyperbola (of the form $1/x^2$ rather than $1/x$ does not detract from the value of this activity. The properties of the two functions are sufficiently similar that the main features are preserved.

Students should be given a variety of situations which may be modeled hyperbolically: such practical situations as the effects of price upon ticket sales, or the effect of the number of workers upon the planting of seeds lead naturally to hyperbolic functions. Experience with real-world examples will help students to feel confident in their dealings with such functions.

LEARNING ACTIVITY 30: TURN UP THE HEAT
DATAMATE APP

AIM   To model the exponential function using real-world data..

METHOD   Students use the data gathered from a previous activity (Data Learning Activity 11: *Can you feel the warmth?*) to investigate the properties of exponential functions.

PROCESS   In this previous activity, students used two temperature probes together to collect data in both degrees Centigrade and degrees Fahrenheit. Here we only need one of those sets of data – it does not matter which, but for convenience we will use degrees Centigrade.

The experiment consisted of measuring the air temperature first for approximately 5 seconds, then 20 seconds into hot water, followed by another 20 seconds in cold water. Using the **WINDOW** menu, it is a simple matter to focus on the last section of data, between approximately 25 and 45 seconds, as shown.


This data provides an excellent exponential model which students should observe, discuss and, eventually, attempt to model. The actual finding of correct figures for this model is not the important part of this activity. Rather,
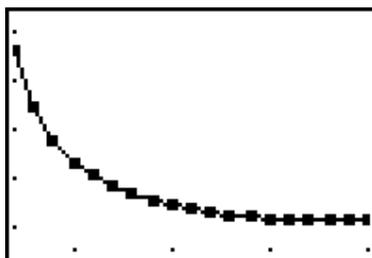
it is the process of becoming familiar with the effects of the various parts of the exponential function that drives it. Beginning with a model of the form $ab^x + c$, students in small groups are free to experiment with different values in search of a reasonable fit.
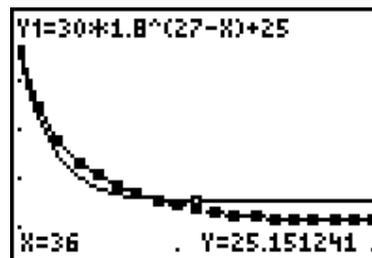
Having attempted this activity, a suitable consolidation exercise would involve doing the same for the data from the first part of the graph: from 5 to 25 seconds, giving the plot shown. Again, the real point of this exercise is the discussion that should ensue concerning the properties of the function and those of the real-life situation, which gave rise to the data.
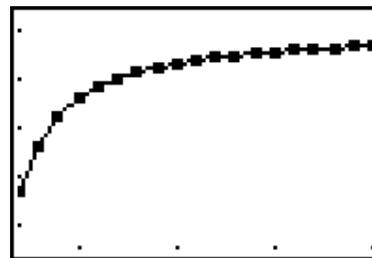
## LEARNING ACTIVITY 31: MODELS IN FASHION

When I first took part in this experiment, the resulting plot looked strangely familiar to me, until I remembered that I had seen it some years previously as part of the Shell Centre's excellent unit on Functions and Graphs. It was given, not as a temperature vs time graph but as velocity vs time, and accompanied by the question: *"What sport is this?"* Listed were over a dozen different sports – archery, pole-vaulting, sky-diving, snooker, high-diving, high jump, golf, fishing, drag-racing, 100-metre sprint and more.
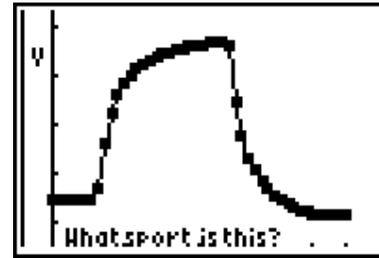
I have used this activity many times over the years with many different classes, and each time I actually come away with a different idea as to the "correct" answer! This makes it a superb learning experience: there is no definitive right answer, but the discussions, reasoning and arguments which ensue guarantee an interesting and worthwhile lesson.

Another interesting discussion starter for seniors beginning their driving careers is the model for fuel consumption. If we assume that the amount of fuel a vehicle uses is related to the speed of the vehicle (discuss this assumption! While there are other variables involved, the process of modelling always involves simplifying and making assumptions) then what would the graph of speed vs petrol consumption look like?

In general, when attempting a modelling task, students should begin with *y = x* and then attempt to isolate ways in which their situation differs from this model! Some simpler examples may include a graph which describes the relationship between the *length* of a race and the *time* it takes to run the race; the *price* of an item and the *profit* that is likely to result (as in the cost of movie tickets); even the old favourite: hoisting the flag: time vs height!
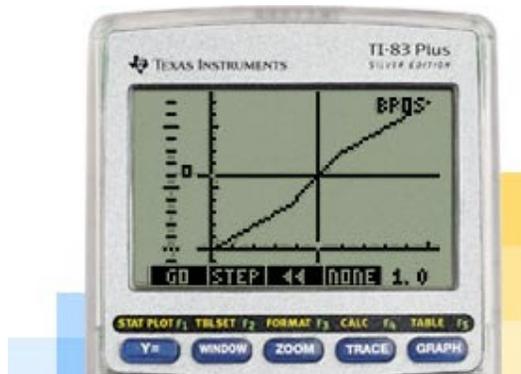
But wait! There is more. One of the more exciting concept Apps developed in the past twelve months is called *MathWorlds* and has been developed as part of the ongoing *SimCalc* project, which seeks to develop tools which allow students to learn much of their mathematics in the context of real activities. The basic premise of this project is that mathematics should always be *about something*. All too often, it is presented to students in a formal decontextualised manner which not only defies their understanding, but bores them silly. *MathWorlds* offers powerful animation tools by which students may observe and investigate real-time motion, and then create their own motions – both directly onto the calculator AND using a CBR™ to introduce their own physical movement to the scene. This tool offers powerful and significant mathematical opportunities to students from an early age to begin building concepts which will form the basis for their later study of calculus.

LEARNING ACTIVITY 32: NUTS AND BOLTS OF ALGEBRA
PRETTYPRINT AND SYMBOLIC APPS

AIM   To assist students in building skills and understanding related to algebraic manipulation and, in particular, solving linear equations.

METHOD   Access to simple symbolic manipulation facilities can prove of great benefit to students who need support in their mastery of algebraic skills. This section describes such a scaffolding option.

PROCESS   The free Apps, *Pretty Print* (from *SoftTheiss Software*) and *Symbolic* (*Detached Solutions*) extend the capabilities of the TI-83 Plus to include basic simplifying and expanding of algebraic forms. When used with two simple BASIC programs (*ALG83* and *EQNSOLVE*) students may easily enter algebraic expressions to be simplified and expanded, or equations which may be solved step-by-step. This activity takes such equation solving as its focus, since the ability to solve linear equations is of key importance in this course.

When students run the *ALG83* program, the opening screen provides three options, as shown. If an expression to be simplified is already entered in **Y1** then it can be picked up from there; otherwise it may be entered now by choosing option 2. Expressions to be entered may be in one or more variables: for example, $5x – 3 – 2x + 11$, $2x(3x – 2)$ or even $2A – 3B – 3A + 5B$.
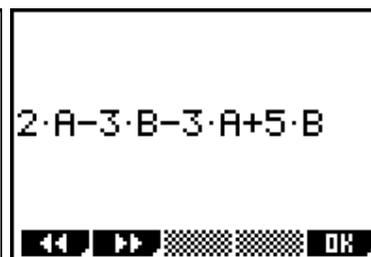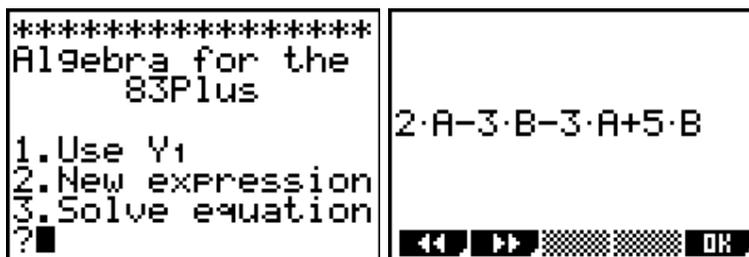
The expression is then displayed, and then a variety of options shown. In this case, we could choose to **SIMPLIFY** and the simplified form is displayed.
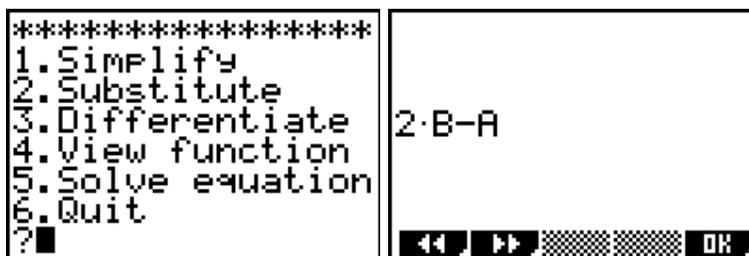
The equation solver begins in a similar way, offering to pick up expressions in **Y1** and **Y2**, inviting a new equation to be entered, or go back to the *ALG83* program. Equations are entered as two distinct parts: left-hand side and right-hand side. It is displayed and then options provided.
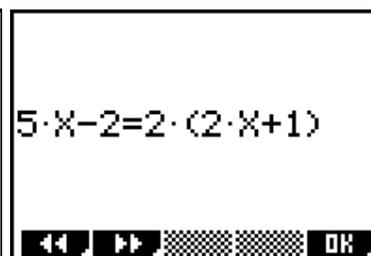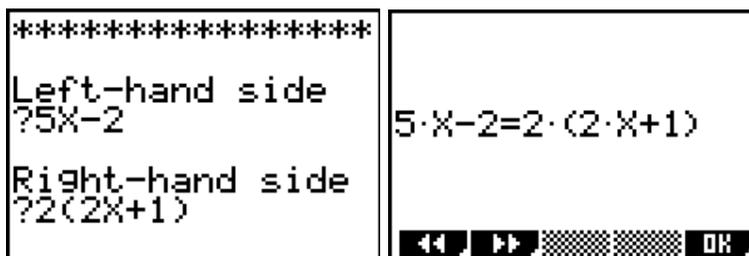
The first option (**Linear Solve**) supports the equation solving process. The graph of the two functions involved may be viewed, or the in-built equation solver may be used to produce a result.

Choosing Linear Solve presents the equation once more, as the user decides on the first action. In this case, that would involve removing the parentheses on the RHS. This is considered a "simplifying" action, and option 2 should be chosen at the next screen.

```
****************
1.Linear solve
2.View graph
3.Fast solve
4.Finish
?
```

```
****************
Enter an action.
e.g. -2X or +3.
Type 1 for menu,
2 to simplify,
0 to finish.
?
```
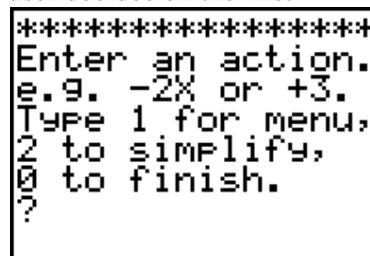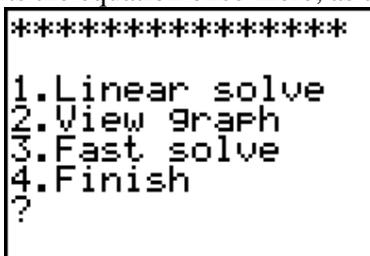
The newly simplified form is displayed, and the next action may be chosen: here, that could be to subtract 4**X** from both sides. Entering **–4X** displays the effects of this action, and pressing **OK (GRAPH)** actually simplifies the equation.

```
5·X-2=4·X+2

◄◄  ►►  ▒▒▒▒▒  OK
```

```
****************
Enter an action.
e.g. -2X or +3.
Type 1 for menu,
2 to simplify,
0 to finish.
?-4X█
```
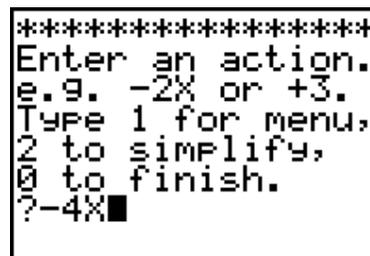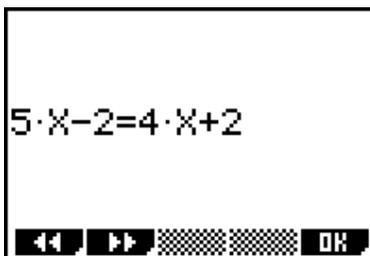
Next, we add 2 to both sides and the final result is achieved.

```
(5·X-2)-4·X=(4·X+2

◄◄  ►►  ▒▒▒▒▒  OK
```

```
X-2=2

◄◄  ►►  ▒▒▒▒▒  OK
```
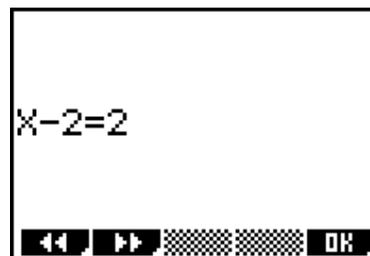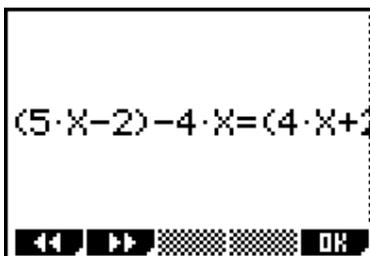
This activity supports students in their learning of the processes required for solving linear equations. It draws their attention directly to both the actions required at each step, and the layout and expected results, which can be recorded as their record of the process.

```
(X-2)+2=(2)+2

◄◄  ►►  ▒▒▒▒▒  OK
```

```
X=4

◄◄  ►►  ▒▒▒▒▒  OK
```
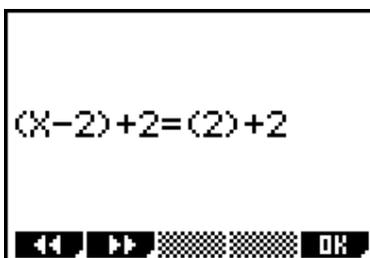
It may be argued that students by Stage 6 should have already mastered these processes, and for many students that may be true. The reality of General Mathematics, however, is that there will be a significant number attempting this course for whom such scaffolding will be of great benefit, even if only as a revision and consolidation sequence.

Such supporting devices suggest once more the true benefits of good technology in teaching and learning: allowing teachers in their instruction to *begin where the student is at* and to work from there, using tools that allow for individual practice, patiently responding to the levels of each student's needs, while at the same time offering them wings on which to soar in their study of, not just the nuts and bolts, but the powerful and important features of mathematics as it applies to their lives and the real world.

AVAILABILITY OF APPS

From *http://education.ti.com*

1.	Topics in Algebra Part 1	$US7
2.	Area Formulas	FREE
3.	CBL™/CBR™	FREE
4.	CabriJr™	FREE
5.	CelSheet	$US15
6.	Decimal Defender	FREE
7.	Fundamental Topics in Science	$US7
8.	GeoMaster™	$US15
9.	Logic Ladder	FREE
10.	SimCalc MathWorlds	Shareware version free
11.	Math by Hand	FREE
12.	Number Curiosities	FREE
13.	ProbSim	FREE
14.	Puzzle Pack	FREE
15.	Rational Rampage	FREE
16.	SciTools	FREE
17.	SolveIt	FREE
18.	StudyCards	FREE
19.	TimeSpan	FREE
20.	What's my Angle?	FREE
21.	World Geography	FREE

From other developers (Search at *http://www.ticalc.org*):

22. LOGO
23. PrettyPrint
24. Symbolic
25. SMILEMathematics

The supporting files for the algebra activities described here may be downloaded from:
http://homepages.ihug.com.au/~arnolds/alg83.zip.

TRADEMARK ACKNOWLEDGEMENT

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