Making Algebra Meaningful With Technology

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Introduction

I begin by observing that this is my thirtieth year as a teacher of mathematics. Soon after I began my teaching career, personal computers also make their introduction to the classroom. I remember being responsible for the purchase of an Apple IIe computer back in 1980, intended to bring my school rapidly into the new “computer age”. It is interesting to look back over that time and, in particular, to ponder what we have learned from both classroom research and the wisdom of practice concerning the use of technology as an aid to learning.

From my perspective, as classroom teacher, researcher and academic, it is possible to make some fairly well-supported and sensible statements at this point in time concerning good teaching and learning, the teaching and learning of mathematics, and of algebra in particular. It is then possible to relate these to the appropriate and effective use of technology for the learning of algebra in a meaningful way.

1. Students learn best when they are actively engaged in constructing meaning about content that is relevant, worthwhile, integrated and connected to their world.
2. Students learn **mathematics** best when
    - ❏ They are active participants in their learning, not passive spectators;
    - ❏ They learn mathematics as integrated and meaningful, not disjoint and arbitrary;
    - ❏ They learn mathematics within the context of challenging and interesting applications.
3. Students learn **algebra** best when
    - ❏ It is not presented as meaningless symbols following arbitrary rules;
    - ❏ The understanding of algebra is based upon concrete foundations, with opportunities for manipulation and visualisation;
    - ❏ Algebra is presented as a vital tool for modeling real-world applications.

And the role of technology in the process?

Technology in mathematics and science learning plays two major roles:

- As a tool for REPRESENTATION, and
- As a tool for MANIPULATION

Good technology supports students in building skills and concepts by offering multiple pathways for viewing and for approaching worthwhile tasks, and scaffolds them appropriately throughout the learning process.

Bearing these principles in mind, it is timely to look now at ways in which they may be integrated using appropriate tools. My vehicle of choice for this exploration is the new TI-Nspire platform from Texas Instruments which, in both handheld and computer software forms, offers a very complete mathematical toolkit, with dynamically linked multiple representations. Such a tool represents the current end-point of thirty years of research and classroom experience in the teaching and learning of mathematics in general, and of algebra in particular.

If algebra is to be taught in an effective and meaningful way, then it must be taught differently than has been the case in general to this point. High school algebra is probably the clearest example of the malaise which affects almost all of school mathematics. We can scarcely claim to be successful in the teaching and learning of a subject in which the vast majority of students, after studying the subject for at least 11 years, leave school not only being unable to apply much of what they have “learned” in any practical or realistic way to their lives, but with an active and often virulent dislike of the subject. Even many of our “success stories” may be very capable “technicians” but can scarcely claim to have any deep mastery or understanding of this discipline. They can make the moves and perform the manipulations, but do they really understand what they are doing?

By most reasonable measures, it is fair to claim that the teaching of mathematics in schools generally has been less than successful. Some might say spectacularly unsuccessful!

We can identify two significant factors which have contributed to this current state:

1. Much of school Mathematics is taught in a decontextualised, fragmented way, with little connection to the lives of students or to the world beyond the classroom and examination.
2. Much of school mathematics is taught in a socially and intellectually isolated way, as a series of routines to be learned rather than processes to be understood. It is algorithmic rather than meaningful, for what is algorithm but a suspension of meaning, designed to break learning down to a memorized series of steps. Efficient? Perhaps. Meaningful? No.

So what might be done?

First, look for opportunities to teach school mathematics within contexts that are rich in meaning and significance for students, engaging them and encouraging them to interact both with the mathematics and with their peers in the learning of that mathematics.

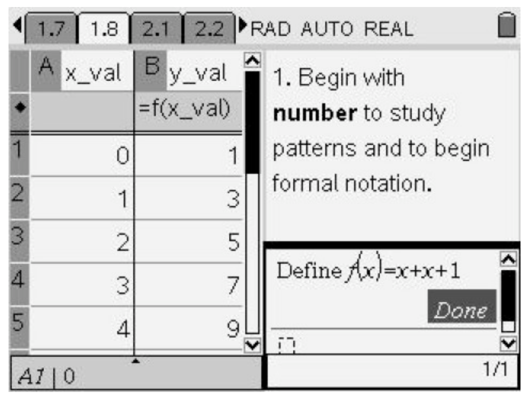
Second, reward informal as well as formal approaches to mathematical thinking. Encourage multiple representations and multiple approaches to problems and to solutions. While algorithmic approaches may be considered efficient in reaching a specified solution, the cost of that efficiency has been high, since it robs students of the opportunities to play with the mathematics they are seeking to learn, to make mistakes (and to learn from those mistakes), and to explore individually and with others in a co-operative learning environment.

There is a clear and highly significant role for good technology in this review of school practice. We may consider the example of the learning of algebra in seeing how such an approach may begin in our classrooms.

Research over the past thirty years points to some clear steps in the process of learning algebra effectively, and the possibilities of new technologies point to some new steps with great potential to assist in bringing meaning to the learning.

Begin with Number

Just as algebra is, most purely, a generalization of the rules by which we operate with numbers, the path to algebra logically grows from students' knowledge and understanding of numbers and their operations. Number patterns, in particular, offer a perfect "jumping off" point by which students may be actively engaged in studying these rules and operations, and tables of values provide a powerful tool for exploring and conjecturing. The simple "guess my rule" games which teachers have used for many years may go well beyond just building simple patterns. They may also be used to introduce the symbolic notation of algebra in a practical and meaningful way.



From simple linear functions such as **y = 2x + 1** students can be challenged to find the rules for variations on the same theme (what about **y = x + x + 1**? **y = 3x + 2 – x – 1**?) – Yes, that rule is correct but it is not what I have – how else could the rule be written?

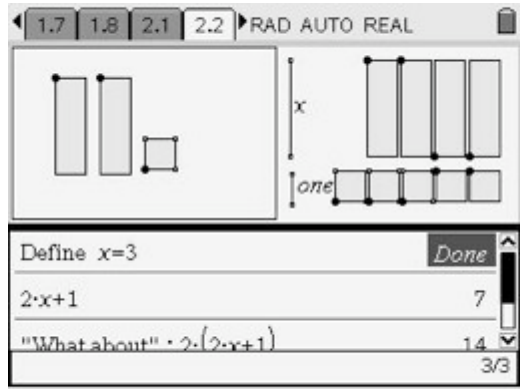
Then on to factors, such as **2(2x + 1)** – stressing the careful use of appropriate language: multiplication is always "lots of" – 3 x 4 is 3 "lots of" 4 and **2(2x + 1)** is **2 lots of 2x + 1**!

We do have much to learn from primary school: subtraction is "how far from?" So 5 – 3 is really "how far from 3 to 5"? Up two steps. Simple?

Then what about -3 – 4? How far from 4 to -3? Clearly, "down 7 steps" if we use a ladder metaphor.

Careful use of correct language is a huge step towards students making their mathematics meaningful, initially with work on number and later, inevitably, with their algebra.

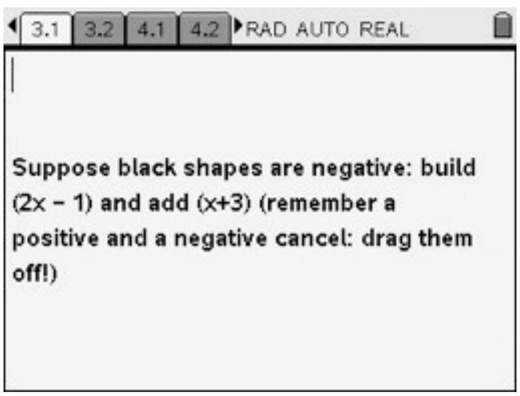
Build firm concrete foundations

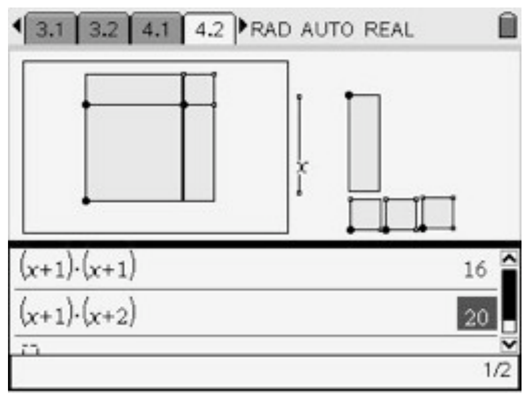


The second "golden rule" from my own teaching experience and also well-grounded in classroom research concerns the appropriate use of concrete materials to provide a firm foundation for the symbolic forms and procedures of high school algebra. "Area models" provide a powerful and robust means for students to interact with symbolic forms in ways both tactile, meaningful and transferable.

Two major limitations may be identified with the use of such concrete materials in this context: there is no direct link between the concrete model and the symbolic form, other than that drawn by the teacher – students working with cardboard squares and rectangles must be reminded regularly what these represent.

Of even greater concern, these concrete models promote a static rather than dynamic understanding of the variable concept. Both these limitations may be countered by the use of appropriate technology to scaffold and support the tactile forms of these models.

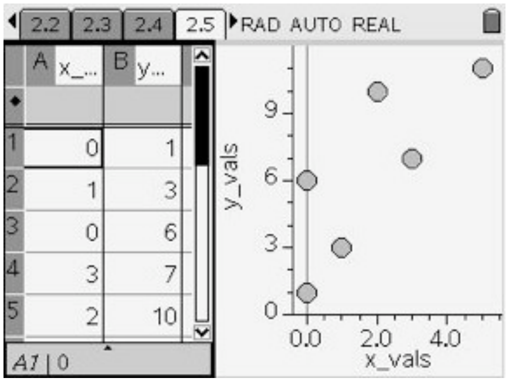


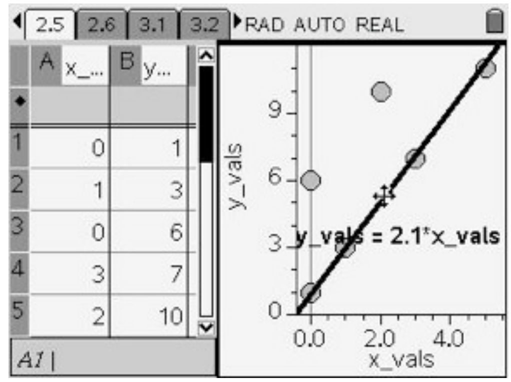


These basic shapes may be readily extended to model negative values (color some of the shapes differently and then these "cancel" out their counterparts) and even to quadratics, using x^2 shapes! After even a brief exposure, students will never again confuse *2x* with x^2 since they are clearly different shapes.

Move carefully into graphs

The introduction of the graphical representation is too often rushed and much is assumed on the part of the students. Like the rest of algebra, the origins of graphs should lie firmly in number.





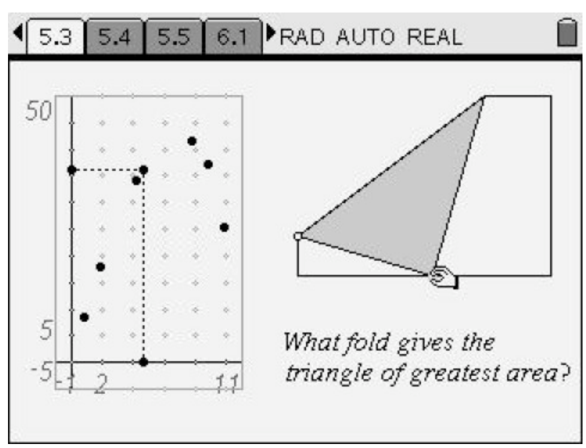
The use of scatter plots of number patterns and numerical data should precede the more usual continuous line graphs, which we use to represent functions. Such conceptual "objects" have little meaning for students, in the same way that symbolic "objects" (such as "2x + 1") need to be conceptually expanded to include more diverse ways of thinking.

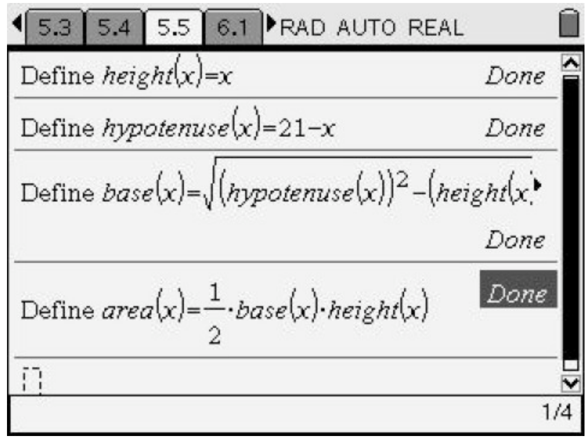
We now have tools which make it easy for students to manipulate scatter plots and so further build understanding of the relationship between table of values and graphical representation. Only then should we encourage the use of the more formal "straight line" representation.

Bring it all together with modeling

Once we have built firm numerical foundations for symbol and graph, our students are ready to begin to *use* algebra – perhaps a novel idea in current classrooms! The real power of algebra lies in its use as a tool for modeling the real world (and, in fact, all possible worlds!) research is clear that students in the middle years of schooling (which is when we introduce algebra) most strongly need their mathematics to be relevant and significant to their lives. Teaching algebra from a modeling perspective most clearly exemplifies that approach, and serves to bring together the symbols, numbers and graphs that they have begun to use.

Opportunities for algebraic modeling abound, especially around such topics as Pythagoras' Theorem. The simple paper folding activity shown - in which the top left corner of a sheet of A4 paper is folded down to meet the opposite side, forming a triangle in the bottom left corner – is a great example of a task which begins with measurement, involves some data collection and leads to the building of an algebraic model. Students measure the base and height of their triangles, use these to calculate the area of the triangle, and then put their data into lists, which can then be plotted.

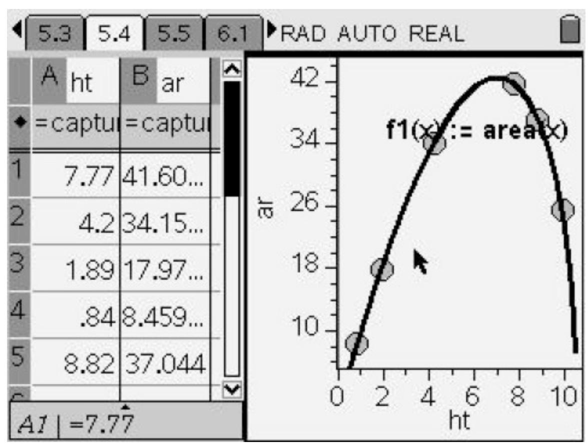




They may then begin to build their algebraic model, but using appropriate technology, may use real language to scaffold this process and develop a meaningful algebraic structure, as shown. In this problem, if we call the height of the triangle "x" (define a function as shown!) then the hypotenuse will be 21 – x, since the width of the sheet of paper is exactly 21 cm. Then apply a little Pythagoras to obtain the function for the base (also dependent on the height – see how the key understandings of variable and function are developed?), and the area follows.

Returning to the graphical representation, students may now plot the graph of their function, *area(x)*, and see how it goes through each of their measured data points – convincing proof that their model is correct – and usually a dramatic classroom moment!

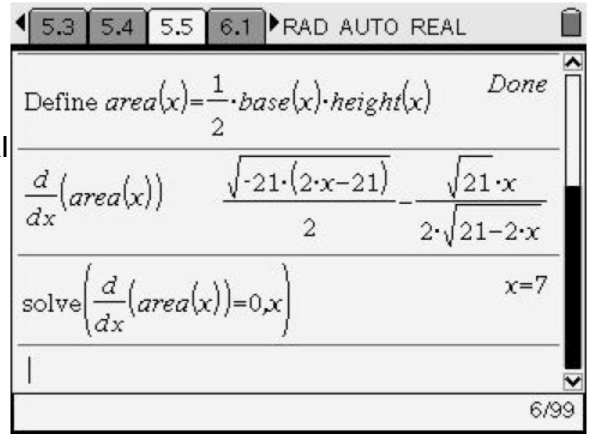
Build algebraic structure using real language



This is powerful, meaningful use of algebraic symbolism. The building of purposeful algebraic structures using real language supports students in making sense of what they are doing, and validates the algebraic expressions which they can then go on to produce. Able students should still be expected to compute the algebraic forms required and perhaps validate them using a variety of means.

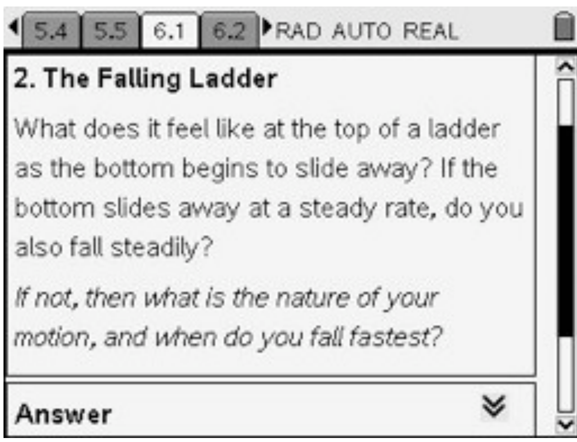
This use of real language for the definition of functions and variables has previously only existed on CAS (computer algebra software) and even there only rarely used. The new TI-Nspire is a numeric platform (non-CAS) and so allowable in all exams supporting graphic calculators, but it supports this use of real language.

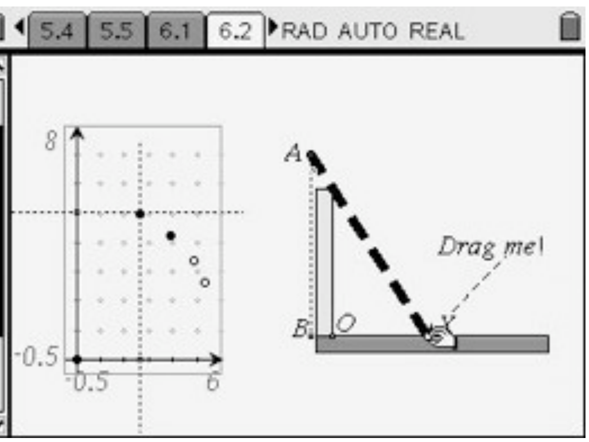
Of course, it is wonderful to have CAS facilities when they are needed. Using CAS we can actually display the function in its symbolic form, and then compute derivative and exact solution, arriving at the theoretical solution to this problem. The best fold occurs when the height of the fold is 7 cm, exactly one third of the width of the page.

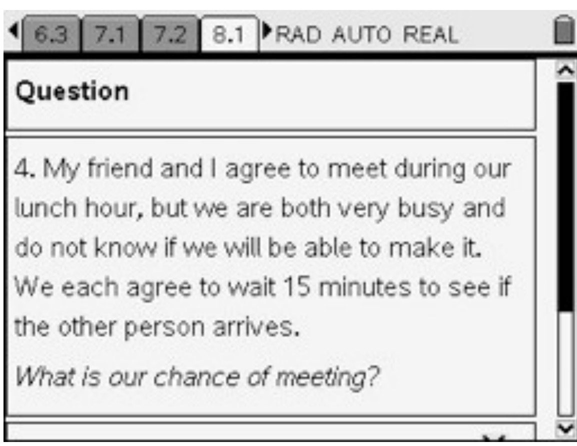


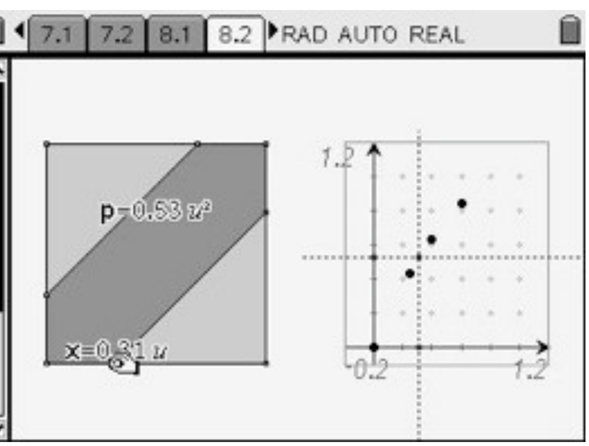
Using non-CAS tools, this same result may be found using the numerical function maximum command, or by using numeric derivative and numeric solve commands.

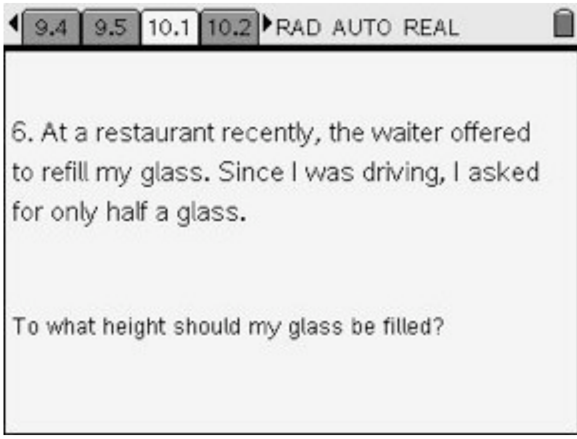
Once we begin looking for such problems, we find that they abound!

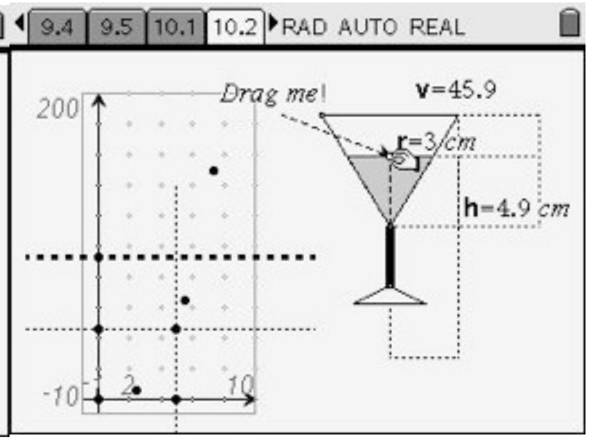






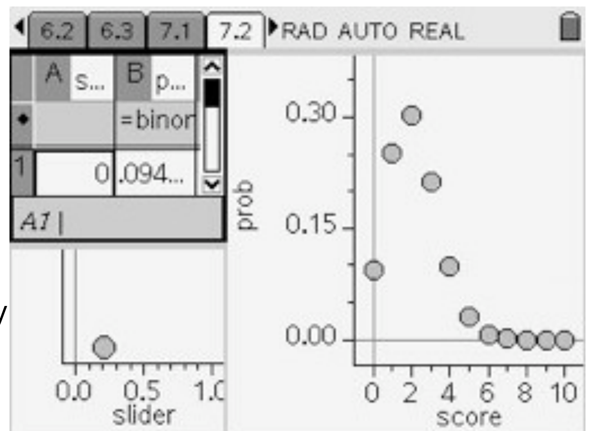


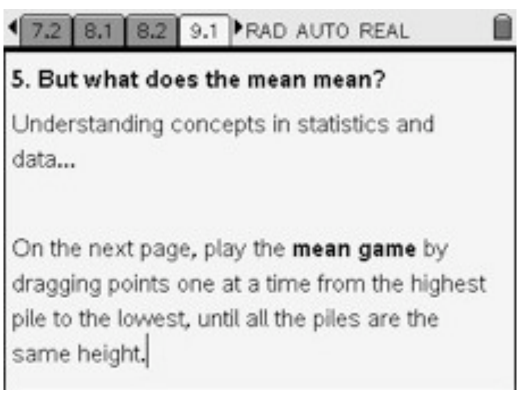


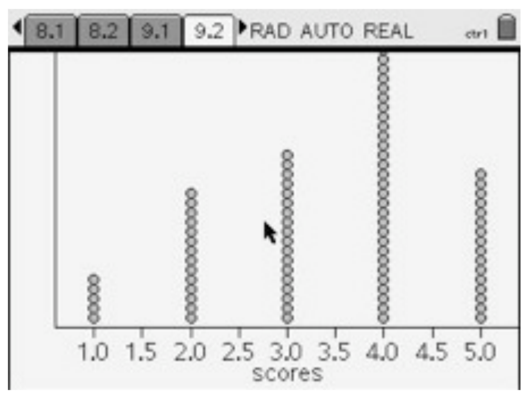


Statistics provides a ready source of good material, and often overlaps with our study of algebra. Consider binomial distribution: if my chance of scoring a bulls-eye at darts is not good (say, 20%) then the distribution of probabilities of scoring between 0 and 10 bulls-eyes will look as shown.

Using appropriate technology, we may vary that chance of a bulls-eye and students may investigate the effect this has upon the possible distributions – in this case, using a slider!







Conclusion

Why do I like to use technology in my Mathematics teaching?

It helps my students to be better learners:

- It scaffolds their learning, allowing them to see more and to reach further than would be possible unassisted
- Good technology extends and enhances their mathematical abilities, potentially offering a more level playing field for all

- ❏ It is inherently motivating, giving them more control over both their mathematics and the ways that they may learn it
- ❏ Good technology encourages them to ask more questions about their mathematics, and offers insight into the true nature and potential of mathematical thinking and knowledge

Good technology also helps me to be a better teacher:

- ❏ It offers better ways of teaching, new roads to greater understanding than was previously possible
- ❏ It encourages me to talk less and to listen more: Students and teacher tend to become co-learners
- ❏ It makes my students' thinking public, helping me to better understand their strengths and weaknesses, and to better evaluate the quality of my own teaching and of their learning
- ❏ It frequently renews my own wonder of Mathematics, helping me to think less like a mathematics teacher and more like a mathematician

Why do I love using technology in my mathematics classroom?

Because, like life, mathematics was never meant to be a spectator sport.